IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

A STUDY OF THE BRAHMASPHUTA SIDDHANTA OF BRAHMAGUPTA

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ABSTRACT

Mathematical sciences developed well in the Asian countries mainly in India, China & Arabian Peninsula during the period from 4th century to 12th century AD. Within this period, a galaxy of mathematicians & astronomers flourished in India who left their indelible mark on the history of Mathematical world. The most celebrated among them was Brahmagupta (598-669AD). His Sanskrit text Brahmasphuta-Siddhanta (628AD) greatly influenced the development of Mathematics and Astronomy not only in India but in the whole world. Brahmasphuta Siddhanta was translated into Arabic in 771AD by Arabian scholar AL-FAZARI under the title SINDHIND-AL-KABIR (the great Siddhanta) by the order of Bagdad Caliph Al-Mansur. This translation of Brahmagupta's text opened the doors of study & development of Mathematics & Astronomy in Arabian regions and then transmitted to western world.

In Brahmasphuta-Siddhanta, Brahmagupta gave some results which are considered unitue throughout the world. He gave formulae for sum to n- terms in A.P. Areas of prism, Cones, Pyramids, Cyclic quadrilateral etc. He also gave rules for operating with zero and infinity. His method of solving linear-indeterminate-equations is almost similar to the present day method. Brahmagupta's method of interpolation of sines of intermediate angles is equivalent to the Newton-Stirling Formula upto second order differences.

In the present paper, we have made efforts to high-light some salient features of the great Sanskrit text of Brahmagupta and have tried to prove that methods & formulae given by Brahmagupta in 628AD are, to some extent, similar and equivalent to the modern & formulae and are useful even today.

Key Word – Life of Brahmagupta, Indeterminate Equation, Quadratic Indeterminate Equation,Brahmgupta Theorem,Cyclic Quadrilateral Equation

Introduction

Brahmagupta was an Indian mathematician and Astronomer. Brahmagupta was born in 598 AD in Sind and flourished at Ujjain, the famous astronomical center of central India. His father's name was Jishnu. No details is available about his early education.

Brahmgupta the author of two early works on mathematics and astronomy. Brahmagupta who wrote his monumental work "Brahma-sphuta- siddhanta" in 628AD and the "Khandakhadyaka" in 665AD. He contributed much to the development of Mathematical sciences. Brahmagupta was the first to give rules to computer with zero. This text is the improved and updated collection of the old astronomical and mathematical work "Brahma-Siddhanta". Brahma-sphuta-siddhanta contain 21 chapters of which chapter 12 relates to Ganita (Arithmetic) and Chapter 18 deals with Kuttaka (Pulverizer).

Brahmagupta (628AD) though well-acquainted with Aryabhata's mathematics and Astronomy, seems, as already pointed out, to have parted company with the school. His geometry contains some remarkable new theorems about the cyclic quadrilateral. But is significant that the elucidation and proof of these theorems were under taken by the Aryabhata school, while Bhaskara II, who closely followed Brahmagupta failed to do so.

Methodology :-

Brahmagupta worked and wrote an almost all branches of Mathematics including geometry, algebra and mensuration.

But the most outstanding contribution of Brahmagupta is his solution of the Indeterminate equation.

$$Nx^2 + 1 = y^2$$

It was Brahmagupta who solved the second degree indeterminate equation for the first time using a general method. Brahamgupta gives the following rule for the solution of a quadratic indeterminate equation involving a factum.

शेषवधाद् द्विकृतिगुणात् शेषान्तरवर्गसंयुतान्मूलम्।

शेषान्तरोनयुक्तं दलितं शेषे पृथगभीष्टे।। (Br. Sp. Si. XVIII.99)

We have gives formula:-

$$x = \frac{1}{2}\left(\sqrt{b^2 + 4c} + b\right),\ y = \frac{1}{2}\left(\sqrt{b^2 + 4c} - b\right)$$

"With the exception of an optional unknown, assume, arbitrary values for the rest of the unknowns, the product of which forms the factum. The sum of the products of these (assumed values) and the (respective) coefficients of the unknowns will be absolute quantities. The continued products of the assumed values and of the coefficients of the factum will be the coefficients of the optionally (leftout) unknown. Thus the solution is effected without forming an equation of the factum. why then was it done so?"

वर्गचतुर्गुणितानां रूपाणां मध्यवर्गसहितानाम्।

मूलं मध्येनोनं वर्ग द्विगुणोद्धृतं मध्यः।। (Br. Sp. Si. XVIII. 44)

Brahmagupta's method prior to Brahmagupts Babylonians solved instances of the quadratic equation using special adhoc method. They have solved equations such as $y = ax^2$, $y = ax^2 + bx$ etc. without providing a general adhoc comprehensive procedure. This is solution :

$$x = \frac{\sqrt{b^2+4ac} - b}{2a}$$

Next Brahmagupta give the following illustrative example :-

Example:-

"On subtracting from the product of signs and degrees of the sun, three and four times (respectively) those quantities, ninety is obtained. Determining the sun within a year (one can pass as a proficient) mathematician."

If x denotes the signs and y the degrees of the sun, then the equation is:

$$xy – 3x – 4y = 90$$

Thus this problem appears to have same relation with that of the "Bhakhshali" work. Prthudakasvami solve it in two ways :

Firstly, he assumes the arbitrary number to be 17, then

$$x = \frac{1}{1}\left(\frac{90.1 + 3.4}{17} + 4\right) = 10 \quad \text{and} \quad y = \frac{1}{1}(17 + 3) = 20$$

Secondly, he assumes arbitrarily,

$$y = 20$$

Now putting this value in above equation we get,

20x - 3x = 170

or, 17x = 170

∴ x = 10

which gives x = 10

In fact Brahmagupta's theorem about the cyclic quardrilatral must have been a hard nut to crack for quite a few earlier Indian mathematician.

There are two other results concerning cyclic quadrilaterals which are known as Brahmagupta's theorem:-

स्थूलफलं त्रिचतुर्भुजबाहुप्रतिबाहुयोगदघातः।

भुजयोगार्धचतुष्टयभुजोनघातात् पदं सूक्ष्म्।। (Br. Sp. Si. XIII .21)

The area of a cyclic quadrilateral with sides a, b, c, d then

P = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where 2s = a + b + c + d

And the one for the area of a triangle:

Q = $\sqrt{s(s-a)(s-b)(s-c)}$ where 2s = a + b + c

The diagonals of a cyclic quadrilateral are given by the following rule:-

कर्णाश्रितभुजघातैक्यमुभयथाऽन्योन्यभाजितं गुणयेत्।

योगेन भुजप्रतिभुजवधयोः कर्णौ पदे विषमे।। (Br. Sp. Si. XII .28)

The sums of the products of the sides about the diagonal should be divided by each other and multiplied by the sum of the products of the opposite side. The square roots of the quotients are the diagonals in a visama quadrilateral.

If the sides are a, b, c, d in order this gives for the diagonals the lengths :-

$x^2 = \frac{(ad+bc)(ac+bd)}{(ab+cd)}$ and $y^2 = \frac{(ab+cd)(ac+bd)}{(ad+bc)}$

This formula is called "Brahmagupta theorem".

Beside the two rules given above, Brahmagupta gives rules corresponding to the formula:-

$2r = \frac{a}{SinA}$ etc.

And If $a^2 + b^2 = c^2$ and $\alpha^2 + \beta^2 = \gamma^2$

Then the quadrilateral's $(a\gamma, c\beta, b\gamma, c\alpha)$ is cyclic and has its diagonals at right angles.

Conclusions:

From the above facts it is proved that Brahmagupta lived some time in between Aryabhata I and Mahaviracarya. After study of works of the great scholar of 7th century and going through the different commentaries made by various scholars of mathematics on the historic Sanskrit work "Brhama-sphuta-siddhanta", we may conclude that Brhamagupta was one of the greatest mathematicians of the world. His work on geometry, particularly on "cyclic quadrilaterals" and on "Indeterminate equation" was outstanding and influenced the later writer to a great extent.

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