



# POLAR STRUCTURES EXTENDING CATMULL-CLARK SUBDIVISION AND PCCM

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## Abstract

Surface modelling and design have embraced subdivision surfaces as a compelling representation. They solve some of the major drawbacks of classic spline-based approaches, such as the inability to accommodate arbitrary topologies and the lack of flexibility. To allow multiscale editing procedures to be supported in this article. Methods of subdivision-based modelling with a focus on interactive tools for 3D model style and embellishment. We complete and unite two sets of surface developments that utilize polynomial bits of degree (3,3) to partner a smooth surface with a lattice. The two sets complete one another in that one broadens the subdivision modelling worldview, the other the NURBS fix approach to freestyle displaying. Both Catmull-Clark and polar development generalize the bi-cubic spline region. Together, they structure a powerful blend for a smooth item plan: while the Catmull-Clark region is more appropriate where not many aspects join, polar development pleasantly models areas where numerous aspects join, as while covering expelled highlights. We tell the best way to effectively join the cross-sections of these two speculations of bi-cubic spline development. A related however unique speculation of bi-cubic splines is to demonstrate non-tensor-item designs by a limited set of flawlessly associated bi-cubic patches. PCCM does as such for designs where Catmull-Clark would apply. We show that a solitary NURBS fix can be utilized where polar development would be applied. This spline is independently parametrized, yet, utilizing a clever strategy, that's what we show the surface is C1 and has limited shapes. Modelling non-tensor-product setups using a finite collection of smoothly linked bi-cubic patches is a related but distinct generalization of bi-cubic splines. PCCM does so in cases when Catmull-Clark would be appropriate. We demonstrate that a single NURBS patch may be utilized in place of a polar subdivision. This spline is unique parametrized, but we demonstrate using a unique method that the surface has restricted curvatures and is C1.

**Keywords:** Catmull Clark region, subdivision-based modelling, bi-cubic spline region, NURBS.

## 1. INTRODUCTION

Region surfaces offer a few benefits over both unpredictable cross-sections and spline patches, two of the most generally utilized surface portrayals today. Subdivision offers a smaller method for addressing calculation with negligible network data. It sums up the old-style spline fix way to deal with inconsistent topology, it normally obliges numerous degrees of detail, and creates networks with all around molded components organized in practically customary designs, appropriate for advanced processing. When joined with a multiresolution examination, the region offers a strong displaying apparatus, considering complex altering activities to be applied proficiently at various goals. As of late, the arrangement of apparatuses accessible for controlling region surfaces has been developing consistently. Calculations for direct assessment [Sta98, ZK02], editing [BKZ01, BMBZ02, BMZB02, BLZ00], finishing [PB00], and transformation to other well-known portrayals [Pet00] have been contrived and equipment support for delivering of region surfaces has been proposed [BAD+01, BKS00, PS96].

This review centers around the utilization of region-based portrayals for styling, and what's more, a theoretical plan. We investigate different strategies for controlling development surfaces and, whenever the situation allows, we delineate the development of such techniques from related portrayals. We give specific consideration to intelligent instruments which are appropriate for a plan as they permit the originator to assess results quickly. While we are attempting to give an exhaustive outline of the area and incorporate the most pertinent strategies, we understand that the volume of distributed work works out positively past that canvassed in this study which is in no way, shape, or form comprehensive (see additionally [DL02, Sab02] for extra overviews). A considerable lot of the points introduced to connect with issues we have tended to in our own work which we trust will give a few bits of knowledge to those pursuing comparative interests.

## 2. BACKGROUND

The fundamental thought of utilizing development to deliver smooth bends and later, smooth surfaces, has been around for a long time (see [ZSD+00] for a short invasion into the historical backdrop of the region). In any case, it is as of late that strong plan apparatuses in view of this portrayal have arisen. This is somewhat because of the new approach of multiresolution strategies that work with catching of non-inconsequential shapes and halfway due to considerably later advances in region hypothesis and strategies for direct furthermore, proficient assessment of development surfaces. With the end goal of this overview, we give a short audit of the essential ideas relating to development surfaces. For extra subtleties, we allude to the peruseto [ZSD+00, WW01]. Region characterizes a smooth surface recursively as the constraint of a grouping of networks (see Figure 1). Each better lattice is acquired from a coarse cross-section by utilizing a bunch of refinement rules which characterize a region conspire.



Fig 1: Subdivision recursively defines a smooth surface as the end of a succession of meshes.

Many plans have been proposed in the writing. Models incorporate Doo-Sabin [DS78], Catmull-Clark [CC78], Loop [Loo87], Butterfly [DLG90, ZSS96], Kobbelt [Kob96a], Midedge [PR97]. Various plans lead to restricting surfaces with various perfect qualities. For configuration purposes, the Catmull-Clark [CC78], and Loop [Loo87] plans are most frequently utilized as they are firmly connected with splines (an accepted standard in demonstrating today) and create C2-persistent surfaces over erratic. by applying

a smoothing filter to points on a level I on a coarse level I 1. The discrepancies between the two levels are used to compute multiresolution details on level I. Synthesis, on the other hand, reconstructs the data on the level I by subdividing the level I 1 control mesh and adding the details [ZSS97]. Subdivision surfaces have the advantage of being easily read as functions on the domain specified by the base mesh. This parametric interpretation is useful in a variety of design situations, ranging from the derivation of differential variables to dealing with restrictions along arbitrary curves.

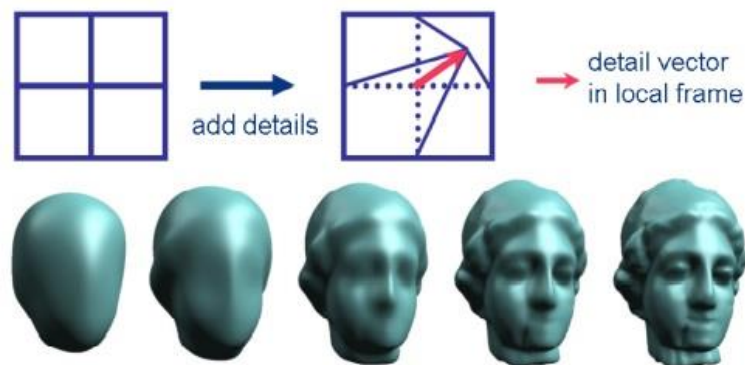


Fig 2: multiresolution subdivision introduces detail vectors at each level, extending the idea of subdivision. Bottom: depending on the quantity of detail introduced and the level at which it is introduced, surfaces created by subdivision of the same coarse mesh seem considerably different. From left to right, no details are added, then finer details are added.

**Table 1. Bi-cubic surface constructions**

patches	quadrilateral	polar
subdivision	Catmull-Clark [3]	bi-cubic polar [7]
finite	PCCM [12]	new (Section 4)

Surfaces created from finitely many NURBS patches are preferred in CAD programmes and suitable for GPU implementations, whereas mesh-based subdivision representation provides an intuitive display for interactive modelling. In this example, four bi-cubic choices (see Table 1) exist in tandem and complement one another.

### 3. SURFACE MODELLING TOOLS

#### 3.1. Freestyle Editing.

Freestyle control of 3D models is a well-known strategy for changing existing shapes which endeavours to copy somewhat the method involved with displaying or chiselling an actual article the hard way. The applications are various, from vivified character creation to virtual reclamations, to modern plans. The chiselling similitude for mathematical displaying has its foundations in the parametric surface works of Sabin and Bezier which contain early notices of surface disfigurements. The resulting work has crossed over thirty years furthermore, keeps on being explored with regard to current frameworks and surface portrayals (e.g., [Bar84, SP86, Coq90, HKD93, CR94, MJ96, SF98, Kob96b, ZSS97, PL97, QMV98, Tak98, WW98, MQ00, TO02, GS01, BMRB04]). The fundamental thought of freestyle demonstrating is to present a level of straightforwardness between the planner and the numerical model of the surface being formed. Rather than controlling the shape through a bunch of non-natural surface parameters, freestyle mishappenings permit the shape to be controlled through control of the actual surface or the space encompassing it. The primary challenge is to play out the control through a restricted arrangement of controls and to characterize normal mishappenings of the surface away from the control positions. Different varieties of this worldview have been created, including pivotal deformations [Bar84, CST94, LCJ94] which adjust the hub of a shape to initiate its deformation and grid distortions [SP86,

Coq90, MJ96] which work on the cells of a space cross-section to disfigure the volume inside the grid, controls on scalar field embeddings [HQ03], control network altering strategies which shape parametrically defined surfaces by forcing requirements on their control networks [ZSS97], and variational techniques which work by improving energy utilization over the surface under requirements [Tak98, BMRB04]. We concentrate on strategies that exploit region representations and among these, we stress those that help intuitive multiscale displaying. Development portrayals are especially reasonable for freestyle editing because of their progressive nature which effectively obliges multiscale alters, as well as their productivity concerning capacity and access. For a study of deformable models in light of different portrayals see [GM97].

### 3.2. Control network controls.

Controlling control networks offers a straightforward interface that upholds intelligent shape distortions. This approach has been broadly utilized in spline-based demonstrating [CRE01] and can be normally stretched out to region surfaces. Assortments of control network vertices, edges, and faces are re-situated to actuate alterations as far as the possible surface. Also, control focuses can be added and edges and faces can be parted to increase the intricacy of the shape as altering advances. This sort of manipulation is widely used and is used as the foundation for commercial modelling products that support subdivision surfaces. It's widely used for animated character design (e.g., in Discreet 3D Studio Max [dsm], in Alias' Maya [may]) and is gaining traction in industrial modelling (e.g., in Dassault Systèmes' Catia [cat]). Figure 4 shows how to control point manipulation may be used to model shapes.

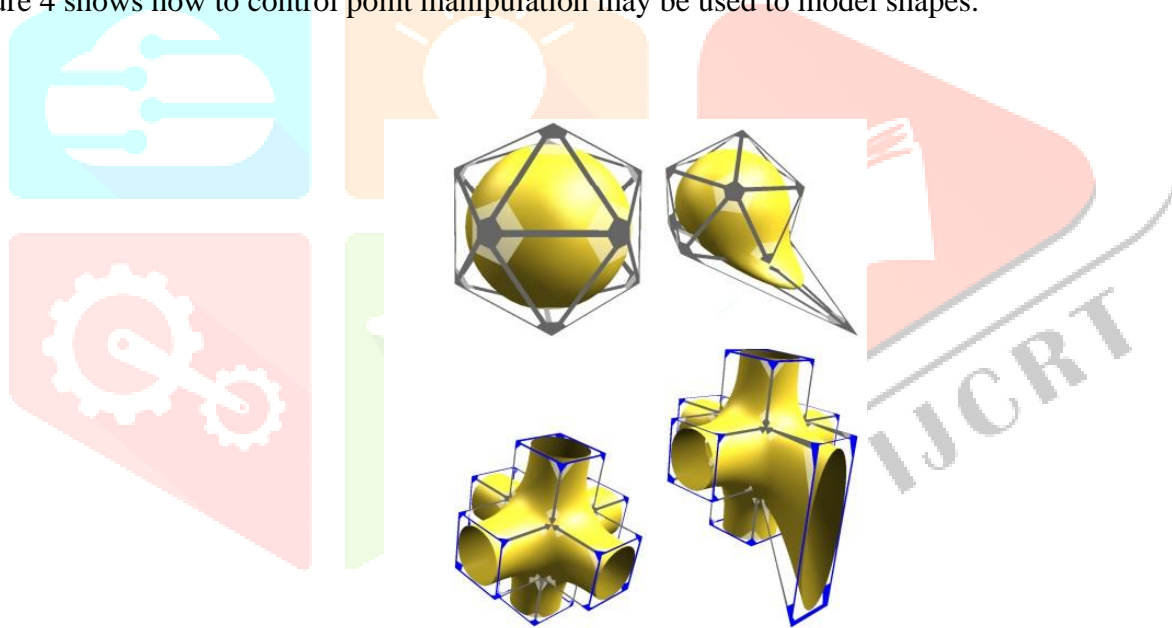


Fig 3: Control point manipulation for shape modelling

Single goal control network controls offer just restricted adaptability in planning shapes: just coarse shape misshapenings can be obliged. Multiresolution development surfaces are a significantly more impressive portrayal that loans itself normally to multiscale altering. Contingent upon the level at which the editing happens, either a worldwide twisting (coarse level) or a neighborhood deformity (fine level) is actuated. This thought was taken advantage of, for example, in [ZSS97, PL97] for the interactive multiresolution altering of Loop surfaces and in [DKT98] for Catmull-Clark ones. Utilizing a mix of regions (i.e., changing a coarse cross-section into a better one) and smoothing (i.e., changing a fine cross-section into a coarser one), alters performed at various degrees of development can be spread through the hierarchy while monitoring the extent of multiresolution subtleties. Figure 5 shows alterations at different scales performed on the Armadillo model. Varieties of this approach incorporate displaying with uprooted development surfaces [LMH00] and region surface fitting [STKK99, LLS01a, MZ00]. The dislodged portrayal can be seen as a limited type of multiresolution subdivision comprising of a control network and a solitary degree of scalar subtleties. An area surface is produced from the control network

utilizing the Loop region [Loo87]. A removal map figured from the scalar relocation is then applied over the area to create the last surface. The relocations can be altered to make fine-level highlights on a superficial level, while control network alters lead to worldwide shape alterations. In surface fitting, a surface is twisted to adjust to the state of another given informational index (e.g., focuses, bends, another surface). This approach is to some degree unique in relation to those talked about such a long way in that it is less reasonable for intelligent manipulation. Ordinarily, some improvement of the surface being fitted is acted in request to decide ideal control point positions which lead to the best fit between the surface and the objective. The precision of the fit is controlled through an edge boundary that limits the mistake between the objective and the fitted surface.

### 3.4. Variational plan

The variational surface plan works on the guideline of altering a shape so that its decency is enhanced. Surface reasonableness is commonly estimated as far as its energy and the thought is to observe a base energy state which, thusly, compares to the most attractive conceivable shape. In Computer Graphics, energy-limiting surfaces became famous with regards to mimicking the physical properties of materials [Bar84, TF88, WW92]. Celniker and Gossard [CG99] furthermore, later Welch and Witkin [WW92] brought up the connection between fair surface plan and energy minimization. Most usually, reasonableness is communicated as an essential of an actual boundaryrelated to a genuine item bearing the state of the surface [Hal96]. A broadlyutilized proportion of reasonableness is the mix of extending and bowing energies

## 4. COMPATIBLE POLAR MESH REFINEMENT

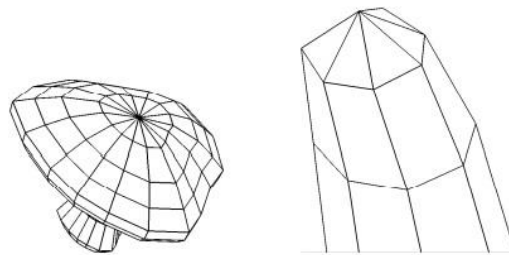


Fig4 : Polar vertices and structures

When the Catmull-Clark mesh is enhanced with polar structures, compatible polar mesh refinement is developed from [7] to provide a consistently  $k$ -times subdivided mesh. Consider the meshes in Figures 3 left and 7 with the sphere's latitude-longitude connectedness. Two polar vertices result from this. The refinement process in [7] uses cubic spline refinement only in the longitudinal (radial) direction, not in the latitudinal (circular) direction. For Figure 4, left, the stencil weights for the polar vertex and associated 1-link are as follows:

$$\alpha := \beta - \frac{1}{4}, \quad \beta := \frac{5}{8}, \quad c_n^k := \cos\left(\frac{2\pi k}{n}\right),$$

$$\gamma_k := \frac{1}{n} \left( \beta - \frac{1}{2} + \frac{5}{8}c_n^k + (c_n^k)^2 + \frac{1}{2}(c_n^k)^3 \right) \quad (1)$$

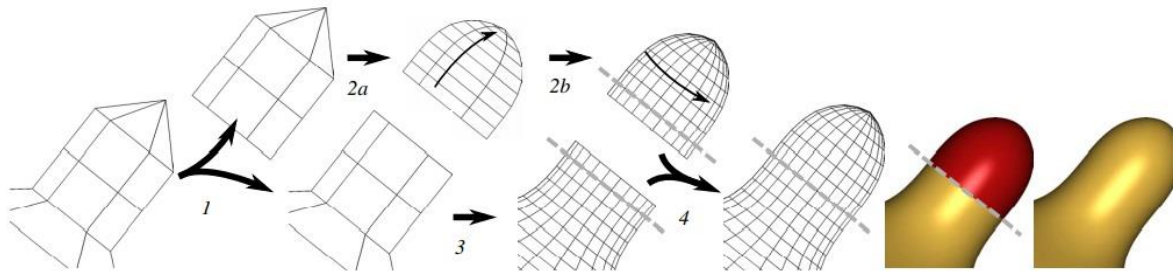


Fig 5: Steps for generalized bi-cubic subdivision (1) The input mesh is separated. (2) Radially (2a) then circularly subdividing the polar structure (2b). (3) The remaining is divided. (4) After removing overlapping facets, join the improved meshes. The limit surface (right).

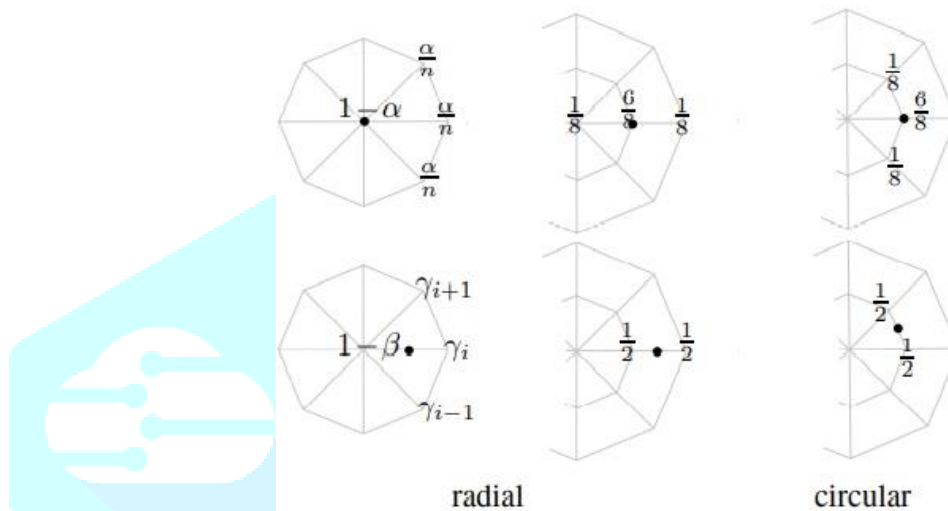


Fig 6: Polar subdivision refinement stencils ([7]). Radial subdivision at the polar vertex (left), radial subdivision everywhere else (center), and circular subdivision everywhere else (right).

a wherever C2 surface besides at as far as the possible point. The main issue with the surface is C1 with limited curvature. Additionally, the wave and seat curios of Catmull-Clark development don't show up. As outlined in Figure 6, absolutely outspread refinement results in a confound or a lattice with T-corners at the progress to the Catmull-Clark region since Catmull-Clark development at the same time partitions radially and circularly. To use and safeguard the great consequences of radial development regardless showcase a reliable control net after k advances, we continue as represented in Figure:6 we don't substitute spiral and roundabout regions in the k advances yet utilize viable polar cross-section refinement.

(a) Apply k strides of the spiral region and save the level

k polar design on the off chance that we proceed with development later.

(b) Apply k round development steps.

Since step (a) jelly the valence and thus the analysis of reference [7], we base any proceeded with refinement on the saved polar design. Rotating outspread and round region makes neighborhood arch changes By contrast, applying step (b) just deduced is essentially a tie addition that doesn't change the surface. In this way, the basic plan illustrated above is best.

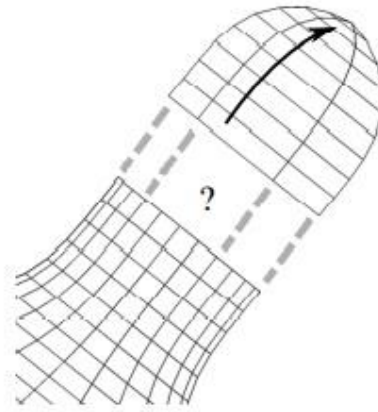


Fig 7: Mesh mismatch between radial subdivision (a) and Catmull-Clark subdivision (b)

#### 4. 1. Limited bi-cubic developments

A connected, different however correlative speculation of the bi-cubic setting is to display regions digressing from the tensor-item setting by a couple of bi-cubic NURBS patches. Since PCCM [12] gives a development for Catmull-Clark designs, we center here around developing a solitary bi-cubic spline for a polar design. Similarly, as PCCM yields a limited bi-cubic surface that is no less than C1 all over, the single bi-cubic NURBS surface will be C1. In spite of its particular parametrization at the focal polar point, it tends to be shown to have a limited arch. In the contribution of a polar design, the limited bi-cubic NURBS development has the accompanying steps (cf. Figure 8).

(I) (suggested for better shape) Subdivide the polar structure. Partition radially, two times for prolonged models like tips of fingers. The subsequent cross-section  $p$  is named as in Figure 8, center.

(ii) Convert the polar design to a spline network. Introduce  $c_{ij} := p^{(i-1)n+j+1}$  for  $I > 1$  and  $j = 0, \dots, n-1$ .

Both  $u$  and  $v$  bunch arrangements are uniform. The roundabout course with boundary  $u$  is occasional.

(iii) Interpolate as far as a possible place of the bi-cubic polar region. For  $I = 0, \dots, n-1$ , set

$$c_{0i} := \eta p_0 + (1 - \eta) \frac{1}{n} \sum_{j=1}^n p_j, \quad \eta := \frac{4(1 - \beta)}{3}, \quad (2)$$

the breaking point equation inferred in [7], and change the beginning of the spiral bunch arrangement to a 4-crease hitch related with  $c_{0i}$ .

(iv) Match the breaking point ordinary of bi-cubic polar region We project the neighbors of the main issue into a normal plane. For  $I = 0, \dots, n-1$

$$c_{1i} := c_{0i} + 2\sigma \sum_{j=0}^{n-1} \Gamma_{j-i} p_{j+1}, \quad \sigma_{\text{default}} := \frac{3}{4}, \quad (3)$$

$$\Gamma_k := \frac{1}{n} \cos\left(\frac{2\pi k}{n}\right).$$

The projection of the spline coefficients doesn't modify the intrinsic C2 congruity separated from the peculiarity as far as possible point; and the projection maps all spiral digressions into a similar plane with a typical direction  $(c_{11} - c_{00}) \times (c_{12} - c_{00})$  at the phenomenon limit point.

(v) (discretionary) Additional bunch inclusion. It is normal

to have cubic NURBS patches with four-overlap endpoints. The addition at the external limit yields for example  $0, 0, 0, 0, 1, 2, \dots, m-1, m, m, m, m$  for the radial hitches. The roundabout bunch grouping remains uniform because of periodicity. Figures 10 and 11, right, show instances of the NURBS construction. The spline surface is  $C^0$  because of the normal interpolated control vertex  $c_{0i}$  that addresses an imploded edge  $c_{0i} = c_{00}, i = 0, \dots, n-1$ . The surface is independently parametrized. Since independently parametrized surfaces are generally utilized in CAD applications, such bundles handle and show the NURBS fix without issues. However, independently parametrized surfaces are interesting to dissect [10, 11, 14, 2, 16]. The traditional methodology is an arithmetical reparametrization of the surface in the solitary point. In the Supplement, we utilize an original methodology that just becomes natural because of working on comprehension of development surfaces: we re-parametrize by a region plot that follows out a similar surface as the NURBS fix. We see that as the surface is  $C^1$  and bend limited.

## CONCLUSION

Bi-cubic polar region expands the abilities to existing Catmull-Clark executions. The expansion is especially significant for expelled highlights and normally supplements Catmull-Clark in locales of high valence. We propose viable polar lattice refinement to insignificantly adjust the current foundation and add the great shape furthermore, the effortlessness of bi-cubic polar development. We likewise fostered limited polar spline speculation of standard bi-cubic splines. Enjoyably, this development consists of a solitary NURBS fix. The portrayal is simple to add to existing CAD and activity displaying bundles, what's more, is appropriate for assessment on the GPU. The focal singularity presents no issue for delivering since the unequivocal typical is known and the Appendix shows that the surface arches are limited. The investigation of the limited development in the Appendix characterizes and utilizes another polar development conspire, called pbs. This brings up the issue of whether we could involve pbs in the spot of the bi-cubic polar region and along these lines get a unified limited in addition to development portrayal. We think about pbs less useful since it has an enormous region impression, with unique guidelines for each  $I$ -connect for  $I = 0, 1, 2, 3$ . Besides, the producing capacities related to the 1-connect vertices are reliant and a unique initial step is expected without which the curved body property isn't ensured. Each of the four surface sorts of Table 1 is viable with each other in that their changes are indistinguishable from bi-cubic splines. The subsequent surfaces are piecewise bi-cubic,  $C^2$  all over, and  $C^1$  at disengaged places (bends in the instance of PCCM). Both the development and the NURBS development give similarly substantial importance to the info network made and controlled by the planner. Furthermore, by increasing the development level, the subsequent surfaces can be made randomly near permit changing from one modeling worldview to the next. Advantageously, the polar bits of each approach can be carried out as a straightforward expansion of existing demonstrating instruments.

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