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Change Detection And Bayesian Preliminary Test Estimation In Weibull Model

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Abstract

Physical system manufacturing the items are often subject to the random fluctuations. It may happen at some point of time of instability in the sequence of lifetimes is observed. Such observed points are known as change point in the inferential problem. For a given sequential data such as stock market prices or streaming stories in a newswire, we might be interested in when these data change in some way, such as a stock price falling or the document topics shifting. This abrupt change can cause shift in sequence and divide the sequence into two parts. Such change point problem is used in statistical quality control to study the change in the process mean, Linear time series models related to the econometrics. The Bayes Estimates of Change point is studied through numerical calculation by using "R" programming and also it is compared with real data. The results are also discussed in this paper.

For a given sequential data such as stock market prices or streaming stories in a newswire, we might be interested in when these data change in some way, such as a stock price falling or the document topics shifting. This abrupt change can cause shift in sequence and divide the sequence into two parts. For example, If n items are put to test their lives then their lives will be $x_1, x_2, \dots\dots, x_n$. If there is one break in sequence, then sequence is divided into two parts. Suppose the change occurs at point mth, the sequence will be $x_1, x_2, \dots\dots, x_m$. and $x_m, x_2, \dots\dots, x_{n+1}$. If we view our data as observations from a generative process, then we care about when the generative parameters change. Now the problem is how to detect and estimate the break point Such change point problem is used in statistical quality control to study the change in the process mean, Linear time series models related to the econometrics. The Bayes Estimates of Change point in Weibull Model are studied through Bayesian preliminary test Estimation (BPTE) method. The numerical calculation by using "R" programming and also it is compared with real data.

Keywords. Change-point analysis, Bayesian Estimation, Preliminary test estimator, SELF, R Software.

1. Introduction.

In statistical quality control such studies are very much useful for the shifting in process mean for example cumulating sum (CUSUM) control chart are used in production process to detect in shift in target value, when small shift or change (<1.5σ) of interest occur, the CUSUM chart and the exponentially weighted chart are used. Montgomery (2001) and Wu et. al. (2004), discussed the procedure of CUSUM control in shifting in target value.

Lim et. al. (2002), Wu and Tiau (2005) and Zhang and Wu (2005) considered the applications of CUSUM control charts.

Change point models are used to describe discontinuous behavior in stochastic phenomena. The change point indexes where or when the shift occurs. It is a discrete random variable. The prior probability mass function of the shift point gives the nature of the change to be expected.

The Bayesian inferential applications can play an important role in study of such problem of change points. Many of statisticians like Chin and Broemeling (1980), Calabria and Pulcini (1994), Zacks (1983), Pandya and Jani (2006), Shah and Patel (2007,2009), Chib (1998), Altissemo and Corradi (2003) and Fiteni (2004) studied the change point Models in Bayesian framework. Broemeling (1985) and Broemeling and Tsurume (1987)are the useful references on structural change .

When a point estimate is required and alternative hypotheses lead to different estimates, an optimal Bayes estimate is obtained by minimizing posterior expected loss averaged over the hypotheses, with posterior probabilities used as weights. In order to reflect uncertainty regarding the validity of different hypotheses, Zellner and Vandaele (1975) suggested preliminary test estimation of the parameter under a specified loss function in Bayesian framework. Such a Bayesian preliminary test estimate (BPTE) incorporates prior information and is optimal relative to a given loss function. However, so far, no attempt has been made to study BPTE of the change point. Some of the literature includes Dey et al. (1998), Martin et al. (1988), Dey and Micheas (2000), Rios ,Insua and Ruggeri (2000), Micheas and Dey (2004), and the references therein.

The aim of this paper to discuss the Bayesian Preliminary Test Estimation (BPTE) Method of a change point in Weibull sequence under squared loss function and examine its robustness through numerical simulation.

2. Statistical Model and Loss Function

The probability density function of Weibull distribution is given as

$$g(y) = \frac{\theta}{\sigma} y^{(\theta-1)} \exp\left(-\frac{y^{\theta}}{\sigma}\right) \quad ; \qquad y, \theta, \sigma > 0 \quad , \qquad (2.1)$$

Where $'\sigma'$ is the scale and 'θ' is shape parameters. Weibull distribution has extensively been used in life testing and reliability problems. The Weibull distribution is a continuous probability distribution. It is named after Waloddi Weibull(1931,1951), who described it in detail in 1951, although it was first identified by Fréchet (1927) and first applied by Rosin & Rammler (1933) to describe the size distribution of particles in connection with his studies on strength of material.

The most widely used loss function in estimation problems is quadratic loss function given as $(\hat{\sigma}, \sigma) = k(\hat{\sigma} - \sigma)^2$, where $\hat{\theta}$ is the estimate of $\theta,$ the loss function is called quadratic weighed loss function. If k=1, we have

$$L(\hat{\sigma}, \sigma) = (\hat{\sigma} - \sigma)^2 \quad , \qquad (2.2)$$

known as squared error loss function (SELF). This loss function is symmetrical because it associates the equal importance to the losses due to overestimation and under estimation with equal magnitudes however in some estimation problems such an assumption may be inappropriate. Overestimation may be more serious than underestimation or Vice-versa Ferguson(1985). Canfield (1970), Basu and Ebrabimi(1991). Zellner (1986) Soliman (2000) derived and discussed the properties of varian's (1975) asymmetric loss function for a number of distributions.

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the Entropy loss

$$L(\delta)\alpha[\delta^P - p\ log_e(\delta) - 1 \quad ; \text{Where } \delta = \frac{\hat{\theta}}{\theta},$$

and whose minimum occurs at $(\hat{\theta} = \theta)$,where p>0, a positive error $(\hat{\theta} > \theta)$ causes more serious consequences that a negative error and vice-versa. For small |p| value the function is almost symmetric, when both $\hat{\theta}$ and θ are measured in a logarithmic scale and is approximately.

$$L(\delta) = b[\delta - log_e(\delta) - 1]; \quad b > 0; \qquad \text{where } \delta = \frac{\hat{\theta}}{\theta}. \tag{2.3}$$

3. The Detection of Change Point.

Suppose $y_1, y_2, \dots, y_m, y_{(m+1)}, \dots, y_n$ is a sequence of independent random variables such that

$$y_i = \begin{cases} g_1(y_i\,; \sigma_1, \theta_1); i = 1,2, \dots\dots\dots\dots\dots, m \\ g_2(x_i\,; \sigma_2, \theta_2), i = (m+1), \dots\dots\dots, n \end{cases} \tag{3.1}$$

Here $y_1, y_2, \dots, y_n$ ($n \geq 3$) be a sequence of observed life times. First let observations $y_1, y_2, \dots, y_n$ have come from Weibull distribution with probability density function (pdf) as

$$g(y) = \frac{\theta}{\sigma} y^{(\theta-1)} \exp\left(-\frac{y^\theta}{\sigma}\right) \quad ; \qquad y, \theta, \sigma > 0 \quad , \tag{3.2}$$

Let 'm' is change point in the observation, which breaks the distribution in two sequences as ($y_1, y_2, \dots, y_m$) & ($y_{m+1}, \dots, y_n$).

The probability density functions of the above sequences are

$$\mathrm{g}_1(\mathrm{y}) = \frac{\theta_1}{\sigma_1} y_i^{\theta_1 - 1} \exp\left(-\frac{y_i^{\theta_1}}{\sigma_1}\right) \quad ; \qquad y, \sigma_1, \theta_1 > 0 \quad , \tag{3.3}$$

$$\mathrm{g}_2(\mathrm{x}) = \frac{\theta_2}{\sigma_2} y_i^{\theta_2 - 1} \exp\left(-\frac{y_i^{\theta_2}}{\sigma_2}\right) \quad ; \qquad y, \sigma_2, \theta_2 > 0 \quad , \tag{3.4}$$

This can be written with Weibull sequence before and after change point 'm'

$$y_i = \begin{cases} \frac{\theta_1}{\sigma_1} y_i^{\theta_1 - 1} \exp\left(-\frac{y_i^{\theta_1}}{\sigma_1}\right) & i = 1, \dots\dots\dots\dots, m \\ \frac{\theta_2}{\sigma_2} y_i^{\theta_2 - 1} \exp\left(-\frac{y_i^{\theta_2}}{\sigma_2}\right) & i = (m+1), \dots\dots\dots, n \end{cases} \tag{3.5}$$

4. Likelihood, Prior and Posterior.

The joint likelihood function of the Weibull sequences of before and after change point 'm' is given by

$$l(\sigma_1, \sigma_2, p|y) = \prod_{i=1}^m g_1(y_i|\sigma_1) \prod_{(m+1)}^n g_2(y_i|\sigma_2), \tag{4.1}$$

$$l(\sigma_1, \sigma_2, p|y) = \prod_{i=1}^m \frac{\theta_1}{\sigma_1} y_i^{\theta_1 - 1} \exp\left(-\frac{y_i^{\theta_1}}{\sigma_1}\right) \prod_{(m+1)}^n \frac{\theta_2}{\sigma_2} y_i^{\theta_2 - 1} \exp\left(-\frac{y_i^{\theta_2}}{\sigma_2}\right) \tag{4.2}$$

The joint prior for 'm' is given by

$$q(m|x) = \iint_{\sigma_1, \sigma_2} q(\sigma_1, \sigma_2, m|y) d\sigma_1 d\sigma_2\,; \tag{4.3}$$

$$s.t. \quad \sigma_1 \in \Theta_1 \ ; \quad \sigma_2 \in \Theta_2 \quad and \quad m = 1,2, \dots .(n-1).$$

With a change point at 'm', where m is unknown, using the equations (4.2) and (4.3), the joint posterior distribution is given by

$$r(\sigma_1, \sigma_2, p|y) = l(\sigma_1, \sigma_2, m). q(\sigma_1, \sigma_2, m); \sigma_1 \in \theta_1, \sigma_2 \in \theta_2, \tag{4.4}$$

such that m=1,2,…(n-1)

Detection of Change Point.

Let the hypothesis for detecting change point 'm' is

$H_o: m = n \quad Vs \quad H_1: m \neq n$

Let us assume that the prior probability mass function of the change point 'm' is

$$g(m) = \begin{cases} p, & if\ m = n \\ \frac{(1-p)}{n-1} & if\ m \neq n \end{cases} \quad ; \quad o < p < 1, p \text{ is known probability}; \tag{4.5}$$

Let us assume that the scalar parameters σ_1 and σ_2 and the change point 'm' are independent of each other. Let us take prior of scalar parameter σ_1 as natural conjugate gamma prior given by,

$$q(\sigma_1) = \begin{cases} \frac{b_1^{a_1}}{\Gamma a_1} \sigma_1^{-(a_1+1)} e^{-b_1/\sigma_1}; & \sigma_1 > 0, (a_1, b_1) > 0 \\ 0 \quad , & \text{Otherwise} \end{cases}, \tag{4.6}$$

The prior of scalar parameter σ_2 as natural conjugate gamma prior given by

$$q(\sigma_2) = \begin{cases} \frac{b_2^{a_2}}{\Gamma a_2} \sigma_2^{-(a_2+1)} e^{-b_2/\sigma_2}, & \text{where } \sigma_2 > 0 \text{ and } (a_2, b_2) > 0 \\ 0 \quad , & \text{Otherwise} \end{cases}, \tag{4.7}$$

Again with independent σ_1, σ_2 and 'm', we have under null hypothesis H_0, the joint prior as

$$q(\sigma_1, \sigma_2, m) = q(\sigma_1).q(m) \quad , \tag{4.8}$$

However under alternative hypothesis H_1, the joint prior is given by

$$g(\sigma_1, \sigma_2, m) = g(\sigma_1)\, g(\sigma_2) g(m), \tag{4.9}$$

Now the joint likelihood is given by

$$l\left(\sigma_1, \sigma_2, m \middle| \underline{y}\right) = \begin{cases} \prod_{i=1}^{n} g_1(y_i; \sigma_1) \; ; & if\ m = n \\ \prod_{i=1}^{m} g_1(y_i; \sigma_1) \prod_{i=m+1}^{n} g_2(y_i; \sigma_2); & if\ m \neq n \end{cases}; \tag{4.10}$$

This is derived as

$$l\left(\sigma_1, \sigma_2, m \middle| \underline{y}\right) = \begin{cases} \prod_{i=1}^{n} \frac{\theta_1}{\sigma_1} y_i^{(\theta_1 - 1)} \exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_1}\right); \\ \prod_{i=1}^{n} \frac{\theta_1}{\sigma_1} y_i^{(\theta_1 - 1)} \exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_2}\right) \prod_{i=m+1}^{n} \frac{\theta_2}{\sigma_2} y_i^{(\theta_2 - 1)} \exp\left(-\frac{\sum y_i^{\theta_2}}{\sigma_2}\right) \end{cases} \tag{4.11}$$

Combining the equations(4.5) ,(4.8),(4.9) and (4.11),we get the joint posterior of $\sigma_1,\ \sigma_2\ \ and\ \ m$ as

$$h\left(\sigma_1, \sigma_2, m \middle| \underline{y}\right) = \begin{cases} p\, g(\sigma_1) \prod_{i=1}^{n} g_1(y_i; \sigma_1)\, d\sigma_1 \; ; & if\ m = n \\ \frac{(1-p)}{(n-1)} \prod_{i=1}^{m} g_1(y_i; \sigma_1) \prod_{i=m+1}^{n} g_2(y_i; \sigma_2)\, q\left(\sigma_1, \sigma_{2_1}\right) d\sigma_1 d\sigma_2; & if\ m \neq n \end{cases} \tag{4.12}$$

And the marginal posterior of 'm' is given by

$$h\left(m \middle| \underline{y}\right) = \begin{cases} P \int g(\sigma_1) \prod_{i=1}^{n} g_1(y_i; \sigma_1) d\sigma_1 \; ; & if\ m = n \\ \frac{(1-P)}{(n-1)} \iint \prod_{i=1}^{m} g_1(y_i; \sigma_1) \prod_{i=m+1}^{n} g_2(y_i; \sigma_2) d\sigma_1 d\sigma_2; & if\ m \neq n \end{cases}; \tag{4.13}$$

with constant of proportionality

$$[D(y)]^{-1} = P \int q(\sigma_1) \prod_{i=1}^{n} g_1(y_i; \sigma_1) d\sigma_1 + \frac{(1-P)}{(n-1)} \sum_{m=1}^{n-1} \iint \prod_{i=1}^{m} g_1(y_i; \sigma_1) \prod_{i=(m+`1)}^{n} g_2(y_i; \sigma_2) q(\sigma_1 \sigma_2) d\sigma_1 d\sigma_2$$

(4.14)

Which is derived as

$$h(m|\underline{y}) = \begin{cases} p\int\left[\frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{\theta_1}{\sigma_1}\prod_{i=1}^{n} y_i^{(\theta_1-1)}\exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_1}\right)\right]d\sigma_1; \\ d\sigma_1 \\ \frac{(1-p)}{(n-1)}\iint\left[\left\{\prod_{i=1}^{m}\left\{\frac{\theta_1}{\sigma_1}y_i^{(\theta_1-1)}\exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_2}\right)\frac{\theta_2}{\sigma_2}\prod_{i=(m+1)}^{n} y_i^{(\theta_2-1)}\exp\left(-\frac{\sum y_i^{\theta_2}}{\sigma_2}\right)*\frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{b_2^{a_2}}{\Gamma a_2}\sigma_2^{-(a_2+1)}e^{-b_2/\sigma_2}\right\}\right]d\sigma_1 d\sigma_1 \end{cases}$$

(4.13)

On simplifying we get

$$h(m|\underline{y}) = \begin{cases} \frac{p\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{n} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} \\ \frac{(1-p)}{(n-1)} * \frac{\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{m} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2}\prod_{i=(m+1)}^{n} y_i^{(\theta_2-1)}}{\left(b_2+\sum y_i^{\theta_2}\right)^{(a_2+1)}} \end{cases}; \quad (4.15)$$

The posterior in favour of the null hypothesis H_0 is

$$O(H_0|y) = p[(m=n|y)]/p\{m \neq n|y\}, \quad (4.16)$$

$$= \frac{p\int q(\sigma_1)\prod_{i=1}^{n} g_1(y|\sigma_1)d\sigma_1}{(1-p)/(n-1)\sum_{m=1}^{(n-1)}\iint\prod_{i=1}^{m} g_1(y|\sigma_1)\prod_{i=(m+1)}^{n} g_2(y|\sigma_2)q(\sigma_1,\sigma_2)\,d\sigma_1 d\sigma_2}$$

$$= \frac{p\int\frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{\theta_1}{\sigma_1}\prod_{i=1}^{m} y_i^{(\theta_1-1)}\exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_1}\right)d\sigma_1}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)}\iint\left\{\frac{\theta_1}{\sigma_1}\prod_{i=1}^{m} y_i^{(\theta_1-1)}\exp\left(-\frac{\sum y_i^{\theta_1}}{\sigma_2}\right)*\frac{\theta_2}{\sigma_2}\prod_{i=m+1}^{n} y_i^{(\theta_2-1)}\exp\left(-\frac{\sum y_i^{\theta_2}}{\sigma_2}\right)\frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{b_2^{a_2}}{\Gamma a_2}\sigma_2^{-(a_2+1)}e^{-b_2/\sigma_2}\right\}d\sigma_1 d\sigma_1}$$

(4.17)

$$= \frac{\frac{p\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{n} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}}}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)}\left\{\frac{\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{m} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2}\prod_{i=(m+1)}^{n} y_i^{(\theta_2-1)}}{\left(b_2+\sum y_i^{\theta_2}\right)^{(a_2+1)}}\right\}}; \quad (4.18)$$

$$O(H_0|y) = \frac{\frac{p\prod_{i=1}^{n} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}}}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)}\left\{\frac{\prod_{i=1}^{m} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2}\prod_{i=(m+1)}^{n} y_i^{(\theta_2-1)}}{\left(b_2+\sum y_i^{\theta_2}\right)^{(a_2+1)}}\right\}}; \quad (4.19)$$

The hypothesis H_0 is not accepted, if the Posterior odds are less than 1.

5. Bayesian Preliminary Test Estimation (BPTE) of the Change Point

Suppose $y_1, y_2, \dots\dots, y_m, y_{(m+1)}, \dots\dots, y_n$ is a sequence of independent random variables such that

$$y_i = \begin{cases} g_1(y_i;\sigma_1,\theta_1); i = 1,2,\dots\dots\dots\dots m \\ g_2(y_i;\sigma_2,\theta_2), i = (m+1),\dots\dots\dots n \end{cases}; \quad (5.1)$$

The change point 'm' is an unknown discrete random parameter. Further suppose that the scalar parameters σ_1, σ_2 and 'm' are independent of each other.

Let p_0 denote the posterior probability of the hypothesis $H_0: m = n$ of no change so that $(1 - p_0)$ is the posterior probability of the alternative hypothesis $H_1: m \neq n$ of a change.

The posterior expected loss under the Squared Error loss function $L(m,\widehat{m})$ with change point 'm' is given by

$$E(L(m,\widehat{m}|y) = P_0 E(L(m,\widehat{m}|H_0 y) + (1-P_0)E\big(L(m,\widehat{m}|H_1 y)\big) \quad (5.2)$$

$$= P_0 L(n,\widehat{m}) + (1-P_0)E\big(L(m,\widehat{m}|H_1 y)\big) \quad (5.3)$$

Thus the BPTE $\widehat{m}$ of change point 'm' under Squared error loss function is

Again under SELF, BPTE of the change point 'm' is

$$\widehat{m}_s = p_0\, E\big(m\big|H_{0,}y\big) + (1-p_0)E(m|H_{1,}y) \;\; ; \quad (5.6)$$

This will give

$$\widehat{m}_s = p_0 \frac{p\theta_1 a_1 b_1^{a_1} \prod_{i=1}^{n} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} + (1-p_0)\frac{(1-p)}{(n-1)} * \frac{\theta_1 a_1 b_1^{a_1} \prod_{i=1}^{m} y_i^{(\theta_1-1)}}{\left(b_1+\sum y_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2} \prod_{i=(m+1)}^{n} y_i^{(\theta_2-1)}}{\left(b_2+\sum y_i^{\theta_2}\right)^{(a_2+1)}} \quad (5.7)$$

For squared error loss function, the posterior means are optimal, the minimizing value $\widehat{m}_s$ for $\widehat{m}$ is a weighted average of the posterior means under the two hypotheses.

Let p_0 denote the posterior probability of the hypothesis $H_0 : m = n$ of no change so that $(1-p_0)$ is the posterior probability of the alternative hypothesis $H_1 : m \neq n$ of a change.

Provided expectation exists. Here $K_{01} = \frac{p_0}{(1-p_0)}$ is the posterior odds ratio (POR) in favour of H_0 . It is to note that K_{01} close to 1 suggests that H_0 is more or less as likelihood as H_1 a posteriori while if this ratio is large, we regard H_0 as relatively more likely than H_1.

For $K_{01} = 0$, that is the posterior odds ratio indicates a change in the sequence. BPTE $\widehat{m_u}$ will reduce to the Bayes estimate under linex loss. However, for large values of $K_{01}, \widehat{m_u}$ would be close to n.

As observed by Zeller and Vandale (1975), it may interest to recall that (i) $\widehat{m}_u$ is a continuous function of the observations (ii) prior information about m under H_1 can be induced through use of an appropriate prior probability mass function and (iii) there is no arbitraries in the choice of the classical significance level.

References:

1. Altissimo, F., Corradi, V.(2003).Strong rules for detecting the number of breaks in a time series, J. Econometrics 117(2):2007-244.
2. Basu, A.P. & Ebrahimi, N. (1991): "Bayesian approach to life testing and Reliability estimation using asymmetric loss function" Jour. Of Stat. Planning & Inf. 29 (1991) 21-31. Dekker Inc., New York.
3. Broemeling and Tsurume (1987):
4. Calabria, R., and Pulcini, G. (1994): An engineering approach to Bayes estimation for the weibull distribution. Microelectron. Reliab, 34, No. 5, pp 789-802.
5. Canfield (1970): A Bayesian Approach to Reliability estimation Using a Loss function , IEEE Tans. Reliab., R-19, 13-16.
6. Chin Choy, J.H. Broemeling, L.,D.(1980). Some Bayesian inferences for a changing linear model. Technometrics 22;71-78.
7. Chib, S.(1998). Estimation and comparison of multiply change point models J. Econometrics 86(2); 221-241.
8. Dey D. K., Lu, K., and Bose, S. (1998) :A Bayesian approach to loss robustness,statistical decisions,16,65-87.
9. Dey, D.K. and Micheas, A.C. (2000):Ranges of posterior expected losses and epsilonrobust actions. In: Robust Bayesian analysis, Lecture notes monograph series, Series no. 152 (eds. D. Rios Insua and F. Ruggeri), Springer-Verlag, 145-160.

10. Ferguson, Thomas S. (1967): Mathematical Statistics: A Decision theoretic Approach. Academic Press, New York.
11. Fiteni, I.(2004). T-estimators regression models with structural change of unknown location. J.Econometrics 119(1);19-44.
12. Fréchet, Maurice (1927): "Sur la loi de probabilité de l'écart maximum", Annales de la Société Polonaise de Mathematique, Cracovie 6: 93–116.
13. Johnson, Norman L.; Kotz, Samuel; Balakrishnan, N. (1994): Continuous univariate distributions. Vol. 1, Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics (2nd ed.), New York: John Wiley & Sons, ISBN 978-0-471-58495-7, MR1299979
14. Kao, J.H.K (1959): "A graphical estimation of mixed Weibull parameters in life-testing of electron tubes." Technometrics 7: 639-643.
15. Lim, T.O.,Soraya, A., Ding, L.M., Morad, Z(2002), Assuming doctor's competence; application of CUSUM techique in monitoring doctor's performance. lnt.J.Qual. Health Care 14(3);251-258.
16. Lieblein, J. and Zelen, M. (1956): "Statistical Investigation of the fatigue life of deep-grove ball bearing." J. Res. Nat. Bur. Stand. 47 : 273-316.
17. Mann.(1968):"Results on statistical estimation and hypothesis testing with application to the Weibull extreme value distribution,Aerospace Research Laboratories, Wright Patterson. Airforce Bax Chio.
18. Micheas, A.C. and Dey, D.K. (2004): On measuring loss robustness using maximum a posteriori estimate, Communications in Statistics, Theory and Methods, 33, 1069-1085.
19. Montgomery,J.G.(2001), Introduction to statistical quality control. New York; John Wiley and Sons.
20. Pandya, M., Jani, P.N.(2006). Bayesian estimation of change point in inverse Weibull sequence. Commun, Statist. Theor. Meth. 35(10);2223-2237.
21. Rios Insua, D. and Ruggeri, F. (2000): Robust Bayesian analysis, Lecture notes monograph series, Series no. 152, Springer-Verlag..
22. Shah, J.B., Patel, M.N.(2007), Bayes estimation of shift point in geometric sequence. Commun. Statist. Theor. Meth.36(6);1139-1151.
23. Soliman, A.A. (2000): Comparison of Linex and Quadratic Bayesian Estimators for Rayleigh Distribution. Commun. Statist. Theor. Meth, 29(1), 95-107.
24. Varian, Hal R. (1975): A Bayesian Approach to Real Estate Assessment. In: Studies in Bayesian Econometrics and Statistics in Honour of Leonard J. Savage (eds. S.E.Fienberg and A. Zellner), Amsterdam: North-Holand, 195-208.
25. Weibull, W. (1939): "A statistical theory of the strength of materials." Ing. Vele. Ak. H.151 : 1-45.
26. Weibull, W. (1951): "A Statistical distribution function of wide applicability J.Appl.Mech.18,pp 293-297.
27. Wu,Z.Tian,Y.(2005): Weighted-loss-function CUSUM chart for monitoring mean and variance of a production process. Int. J. Product. Res. 43:3027-3044.
28. Wu,Z., Shamsuzzaman, M.,Pan, E.S.(2004): Optimization design of the control chart based on Taguchi's loss function and random process shifts. Int.J. Product. Res.42:379-390.
29. Zack, S. (1983). Survey of classical and Bayesian approaches to the change point problem: fixed sample and sequential procedures for testing and estimation. Recent Advances in Statistics. Herman Chernoff Best Shrift. New York : Academic Press. growth of nestling Tree Swallows. Can. J. Zool. 60: 1080-1090.
30. Zhang, S.Wu, Z.(2005). Design of the control charts with supplementary runs ruls. Comput. Indust, Eng. 49(1); 76-97.
31. Zellner A. and Vandaele, W. (1974): Bayes-Stein estimators for k-means, regression and simultaneous equation models, Studies in Bayesian Econometrics and Statistics, North-Holland, 317-343.

32. Zellner, A. (1986): Bayesian estimation and prediction using asymmetric loss functions,Journal of American Statistical Association, 81(394), 446-451.