



SOLVING FUZZY TRANSPORTATION THROUGH PASCAL'S TRIANGULAR GRADED MEAN APPROACH

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Abstract: In this paper, we proposed new ranking function for solving fuzzy transportation problem by using Pascal's triangular graded mean approach. The transportation cost, demand and supply of the problem are addressed as nanogonal fuzzy numbers to represent all transportation parameters. By using this approach we find out a crisp problem. Illustrated examples of fuzzy transportation problem are given to clarify our proposed technique.

Index Terms: Pascal's Triangular Graded Mean, Membership Function, Nanogonal Fuzzy Number, Fuzzy Transportation Problem

I. INTRODUCTION

The transportation problem a sub classes of linear optimization problem that arrange with delivery products from sources to objections in the broad area of operation research. The objective of this problem is to maximize the profit or minimize the transportation cost. A fuzzy concept has been applied in different areas of management, science and Engineering and fuzzy transportation problem is a transportation problem in which parameters, supply and demand quantities are fuzzy numbers. First of all the transportation related idea was presented by F. L. Hitchcock in 1941 after that in 1947, T. C. Koopmans was implemented this idea for the transportation frame work. R. Bellman and L. A. Zadeh[7] introduced the concept of decision making in fuzzy environment and the fuzzy assertion imprecise by virtue of the fuzziness of the terms. S. Divya Bharathi and P. Saraswathi[10] purposed a ranking function whose based on Pascal's triangular graded mean approach for solving fuzzy game problem and the quantities pay off matrix in which the transportation cost, demand and supply are addressed as a octagonal fuzzy number. Jain[8] first introduced the ranking function of fuzzy numbers in fuzzy logic programming and decision making etc. P. Malini and M. Ananthanarayanan [6] considered a new technique for solving fuzzy transportation problem by using ranking function of octagonal numbers using MODI method and compare it with Vogel's approximation method. U. Jayalakshmi and K. A. Mohana[13] developed a new method for optimal solution of fuzzy transportation problem using by hexadecagonal fuzzy numbers and solved by Vogel's approximation method. S. Muruganandam and R. Srinivasan[11] introduced a new algorithm for solving fuzzy transportation problem with trapezoidal fuzzy number and compare the solution with different existence methods Least cost method, North west corner method and Vogel's approximation method. M. S. Annie Christi and B. Kasthuri[5] purposed a solution methodology for transportation problem with pentagonal intuitionistic fuzzy number and solved it by using a ranking technique and Russell's method D. Kumar and J. Singh[2] purposed a measure of central tendency approach to obtain optimal solution for pentagonal intuitionistic fuzzy and to compare other traditional methods. A. Felix et al[1] introduced Nanogonal fuzzy number with its membership function and defined aithmetic operations as addition, subtraction multiplication with help of alpha cut. K. Deepika and S.Rekha[3] developed a new ranking function for

obtaining optimal solution of nanogonol fuzzy transportation problem and solved by the highest cost method. L. Sudha et.al[4] purposed a new ranking function for solving Nanogonol fuzzy transportation problem and compared by NWC, LCM and VAM. R. Saravanan, and M.Valliathal[9] developed a new ranking function to solve fuzzy travelling salesman problem in which all parameters are represent to nanogonol fuzzy number. In this paper we extend above work for nanogonol fuzzy numbers for solving fuzzy transportation problem by using the Pascal's triangular graded mean approach.

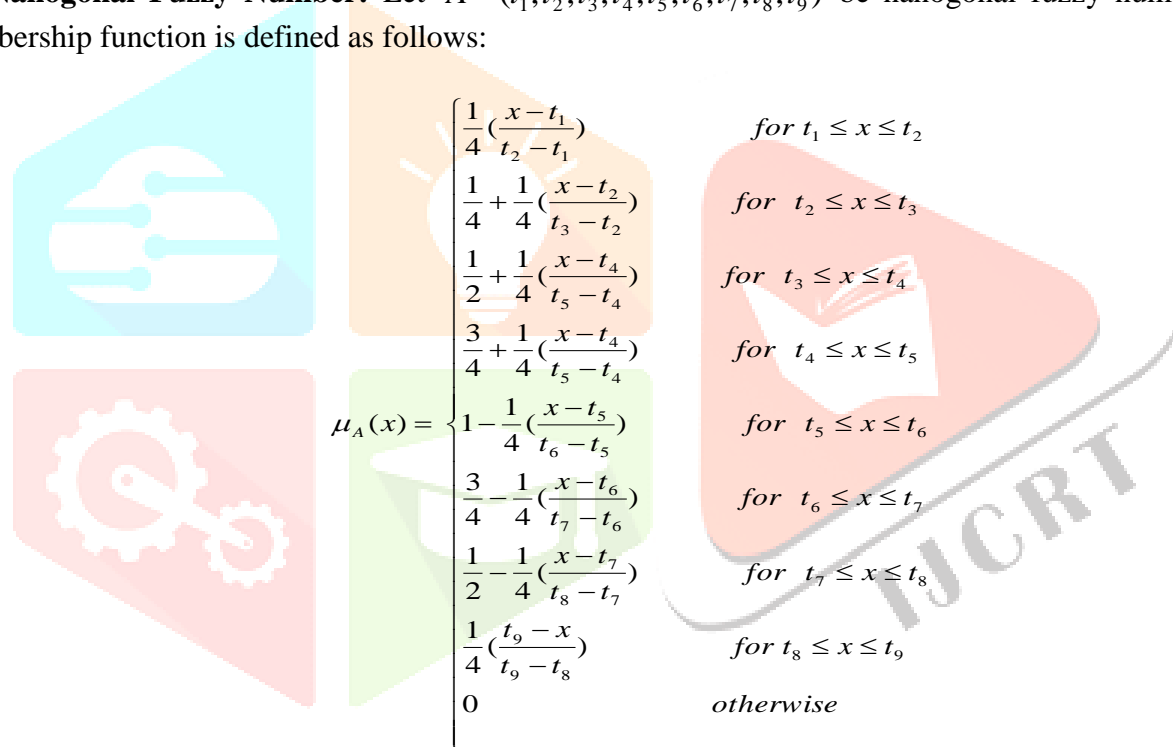
II. PRELIMINARIES

2.1 Fuzzy Set: Let T be real number set. A fuzzy set A of T is defined as $A = \{(t, \mu_A(t)) / x \in T\}$, where the membership function of t in A which is $\mu_A(t) : T \rightarrow [0, 1]$.

2.2 Fuzzy Number: The fuzzy set A is called fuzzy number if its membership function $\mu_A(t)$ must satisfy following characteristics:

- A is normal i.e. $\exists t_0 \in T$ such that $\mu_A(t_0) = 1$.
- $\mu_A(t)$ is piecewise continuous.
- A is convex i.e. $\mu_A\{\lambda t_1 + (1 - \lambda)t_2\} \geq \min\{\mu_A(t_1), \mu_A(t_2)\}$, for each $t_1, t_2 \in T$ & $\lambda \in [0, 1]$.
- The support of $A, S(A) = \{t \in X / \mu_A(t) > 0\}$ is bounded in T .

2.3 Nanogonol Fuzzy Number: Let $A = (t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9)$ be nanogonol fuzzy number then its membership function is defined as follows:



$$\mu_A(x) = \begin{cases} \frac{1}{4} \left(\frac{x-t_1}{t_2-t_1} \right) & \text{for } t_1 \leq x \leq t_2 \\ \frac{1}{4} + \frac{1}{4} \left(\frac{x-t_2}{t_3-t_2} \right) & \text{for } t_2 \leq x \leq t_3 \\ \frac{1}{2} + \frac{1}{4} \left(\frac{x-t_4}{t_5-t_4} \right) & \text{for } t_3 \leq x \leq t_4 \\ \frac{3}{4} + \frac{1}{4} \left(\frac{x-t_4}{t_5-t_4} \right) & \text{for } t_4 \leq x \leq t_5 \\ 1 - \frac{1}{4} \left(\frac{x-t_5}{t_6-t_5} \right) & \text{for } t_5 \leq x \leq t_6 \\ \frac{3}{4} - \frac{1}{4} \left(\frac{x-t_6}{t_7-t_6} \right) & \text{for } t_6 \leq x \leq t_7 \\ \frac{1}{2} - \frac{1}{4} \left(\frac{x-t_7}{t_8-t_7} \right) & \text{for } t_7 \leq x \leq t_8 \\ \frac{1}{4} \left(\frac{t_9-x}{t_9-t_8} \right) & \text{for } t_8 \leq x \leq t_9 \\ 0 & \text{otherwise} \end{cases}$$

2.4 Function based on Triangular of Pascal's Graded Mean:

If X be a universal discourse set and $\tilde{A}^r = (a_1, a_2, \dots, a_r)$ is fuzzy number as row vector then the ranking function of Pascal's triangular is defined as follows:

$$R_{pac}(\tilde{A}^r) = \frac{C_r \cdot (\tilde{A}^r)^T}{(2)^{r-1}}$$

Where C_r be coefficients of triangular of Pascal row vector for the value of $r (> 2) \in N$. Let

$A = (t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9)$ be nanogonol fuzzy number then the triangular of Pascal's graded mean of this number is defined as:

IV. NUMERICAL EXAMPLES

Example 4.1 (L. Sudha[4]) Consider the Nanogonal fuzzy transportation problem:

Table 4.1 (Fuzzy Transportation Problem)

| | S_1 | S_2 | S_3 | Supply |
|--------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| D_1 | [8,10, 12, 14, 16, 18, 20, 22, 24] | [50, 52, 54, 56, 58, 60, 62, 64, 66] | [38,40,42, 44, 46, 48, 50, 52, 54] | [60, 62, 64, 66, 68, 70, 72, 74, 76] |
| D_2 | [76, 78, 80, 82, 84, 86, 88, 90, 92] | [24, 26, 28, 30, 32, 34, 36, 38, 40] | [8,10, 12, 14, 16, 18, 20, 22, 24] | [32, 34, 36, 38, 40, 42, 44, 46, 48] |
| D_3 | [80, 82, 84, 86, 88, 90, 92, 94, 96] | [20, 22, 24, 26, 28, 30, 32, 34, 36] | [80, 82, 84, 86, 88, 90, 92, 94, 96] | [80, 82, 84, 86, 88, 90, 92, 94, 96] |
| Demand | [32, 34, 36, 38, 40, 42, 44, 46, 48] | [60, 62, 64, 66, 68, 70, 72, 74, 76] | [80, 82, 84, 86, 88, 90, 92, 94, 96] | |

By using proposed ranking function

$$P(A) = \frac{(t_1 + 8t_2 + 28t_3 + 56t_4 + 70t_5 + 56t_6 + 28t_7 + 8t_8 + t_9)}{256}$$

We obtain the values of the cost of Nanogonal fuzzy transportation problem as below:-

$$\begin{aligned} P(\tilde{c}_{11}) &= P [8, 10, 12, 14, 16, 18, 20, 22, 24] = 16, & P(\tilde{c}_{12}) &= P [50, 52, 54, 56, 58, 60, 62, 64, 66] = 58, \\ P(\tilde{c}_{13}) &= P [38, 40, 42, 44, 46, 48, 50, 52, 54] = 46, & P(\tilde{c}_{21}) &= P [76, 78, 80, 82, 84, 86, 88, 90, 92] = 84, \\ P(\tilde{c}_{22}) &= P [24, 26, 28, 30, 32, 34, 36, 38, 40] = 32, & P(\tilde{c}_{23}) &= P [8, 10, 12, 14, 16, 18, 20, 22, 24] = 16, \\ P(\tilde{c}_{31}) &= P [80, 82, 84, 86, 88, 90, 92, 94, 96] = 88, & P(\tilde{c}_{32}) &= P [20, 22, 24, 26, 28, 30, 32, 34, 36] = 28, \\ P(\tilde{c}_{33}) &= P [80, 82, 84, 86, 88, 90, 92, 94, 96] = 88. \end{aligned}$$

The costs of Demand are $P(\tilde{b}_1) = P [32, 34, 36, 38, 40, 42, 44, 46, 48] = 40$, $P(\tilde{b}_2) = P [60, 62, 64, 66, 68, 70, 72, 74, 76] = 68$ and $P(\tilde{b}_3) = P [80, 82, 84, 86, 88, 90, 92, 94, 96] = 88$.

The costs of Supply are $P(\tilde{a}_1) = P [60, 62, 64, 66, 68, 70, 72, 74, 76] = 68$, $P(\tilde{a}_2) = P [32, 34, 36, 38, 40, 42, 44, 46, 48] = 40$ and $P(\tilde{a}_3) = P [80, 82, 84, 86, 88, 90, 92, 94, 96] = 88$.

The total demand is equal to the total supply, so the Nanogonal fuzzy transportation problem is balanced. After using Proposed function fuzzy transportation problem is transformed to the crisp transportation problem as below

Table 4.2 (Crisp Transportation Problem)

| | D_1 | D_2 | D_3 | Supply |
|--------|-------|-------|-------|--------|
| D_1 | 16 | 58 | 46 | 68 |
| D_2 | 84 | 32 | 16 | 40 |
| D_3 | 88 | 28 | 68 | 88 |
| Demand | 40 | 68 | 88 | |

To solve this problem by using North West corner Method

Table 4.3 (North West corner Method)

| | D_1 | D_2 | D_3 | Supply |
|--------|---------------|---------------|---------------|--------|
| D_1 | 16(40) | 58(28) | 46 | 68 |
| D_2 | 84 | 32(40) | 16 | 40 |
| D_3 | 88 | 28 | 68(88) | 88 |
| Demand | 40 | 68 | 88 | |

The transportation cost by using North West corner Method is $\text{Min } Z = 9528$

To solve this problem by using Least Cost Method

Table 4.4 (Least Cost Method)

| | D ₁ | D ₂ | D ₃ | Supply |
|----------------|----------------|----------------|----------------|--------|
| D ₁ | 16(40) | 58 | 46(28) | 68 |
| D ₂ | 84 | 32 | 16(40) | 40 |
| D ₃ | 88 | 28(68) | 68(20) | 88 |
| Demand | 40 | 68 | 88 | |

The transportation cost by using Least Cost Method is $\text{Min } Z = 5832$

To solve this problem by using Vogel's Approximation Method

Table 4.5 (Vogel's Approximation Method)

| | D ₁ | D ₂ | D ₃ | Supply |
|----------------|----------------|----------------|----------------|--------|
| D ₁ | 16(40) | 58 | 46(28) | 68 |
| D ₂ | 84 | 32 | 16(40) | 40 |
| D ₃ | 88 | 28(68) | 68(20) | 88 |
| Demand | 40 | 68 | 88 | |

The transportation cost by using Vogel's Approximation Method is $\text{Min } Z = 5832$

V. COMPARISON

To compare solution by using the proposed Ranking function with Existing ranking function as follow

Table 5.1(Comparison Table of Optimal Solution)

| S. No. | Methods | Optimal Solution | |
|--------|------------------------------------|------------------------------|------------------------------|
| | | By Existing ranking function | By Proposed ranking function |
| 1. | North west corner method (NWC) | 11760 | 9528 |
| 2. | Lest cost method (LCM) | 7515 | 5832 |
| 3. | Vogel's approximation method (VAM) | 7515 | 5832 |

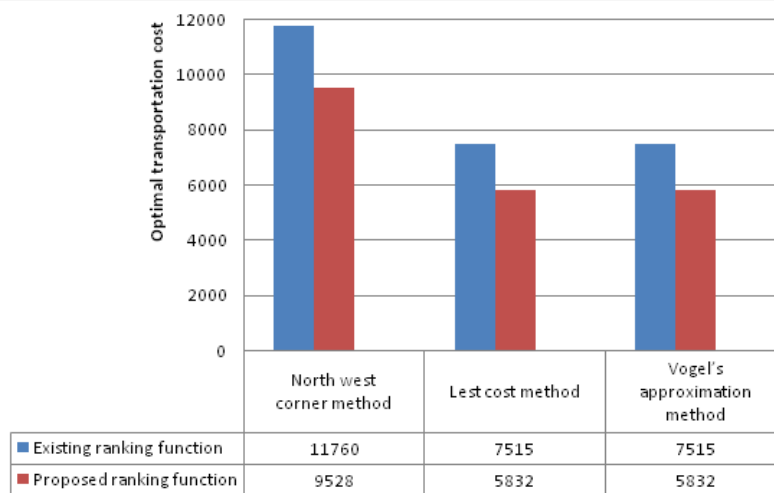


Fig.5.1. Compare optimal solution with all traditional methods by using proposed ranking function

VI. CONCLUSION

The objective of this paper is to propose a ranking function for obtaining minimum transportation cost and maximum profit for nanogonal fuzzy transportation problem. The principle of this function is based on by using the Pascal's triangular graded mean approach in which transportation parameters are represented by this fuzzy number. Fuzzy transportation problem transformed into crisp problem and solved it by the proposed ranking function which gives more minimum transportation cost compare to existing ranking function for solving by NWC, LCM, Vogel's approximation method and other methods. This approach can be used for decagonal fuzzy number and higher fuzzy number in different optimization techniques.

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