OPTIMAL SOLUTION OF LINEAR PROGRAMMING PROBLEMS WITH FUZZY VARIABLES

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Abstract: This paper presents to find the Optimal Solution of Fuzzy Linear Programming problems with inequality constraints by representing all the parameters as fuzzy numbers. By using the proposed method the fuzzy Optimal Solution of Linear Programming problems, occurring in real life situation, can be easily obtained.

Key words - **Fuzzy Linear Programming problems, Fuzzy Optimal Solution, Fuzzy numbers, membership function, ranking function.**

I. INTRODUCTION

Through knowledge, Science and Specialization, man has recognized many of the relationships of his environmental systems. Due to the nature of real world problems, the collected data usually involve some kind of uncertainty. As a matter of fact, many pieces of information cannot be quantified due to their nature. Incomplete information is also another cause for resorting to fuzziness.

This new knowledge has provided man with the opportunity to manipulate environmental conditions to produce desired consequences. Man attempts to select the courses of action, from a set of alternative courses of action, that will achieve his objectives. Decision analysis is a formalized process for increasing man's understanding and control over environmental conditions. In real world any linear programming model involves Parameters whose values are assigned by experts. They usually cannot assign exact values to these important parameters. The decision maker has to deal with uncertainty. In contrast with crisp logic, where binary sets have two valued logic, fuzzy logic variables many have a truth value that ranges in degree between '0' and '1'. Fuzzy logic concept is used widely in many implementations like automatic gear control systems, air conditioners, TV sets, mobile robots and so on.

Fuzzy Linear Programming problem was introduced by Zimmermann. The concept of Fuzzy logic was first conceived by Loft ZADEH. NASSERI et al [1] proposed for solving Fuzzy Linear Programming by using Tsao's method. EBRAHIMMEJAD et al [2] have developed using complementary slackness property to solve L.P.P with Fuzzy parameters. MALEKI et al [3] proposed to solve Linear Programming with Fuzzy variables. BEAULA et al [4] have developed solving Fully Fuzzy Linear Programming using Breaking Points Method. BELLMAN et al [10] proposed the concept of decision making in fuzzy environment. AMITKUMAR et al [11] proposed a new method for solving fuzzy Linear Programming. Li XIAOZHONG et al have developed fuzzy linear programming problems with fuzzy variables.

II. FUZZY LINEAR PROGRAMMING PROBLEM

Definition

A ranking function is a function f: F (R)--→ R, where F (R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real line.

Let A =(a, b, c) be a triangular fuzzy number then

$$f(A) = \frac{a+2b+c}{4}$$

The Objective Function of each LP problem is expressed in terms of decision variables to optimize the criterion of Optimality, it is represented as:

Maximize (or) Minimize = $C^T.X$

Subject to A.X ≤, =,≥b, X is a non-negative fuzzy number,

where $A= (a_{ij})_{mxn}$, $C^T= (c_j)_{lxn}$, $b= (bi)_{mxl}$, $x= (x_j)_{nxl}$ and a_{ij}, c_j, b_i,xj € F(R) is said to be multi objective FLP problem with fuzzy variable and fuzzy constraints.

III. CONVERSION OF INEQUALITY CONSTRAINTS INTO EQUALITY CONSTRAINTS

Check type of all constraints

$$\sum_{j=1}^{n} \text{aij.Xj} = \text{bi} \quad or \quad \sum_{j=1}^{n} \text{aij.X}j \neq bi$$

for all i=1,2 ……,m

Case I : If

$$\sum_{j=1}^{n} \text{aij}.\text{Xj} \leq \text{bi}$$

for some i, then convert inequality constraints into equality constraints by introducing non-negative variables S_i to the left side of the constraints i.e.

$$\sum_{j=1}^{n} \text{aij}.\text{Xj} + \text{Si} = \text{bi}$$

for some i, where S_i is a non- negative fuzzy number.

Case II : If

$$\sum_{j=1}^{n} \text{aij}.\text{Xj} \geq \text{bi}$$

for some i, then convert inequality constraints into equality constraints by introducing non-negative variables S_i to the right side of the constraints i.e.

$$\sum_{j=1}^{n} \text{aij}.\text{Xj} = \text{bi} + \text{Si},$$

for some i, where S_i is a non- negative fuzzy number.

IV. METHOD TO FIND THE FUZZY OPTIMAL SOLUTION OF FLP PROBLEMS

Maximize (or Minimize) $C^T.X$ subject to $A.X \leq, =, \geq b$,
where $A= (a_{ij})_{mxn}$, $C^T= (c_j)_{lxn}$, $b= (b_i)_{mxl}$, $x= (x_j)_{nxl}$

Step 1 Convert all the inequalities of the constraints into equations.
Step 2 Now the FLP may be written as:

Maximize (or Minimize)

$$\sum_{j=1}^{n} \text{Cj}.\text{Xj}$$

Subject to

$$\sum_{j=1}^{n} \text{aij}.\text{Xj} = \text{bi}$$

for all i=1,2….m, X_j is a non-negative triangular fuzzy number.

Step 3 If all the parameters are represented by triangular fuzzy number then FLP can be written as:

Maximize (or Minimize)

$$\sum_{j=1}^{n} .(\text{pj}, \text{qj}, \text{rj}).(\text{xj}, \text{yj}, \text{zj})$$

Subject to $(a_{ij},b_{ij},c_{ij}) . (x_j,y_j,z_j) = (b_i,g_i,h_i)$ for all i=1,2….m

(x_j,y_j,z_j) is a non-negative triangular fuzzy number

Step 4 Assuming $(a_{ij},b_{ij},c_{ij}) . (x_j,y_j,z_j) = (m_{ij},n_{ij},o_{ij})$ then the FLP obtained in step 3 may be written as:

Maximize (or Minimize)

$$f(\sum_{j=1}^{n} .(\text{pj}, \text{qj}, \text{rj}).(\text{xj}, \text{yj}, \text{zj}))$$

Subject to

$$\sum_{j=1}^{n} : (\text{mij}, \text{nij}, \text{oij}) = (\text{bi}, \text{gi}, \text{hi})$$

for all i=1,2, …..m

(x_j,y_j,z_j) is a non-negative triangular fuzzy number

Step 5 Using arithmetic operations, FLP obtained in step 4 is converted into following LP problem:

Maximize (or Minimize)

$$f\left(\sum_{j=1}^{n} .(pj, qj, rj).(xj, yj, zj)\right)$$

Subject to

$$\sum_{j=1}^{n} .\, mij = bi,$$

$$\sum_{j=1}^{n} .\, nij = gi,$$

$$\sum_{j=1}^{n} .\, oij = hi,$$

$y_j - x_j \geq 0, z_j - y_j \geq 0$, for all i=1,2,…..m

Step 6 Find the optimal solution x_j,y_j,z_j by solving the LP problem obtained in step 5 .

Step 7 Find the fuzzy optimal solution by putting the values of x_j,y_j,z_j in $X_j=(x_j,y_j,z_j)$.

Step 8 Find the fuzzy optimal value by putting X_i in X_j.

V. PROPOSED METHOD IS ILLUSTRATED WITH THE HELP OF NUMERICAL EXAMPLES

Example 1: Let us consider the following FLP, problem and solve by the proposed method.

Maximize = $(2,2,3).x_1+(3,3,4).x_2$

Subjected to

$(2,3,0)x_1 + (3,2,1).x_2$ d"(3,6,9)

$(1,3,2).x_1+(0,2,3).x_2$ d"(3,4,26)

x_1, x_2 are non-negative triangular fuzzy numbers.

Solution: Converting all the inequalities of the constraints into equation by adding non-negative fuzzy number FLP problem may be written as:

Maximize = $(2,2,3).x_1+(3,3,4).x_2$

Subjected to

$(2,3,0).x_1 + (3,2,1).x_2 + (1,1,1).s_1 =(3,6,9)$

$(1,3,2).x_1+(0,2,3).x_2 + (1,1,1).s_2 = (3,4,26)$

x_1, x_2, s_1, s_2 are non-negative triangular fuzzy numbers.

Let $x_1=(x_1,y_1,z_1)$, $x_2=(x_2,y_2,z_2)$, $s_1=(s_1,t_1,u_1)$, $s_2=(s_2,t_2,u_2)$ then given FLPP may be written as:

Maximize = $((2,2,3).x_1+(3,3,4).x_2)$

Subjected to $(2,3,0)\,(x_1,y_1,z_1) + (3,2,1)\,(x_2,y_2,z_2)+ (1,1,1)\,(s_1,t_1,u_1) =(3,6,9)$

$(1,3,2)\,(x_1,y_1,z_1) +(0,2,3)\,(x_2,y_2,z_2) + (1,1,1)\,(s_2,t_2,u_2) = (3,4,26)$

Where (x_1,y_1,z_1), (x_2,y_2,z_2), (s_1,t_1,u_1) and (s_2,t_2,u_2) are non-negative triangular fuzzy numbers

Maximize = f $(2x_1+3x_2, 2y_1+3y_2, 3z_1+4z_2)$

Subjected to

$(2x_1+3x_2,+s_1, 3y_1+2y_2+t_1, 0.z_1+2z_1+u_1) = (3,6,9)$

$(x_1+0.x_2,+s_2, 3y_1+2y_2+t_2, 2z_1+3z_2+u_2) = (3,4,26)$

Now by using step 5 the above FLP problem is converted into the following problem.

Maximize = $(1/4\,(2x_1+3x_2+4y_1+6y_2+3z_1+4z_2))$

Subjected to

$2x_1+3x_2+s_1 = 3$

$3y_1+2y_2+t_1 = 6$

$0.z_1+2z_2+u_1 =9$

$x_1+0.x_2,+s_2 =3$

$3.y_1+2.y_2+t_2 \ =4$

$2z_1+3.z_2+u_2 \ =2$

The optimal solution of the above L.P. problem is

$x_1=2$

$x_2=0$

$y_1=2.5$

$y_2=0$

$z_1=6$

$z_2=5$

Using step 7, the fuzzy optimal solution is given by

$x_1=(2,2.5,6)$

$x_2=(0,0,5)$

Hence, using step 8, the fuzzy optimal value of the given FLP problem is=(2,2.5,32)

Example2: Maximize = $(2,4,4).x_1 + (4,6,6).x_2$

Subjected to

$(0,2,4).x_1 + (2,4,0).x_2 \leq (2,16,24)$

$(2,4,6).x_2 + (0,4,8).x_2 \leq (4,24,60)$

x_1, x_2 are non-negative triangular fuzzy numbers.

Solution: Converting all the inequalities of the constraints into equation by adding non-negative fuzzy number FLP problem may be written as:

Maximize = $(2,4,4).x_1 + (4,6,6).x_2$

Subjected to

$(0,2,4).x_1 + (2,4,0).x_2 + (1,1,1)\ s_1 = (2,16,24)$

$(2,4,6).x_1 + (0,4,8).x_2 + (1,1,1)\ s_2 = (4,24,60)$

Let x_1, x_2, s_1, s_2 are non-negative triangular fuzzy numbers.

Let $x_1=(x_1,y_1,z_1)$, $x_2=(x_2,y_2,z_2)$, $s_1=(s_1,t_1,u_1)$ and $s_2=(s_2,t_2,u_2)$ then given FLP problem may be written as :

Maximize = $(2,4,4).x_1 + (4,6,6).x_2$

Subjected to

$(0,2,4).\ (x_1,y_1,z_1)+ (2,4,0).\ (x_2,y_2,z_2)+ (1,1,1)\ (s_1,t_1,u_1) = (2,16,24)$

$(2,4,6).\ (x_1,y_1,z_1)+ (0,4,8).\ (x_2,y_2,z_2)+ (1,1,1)\ (s_2,t_2,u_2) = (4,24,60)$

Maximize = $f\ (2\ x_1+4\ x_2,\ 4y_1+6y_2,\ 4z_1+6z_2)$

Subjected to

$(0.\ x_1+2\ x_2,+s_1,\ 2y_1+4y_2\ +t_1,\ 4z_1+0.z_1+u_1) = (2,\ 16,24)$

$(2\ x_1+0.\ x_2,+s_2,\ 4y_1+4y_2+t_2,\ \ 6z_1+8z_2+u_2) = (4,\ 24,60)$

(x_1,y_1,z_1),(x_2,y_2,z_2),(s_1,t_1,u_1) and (s_2,t_2,u_2) are non-negative triangular fuzzy numbers.

Now by using step 5 the above FLP problem is converted into the following problem.

Maximize = $1/4\ (2\ x_1+4\ x_2+ 8y_1+12y_2+4z_1+6z_2)$

Subjected to

$0.x_1+2\ x_2\ +s_1 = 2$

$2.y_2+4y_2\ +t_1\ \ = 16$

$4z_1+0.z_1+u_1\ \ =24$

$2.x_1+0.\ x_2,+s_2\ \ =4$

$4.y_1+4y_2+t_2\ \ =24$

$6z_1+8z_2+u_2\ \ =60$

The optimal solution of the above L.P. problem is

$x_1=2$

$x_2=1$

$y_1=0$

$y_2=4$

$z_1=0$

$z_2=7.5$

Using step 7, the fuzzy optimal solution is given by

x_1=(2,0,0)

x_2=(1,4,7.5)

Hence, using step 8, the fuzzy optimal value of the given FLP problem is (4,12,22.5)

VI. CONCLUSION

In this method we proposed a method to find the Fuzzy Optimal Solution of Fuzzy Linear Programming Problem with inequality constrains by representing all the parameters as triangular fuzzy members. Two examples were solved and results are discussed to illustrate our proposed method.

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