Mathematical model for behavior of Synovial fluid in slider bearing

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Abstract

Present herein are the analytical study of lubrication mechanism of a slider bearing. The idealized model is to produce the result consistent with those in normal situation. We observe that load bearing capacity decreases with the increases the value h and load capacity increases with the increase value of viscosity. Again it clear that the load capacity increases with the increase the value of L and load capacity increases with the increase the value of viscosity.

Keywords- Lubrication, synovial fluid, synovial joint articular cartilage

Introduction

When to dry metallic surface move relatively in contact, The motion of these surface gives rise to friction. However these undesirable effects can be prevented by the use of substance called lubricant. The science which enable us knee as to reduce friction on bearings is known as lubrication.

The basic mechanism can be explained in the following way, when a viscous lubricant flows in a narrow clearance between the slider and bearing there develops high pressure which supports the load and prevent the contact between the slider and bearing.

Almost a number of scientist and lubrication theory normally starts with the plane inclined pal. Rayleigh, L [1] was first to analyse such type of bearings. He investigated that a combination of flat and inclined parts of a slider is definitely better then a simple slider. Prakash, J. [2] discussed the effect of transverse magnetic field on the ratio of flat and inclined parts under general load condition. Yadav, A.K. and S.C. Pokhriyal [3] Discussed the load capacity of a composite slider for various values of parameters.

Engineers have started studying the problem of bearing lubrication theoretical and experimental aspects. Within last six decades sufficient thought has been given to the study of lubrication mechanism in human joints. Normal Synovial joints Exhibit salient features which are extra ordinary in mechanical sense. A synovial joint may be described as a load carrying system consisting of two mating bones with tangential/or normal motions. The bone ends, which are usually globular in appearance are covered with a soft sponge-like material, called articular cartilage. The space between this cartilaginous extremities of the bones called a joint cavity, is filled with shear dependent fluid synovial fluid. Downson, [5] have confined that the synovial fluid acts as lubricants in the study of lubrications mechanism.
Walker et al.[4] demonstrated that the concentration of molecular weight constituent of synovial fluid increases due to filtration action of suspended medium. It increases the viscosity. Walker et al of observed the similar result on frictional experiments. Downson, [5] provided result for lubrication and cartilage when bearing material as in knee joint is soft it may deform under the hydrodynamic pressure. The knee joints differs from most bearing in mechanical system. Ogston, A.G. and J.E. Stanier [6] observed as result of more fluid will be trapped in the centre of contact

In this paper we have made to study the load capacity of slider bearing.

**Formulation of the problem**

The proposed model may be considered as two dimensional squeeze film of synovial fluid in slider bearing.

The basic equation governing the flow of lubricant under the usual assumption is given below;

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{p} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  

(1)

The equation of continuity;

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(2)

When \((u, v)\) are the component of velocities in x and y direction and \(p\) be the pressure in the fluid region.

The solution of the equation subject to the boundary condition;

\[ u = -U_0 \text{, at } y = 0 \]  

(3)

\[ u = 0 \text{, at } y = h \]  

(4)

If the fluid film is thin, the model equation is

\[ \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{dp}{dx} \]  

(5)

**SOLUTION OF THE PROBLEM**

Since pressure gradient exists along x axis. The integration of equation (5) under boundary condition (3),& (4) is,

\[ U = -U_0 \left( 1 - \frac{y}{h} \right) - \frac{1}{2 \mu} \frac{dp}{dx} h \left( 1 - \frac{y}{h} \right) \]  

(6)

When the x- dependence is boundary condition,

\[ u = 0, \text{ at } y = h \]

The volume flow rate

\[ Q = \int_0^h u \, dy = \text{constant} \]  

(7)

\[ Q = -\frac{h}{2} U - \frac{1}{4 \mu} \frac{dp}{dx} h^2 \]  

(8)

And

\[ \frac{dp}{dx} = -\left( \frac{4\mu Q}{h^2} + h\mu U_0 \right) \]  

(9)

Integrating we get
The load capacity of a slider bearing is

\[ p_L - p_0 = -\left(\frac{4\mu Q L}{h^2} + \mu U_0 h L\right) \]  \hspace{1cm} (10)

The load capacity of a slider bearing is

\[ w = \int_0^L (p_L - p_0) \, dx \]  \hspace{1cm} (11)

\[ w = 2\mu U_0 \frac{L}{h} (1 - h^3) \]  \hspace{1cm} (12)

**Table (1)**

Variation of load carrying capacity for different value of \( h=1, U_0 = 1 \)

<table>
<thead>
<tr>
<th>( h )</th>
<th>( \mu )</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>3.9960</td>
<td>7.9920</td>
<td>11.9880</td>
<td>15.9840</td>
<td>19.9880</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>1.9840</td>
<td>3.9680</td>
<td>5.9520</td>
<td>7.9360</td>
<td>9.9200</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>1.2973</td>
<td>2.5946</td>
<td>3.8920</td>
<td>5.1894</td>
<td>6.4867</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>0.9360</td>
<td>1.8720</td>
<td>2.8080</td>
<td>3.7440</td>
<td>4.6800</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.7000</td>
<td>1.4000</td>
<td>2.1000</td>
<td>2.8000</td>
<td>3.5000</td>
</tr>
</tbody>
</table>

Variation of load capacity \( w \) for different value of \( \mu \) and \( h \)
Table (2)

Variation of load carrying capacity for different value of $L=1, U_0 = 1$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L=1$</td>
<td>0.7000</td>
<td>1.4000</td>
<td>2.1000</td>
<td>2.8000</td>
<td>3.5000</td>
</tr>
<tr>
<td>$L=2$</td>
<td>1.4000</td>
<td>2.8000</td>
<td>4.2000</td>
<td>5.6000</td>
<td>7.0000</td>
</tr>
<tr>
<td>$L=3$</td>
<td>2.1000</td>
<td>4.2000</td>
<td>6.3000</td>
<td>8.4000</td>
<td>10.5000</td>
</tr>
<tr>
<td>$L=4$</td>
<td>2.8000</td>
<td>5.6000</td>
<td>8.4000</td>
<td>11.2000</td>
<td>14.0000</td>
</tr>
<tr>
<td>$L=5$</td>
<td>3.5000</td>
<td>7.0000</td>
<td>10.5000</td>
<td>14.0000</td>
<td>17.5000</td>
</tr>
</tbody>
</table>

Result and discussion

The research paper proposes a more realistic model for explaining the lubrication mechanism occurring normally loaded slider bearing. The result of load bearing capacity have been examined for different value of $h$, $L$ & $\mu$. It is clear that the load capacity decreases with the increase of $h$. We have also observed that the load capacity increases when increase the value of viscosity. Again it is clear that the load capacity increases when increase the value of $L$ and load capacity increases when increase the value of viscosity.

Reference:

6. Ogston, A.G. and J.E. Stanier (1953) “Viscous elastic lubricant properties” Physiol. 119,244-252