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TOPOLOGICAL ROUGHNESS

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Abstract

The method of topological roughness and constructed according to the Andronov-Pontryagin concept of roughness, is under consideration. The method is used for research of roughness and bifurcations of dynamic and synergetic systems of various physical nature, chaos in these systems too.

Introduction

On the basis of concept of roughness according to Andropov- Pontryagin the foundation of ‘a method of topological roughness’ which allows to investigate roughness (robustness) and bifurcations of dynamic systems of various nature, in particular synergetic systems, and also to synthesize rough control systems was laid. In this article ‘the method of topological roughness’ which bases as it is noted above is considered were are put in work and further development of a method was gained by broad application at research of roughness and bifurcations of synergetic systems of various physical nature, in particular, at chaos research in these systems.

Method Bases

In classical statement questions of roughness and bifurcations were put at a dawn of formation of topology as new scientific direction of mathematics by the great French scientist H. Poincare, in particular the term bifurcation is introduced for the first time by it and means literally ‘bifurcation’ or otherwise from solutions of the equations of dynamic systems new decisions branch off. Roughness of dynamic systems thus is defined, how property of systems to keep a qualitative picture of splitting phase space on a trajectory at small indignation of topology, by consideration of relatives by the form the equations of systems. Transition between rough systems is carried out through not rough areas (spaces). Many fundamental results in the theory of roughness and

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bifurcation are received by A. Andronov and his school. In work the concept of roughness is for the first time given and qualitative criteria of roughness which subsequently, is called concept of roughness according to Andronov-Pontryagin are formulated. In multidimensional statement the order n dynamic system is considered

ż (��)=��(��(��)) (1)

where ��(��)∈�� �� - a vector of phase coordinates, F- n-dimensional differentiated a vector - function.

The system (1) is called topological rough according to Andronov – Pontryagin in some area G if initial system and the pretreated system defined in a subarea of G areas G:

z̃ =��z̃) + ��(z̃) (2)
where f (z̃) – differentiable small on any norm ‖•‖ n – a measured vector
function, are ε– identical in topological sense. Systems (1) and (2) ε – are identical if
in n – measured phase space. %there are open areas D, D
∃ ��, �� >0:

ಖಜಿ ‖ಜಿ (ದ˜)‖ &ಟಣ; δ, |ಜಜು (ದ˜) / ಜ ದ˜ಠ | &ಟಣ; δ, ಖ, ಠ = 1 ಟಿ, (3)
that |‖z‖−‖ z̃ ‖| < ε, (4)
, (2)) (D, (1)), (5)%or (D
and D trajectories of systems (2) and (1) ε – are%Otherwise, splitting areas D
identical (have identical topological structures with trajectories close to ε).
If (5) it isn’t carried out, the system (1) isn’t rough according to Andronov-Pontryagin.
The topological structure of dynamic systems is defined by special trajectories and varieties like singular points, the special lines, the closed trajectories, attracting varieties (attractors).
In work [1] on the basis of concept of roughness according to Andronov-Pontryagin bases of ‘a method of topological roughness’ on the basis of a roughness

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measure in the form of conditionality number are offered C{M} – M matrixes - a rated matrix of reduction of system to a canonical diagonal (quasi-diagonal) form in singular points of phase space. Here, the concept of the maximum roughness and the minimum non-roughness on couple relations δ and ε is introduced for the first time.
Definition- 1. Rough in the field of G system (1) is maximally rough on the set of systems N topological identical each other, if dimension of δ – proximity of the systems (1) and (2), bringing to ε – identity, (for everyone ε > 0) is maximal.
Definition- 2. Non-rough in the field of G system (1) is called minimum non-rough on the set of systems N topological identical each other, if dimension ε – identity of systems (1) and (2) at which the roughness condition is still satisfied, (for everyone δ > 0) is minimal.
The condition of approachability of the maximum roughness and the minimum non-roughness in a vicinity of singular points of phase space is defined by the following theorem proved in work [1].
Theorem- 1. In order that the dynamic system in the neighborhood of a hyperbolic singular point (z 0) was maximum rough, and in the neighborhood a non-hyperbolic- is minimum non-rough, it is necessary and to sufficient to have as follows:
M* = argmin C{M}, (6)
where M - a matrix of reduction of linear part A of system (1) in a singular point (z 0) to diagonal (quasi-diagonal) basis, the C{M} - number of conditionality of a matrix of M.
Remark- 1. As appears from definitions 1 and 2, and also theorems 1, exist both minimum rough, and maximum non-rough systems, for which C{M} = ∞. Otherwise, the set of rough and non-rough systems is formed by continuous sets. Thus, systems with a Jordan quasi-diagonal form of matrixes of linear approach A, will be systems with the C{M} = ∞.

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Obviously, the number of conditionality C{M} as a measure of roughness can be used for piecewise-smooth dynamic systems, considering cumulative roughness on areas of smoothness of system if singular points aren't on border of these areas. It should be noted that for non-smooth systems, using any generalized derivative from non-smooth analysis when determining a matrix of linear part, it is possible to generalize this measure of roughness.
In work [7] the measure of roughness of periodic movements with the T period in the form of number of conditionality of CT on a matrix of a monodromy X (T) of these movements is entered

�� �� = {(��)}: (��)��(��) = ��(��)��(��), (7)
Where Λ (T) = diag {μi, i = 1, …, n }, μ – animators (own values) of matrixes X(T), T – the period of oscillations of a cycle.
We will notice that the similar measure of roughness can be entered and for given non-stationary linear systems, considering in quality X(T) a matrix P given systems.
Theoretical results of 'a method of topological roughness', received in works [1, 11, 12], allow to control roughness of synergetic systems. The system is considered
ż = Q (z, u), (8)
where � ∈ �� �� , � ∈ �� �� - according to a vector of phase coordinates and system controls, Q(•) – n – a measured nonlinear differentiable vector function.
Possibilities of control of roughness are defined by conditions of the following theorem which has been also proved in work [1].
Theorem- 2. In order that in the control ability dynamic system (8) described in n – measured phase space by means of matrixes of linear approximately of A, B respectively for phase coordinates and controls, there was the control u (t) providing in a vicinity of the corresponding singular point of closed system the maximum roughness or the minimum non-roughness, it is necessary and sufficiently that conditions of nonsingular resolvability of the matrix equation of Sylvester were satisfied.

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The control � =� (��) ∈ �� is looked in a class of systems with � =−��x, such that the matrix of the closed system �� = �� – ����, near special trajectories, in particular, singular points, conditions were satisfied.

(��) = (Γ),Γ – ��M= − ��H,�� = ��M−1 , (9)
where Γ∈ �� ��×�� – a diagonal (quasi-diagonal) matrix of a condition of canonical model, � ∈ � ��×�� – the matrix which is set randomly with restriction on observability of couple (Γ, H), �� ∈ ��×�� , �� ∈ ��×�� – matrixes of coordinates and controls.
Near a singular point:

F (z (t)) = 0, Z = Az+ Bu (10)
The control � = � (��) ∈ �� is synthesized so that to reach demanded value of an indicator C{M} using any methods of nonlinear programming. 'The method of topological roughness' also allows defining bifurcations of dynamic systems on the basis of criteria developed in works [1, 7]. Moreover, the method presents to possibility of forecasting of bifurcations, and also control of parameters of bifurcations.
Theorem- 3. In order that in the field of G multidimensional (n > 2) DS at value of parameter q = q*, � ∈ �� �� arose any bifurcation of topological structure, is necessary and sufficiently, that:
- or 1), in considered area G, DS there are non-hyperbolic (non-rough) singular

points (SP), or the orbital-unstable limit cycles (LC) for which equality takes place - or 2), in the field of G DS, are available any rough SP or LCs for which the condition is satisfied

$$C\{M(q^*)\}=\infty \quad (11)$$

Remark- 2. The type of bifurcation depends; first, on what (11) is carried out from conditions, secondly, from what special trajectory – SP or LCs meets these conditions. So, for example, chaotic fluctuations (strange attractors), arising because of symmetry loss, happen when (11) meet a condition SP, and the chaotic fluctuations

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arising through sequences of bifurcations of doubling of the period, happen in that case when the condition (11) is answered by LCs.

Conclusion-

Considered in this article 'method of topological roughness' is a method of quantitative research of roughness and bifurcations of dynamic systems of the widest class and various physical nature. Possibilities of a method for researches of roughness and bifurcations of systems can be used for researches of a large number, as synergetic systems of various nature –Lorentz, Chua, 'Prey-Predator' etc., and for researches of dynamic systems of wider class, in particular, at researches of oscillatory systems and Hopf's bifurcations, in particular, at research of an attractor of mapping of Henon.

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