



INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Quin-Terranean Fuzzy Soft Continuity

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ABSTRACT:

This paper introduces the novel concept of **Quin-Terranean fuzzy soft Continuity building notion of quin –Terranean fuzzy soft topological spaces** and also the concept of continuity. Various characterizations, Definitions and basic properties of the **Quin-Terranean fuzzy soft interior, Quin-Terranean fuzzy soft closure, Quin-Terranean fuzzy soft image and pre-image**, and **Quin-Terranean fuzzy soft continuity** are studied. The concept of **Quin – Terranean fuzzy soft continuous mapping** are studied.

KEYWORDS:

Quin-Terranean Fuzzy Soft Set, Quin-Terranean Fuzzy Soft Topological Space, Quin-Terranean Fuzzy Soft Interior, Quin-Terranean Fuzzy Soft Closure, Quin-Terranean Fuzzy Soft Mapping, Quin-Terranean Fuzzy Soft Continuity.

1. INTRODUCTION

A number of algebraic operations were presented by Maji et al. [15] in the context of soft set theory. By suggesting extra operations within soft set theory, Ali et al. [2] built on the previous research. Soft topological spaces were first proposed by Shabir and Naz [25], who also defined a number of concepts related to soft sets in these spaces. Maji et al. subsequently presented a unified framework by combining fuzzy set theory and soft set theory. Fuzzy soft sets were first proposed by Maji et al. [13], and Tanay and Kandemir [29] expanded on this idea to create fuzzy soft topology. Chang [6] expanded on this idea by providing fundamental precepts. A membership function and a non-membership function are two functions that Atanassov [3] added to fuzzy sets to create intuitionistic fuzzy sets (IFS), which are a classification of fuzzy sets. $[0,1]$ is the closed unit interval into which both functions map. In addition to introducing intuitionistic fuzzy topology, Coker [7] investigated analogous forms of traditional topological ideas such as continuity and compactness. The framework of soft sets was expanded to intuitionistic fuzzy soft sets by Maji et al. [14]. Furthermore, the notion of intuitionistic fuzzy soft topological spaces has been investigated by a number of writers [5, 11, 20, 32]. The idea of the Pythagorean fuzzy set was first presented by Yager [30]. It is defined by a membership degree and a non-membership degree, where the square sum of these two degrees is less than or equal to one. Pythagorean fuzzy set theory has been studied by a number of authors [4, 9, 17, 18, 24, 28, 34]. Olgun et al. [19] presented the idea of Pythagorean fuzzy topological spaces in 2019 and examined their continuity as well as a few important characteristics. Pythagorean fuzzy soft set

theory was defined and important properties were analyzed by Peng et al. [23]. Guleria and Bajaj [8] investigated the different feasible types of Pythagorean fuzzy soft matrices. In order to solve decision-making problems, M. Kirisci [12] proposed using a new kind of Pythagorean fuzzy soft set. Several properties of Pythagorean fuzzy soft topological spaces have been studied by Adem Yolcu et al. [1]. According to Senapati and Yager's 2020 [26] proposal, Fermatean fuzzy sets are better able to manage uncertain information during the decision-making process. They used Fermatean fuzzy sets to define basic operations over them.

Fermatean fuzzy topological spaces were introduced and their properties were examined by Hariwan and Ibrahim [9]. Using their sets in the paper, Rajarajeswari and Thirunamakanni [25] introduced the idea of Quin-Terranean fuzzy sets, Quin-Terranean fuzzy soft topological spaces which are supersets of Fermatean fuzzy sets. It is evident from a survey of the literature that we define Quin-Terranean fuzzy soft closure and Quin-Terranean fuzzy soft interior and discuss some of their key characteristics. Quin-Terranean fuzzy soft continuous mappings are examined, and intriguing findings are obtained that may help us develop this area of study.

2. PRELIMINARIES

Definition 2.1

Let X be an universal set. X contains a fuzzy set $\mathcal{F}$, denoted by $\mathcal{F} = \{(\bar{x}, \alpha_{\mathcal{F}}(\bar{x})) : \bar{x} \in X\}$ where $\alpha_{\mathcal{F}} : X \rightarrow [0, 1]$ represents the membership function. The membership of $\bar{x} \in X$ in $\mathcal{F}$ is denoted by $\alpha_{\mathcal{F}}(\bar{x}) \in [0, 1]$. The notation $\mathcal{FS}(X)$ will represent the set of all fuzzy sets over X .

Definition 2.2

In X , an intuitionistic fuzzy set $\mathcal{I}$ is defined as $\mathcal{I} = \{(\bar{x}, \alpha_{\mathcal{I}}(\bar{x}), \beta_{\mathcal{I}}(\bar{x})) : \bar{x} \in X\}$, where $\alpha_{\mathcal{I}} : X \rightarrow [0, 1], \beta_{\mathcal{I}} : X \rightarrow [0, 1]$ are the degrees of membership and non-membership of x to $\mathcal{I}$ respectively which satisfies $0 \leq \alpha_{\mathcal{I}}(\bar{x}) + \beta_{\mathcal{I}}(\bar{x}) \leq 1, \forall \bar{x} \in X$. Let $\mathcal{IFS}(X)$ denotes the set of all intuitionistic fuzzy sets over X .

Definition 2.3

Let X be an universal set and V be a set of parameters. A soft set over X is a pair $(\mathcal{F}, V)$ where $\mathcal{F}$ is a mapping $\mathcal{F} : V \rightarrow \mathcal{P}(X)$. In other words, a soft set is a parameterized collection of subsets of the universal set X .

Definition 2.4

Let X be an universal set and V be a set of parameters. A fuzzy soft set over X is a pair $(\mathcal{F}, V)$, If $\mathcal{F} : V \rightarrow \mathcal{FS}(X)$ is a mapping from V to the set of all fuzzy sets in X , where $\mathcal{FS}(X)$ is the set of all fuzzy subsets in X .

Definition 2.5

Assume that X is an universal set. Let V be a collection of parameters. An intuitionistic fuzzy soft set over X is a pair $(\mathcal{F}, V)$ where $\mathcal{F}$ is a mapping denoted by $\mathcal{F} : V \rightarrow \mathcal{IFS}(X)$, where $\mathcal{IFS}(X)$ is the set of all intuitionistic fuzzy subsets in X .

In general, for every $v \in V$, $\mathcal{F}(v)$ is an intuitionistic fuzzy set of X referred to as the intuitionistic fuzzy value set corresponding to the parameter v . It is evident that $\mathcal{F}(v)$ can be expressed as an intuitionistic fuzzy set as follows $\mathcal{F}(v) = \{(\bar{x}, \alpha_{\mathcal{F}}(\bar{x}), \beta_{\mathcal{F}}(\bar{x})) : \bar{x} \in X\}$.

Definition 2.6

Let V be a set of parameters and X be the universal set. Quin-Terranean fuzzy soft set is defined as the pair $(\mathcal{F}, V)$, where $\mathcal{F} : V \rightarrow \mathcal{QTFSS}(X)$ and $\mathcal{QTFSS}(X)$ represents the Quin-Terranean

fuzzysubsetsofX.Quin-Terraneanfuzzy soft sets degenerate into Pythagoreanfuzzysoftsets.

Definition 2.7

Let V be a set of parameters and X be the universal set and let $M, N \subseteq V$. Consider two Quin-Terranean fuzzy soft sets $(\mathcal{F}, M), (S, N)$ over X . $(\mathcal{F}, M)$ is a Quin-Terranean fuzzy soft subset of (S, N) denoted by $(\mathcal{F}, M) \subseteq (S, N)$ if,

- (1) $M \subseteq N$
- (2) $\forall v \in M, \mathcal{F}(v)$ is the Quin-Terranean fuzzy subset of $S(v)$ that is, $\forall x \in X$ and $\forall v \in M, \alpha_{\mathcal{F}}(v)(x) \leq \alpha_S(v)(x)$ and $\beta_{\mathcal{F}}(v)(x) \geq \beta_S(v)(x)$. If $(\mathcal{F}, M) \subseteq (S, N)$ and $(S, N) \subseteq (\mathcal{F}, M)$, then $(\mathcal{F}, M)$ and (S, N) are said to be equal.

Definition 2.8

Let $(\mathcal{F}, V)$ be a Quin-Terranean fuzzy soft sets over X . Then $(\mathcal{F}, V)^c$ is the notation for the complement of $(\mathcal{F}, V)$ which is defined by

$$(\mathcal{F}, V)^c = \{(v, (x, \beta_{\mathcal{F}}(v)(x), \alpha_{\mathcal{F}}(v)(x))) : x \in X, v \in V\}$$

Definition 2.9

If $\alpha_{\mathcal{F}}(v)(x) = 0$ and $\beta_{\mathcal{F}}(v)(x) = 1 : \forall v \in V, \forall x \in X$. Then a Quin-Terranean fuzzy soft set $(\mathcal{F}, V)$ over the universe X is considered null Quin-Terranean fuzzy soft set. It is represented by $0_{(X, V)}$. If $\alpha_{\mathcal{F}}(v)(x) = 1$ and $\beta_{\mathcal{F}}(v)(x) = 0 : \forall v \in V, \forall x \in X$. Then a Quin-Terranean fuzzy soft set $(\mathcal{F}, V)$ over the universe X is an absolute Quin-Terranean fuzzy soft set. It is represented by $1_{(X, V)}$.

Definition 2.10

- i. Consider two Quin-Terranean fuzzy soft sets over the universe set $X, (\mathcal{F}_1, V)$ and $(\mathcal{F}_2, V)$. Then, the restricted union of $(\mathcal{F}_1, V)$ and $(\mathcal{F}_2, V)$ which is denoted by $(\mathcal{F}_1, V) \cup (\mathcal{F}_2, V) = (\mathcal{F}_3, V)$ and is defined by

$$(\mathcal{F}_3, V) = \{(v, (x, \alpha_{\mathcal{F}_3}(v)(x), \beta_{\mathcal{F}_3}(v)(x))) : x \in X, v \in V\}$$

where $\alpha_{\mathcal{F}_3}(v)(x) = \max\{\alpha_{\mathcal{F}_1}(v)(x), \alpha_{\mathcal{F}_2}(v)(x)\}$ and $\beta_{\mathcal{F}_3}(v)(x) = \min\{\beta_{\mathcal{F}_1}(v)(x), \beta_{\mathcal{F}_2}(v)(x)\}$.

- ii. The notation $(\mathcal{F}_1, V) \cap (\mathcal{F}_2, V) = (\mathcal{F}_3, V)$, indicates the restricted intersection of $(\mathcal{F}_1, V)$ and $(\mathcal{F}_2, V)$ which is defined by

$$(\mathcal{F}_3, V) = \{(v, (x, \alpha_{\mathcal{F}_3}(v)(x), \beta_{\mathcal{F}_3}(v)(x))) : x \in X, v \in V\}$$

Where $\alpha_{\mathcal{F}_3}(v)(x) = \min\{\alpha_{\mathcal{F}_1}(v)(x), \alpha_{\mathcal{F}_2}(v)(x)\}$ and $\beta_{\mathcal{F}_3}(v)(x) = \max\{\beta_{\mathcal{F}_1}(v)(x), \beta_{\mathcal{F}_2}(v)(x)\}$.

Definition 2.11

Let V be a parameter set, $M, N \subseteq V$ and $(\mathcal{F}, M)$ and (S, N) be two Quin-Terranean fuzzy soft sets over the universe set X . Then,

- a) The expression $(\mathcal{F}, M) \cup \mathcal{E}(S, N) = (T, O)$ represents the extended union of $(\mathcal{F}, M)(S, N)$, where $O = M \cup N$ and (T, O) is defined by;

$$(T, O) = \{(v, (x, \alpha_T(v)(x), \beta_T(v)(x))) : x \in X, v \in V\}$$

where

$$\alpha_{\mathcal{F}}(v)(x), \text{ if } v \in M - N$$

$$\alpha T(v)(x) = \begin{cases} \alpha s(x), & \text{if } v \in N - M \\ \max\{\alpha \mathcal{F}(v)(x), \alpha \mathcal{S}(v)(x)\}, & \text{if } v \in M \cap N \end{cases}$$

$$\beta T(v)(x) = \begin{cases} \beta \mathcal{A}(v)(x), & \text{if } v \in M - N \\ \beta s(v)(x), & \text{if } v \in N - M \end{cases}$$



$$\min \{ \beta_{\mathcal{F}}(v)(x), \beta_{\mathcal{S}}(v)(x) \}, \text{ if } v \in M \cap N$$

- b) The extended intersection of $(\mathcal{F}, M)$ and $(\mathcal{S}, N)$ is represented by $(\mathcal{F}, v) \cap \mathcal{E}(\mathcal{S}, N) = (T, O)$ when $O = M \cup N$ and (T, O) is defined by

$$(T, O) = \{ (v, (x, \alpha_T(v)(\bar{x}), \beta_T(v)(\bar{x})), \bar{x} \in X) : v \in V \}$$

where

$$\alpha_{\mathcal{F}}(v)(x), \text{ if } v \in M - N$$

$$\left\{ \begin{array}{l} \alpha_{\mathcal{S}}(v)(x), \text{ if } v \in N - M \\ \min \{ \alpha_{\mathcal{F}}(v)(x), \alpha_{\mathcal{S}}(v)(x) \}, \text{ if } v \in M \cap N \end{array} \right\}$$

$$\alpha_T(v)(x) =$$

$$\beta_T(v)(x) =$$

$$\left\{ \begin{array}{l} \beta_{\mathcal{F}}(v)(x), \text{ if } v \in M - N \\ \beta_{\mathcal{S}}(v)(x), \text{ if } v \in N - M \\ \max \{ \beta_{\mathcal{F}}(v)(x), \beta_{\mathcal{S}}(v)(x) \}, \text{ if } v \in M \cap N \end{array} \right\}$$

Definition 2.12

- a) Let V be a parameter set $M, N \subseteq V$ and let $(\mathcal{F}, M)$ and $(\mathcal{S}, N)$ be two Quin-Terranean fuzzy soft sets over the universe set X . Then the "and" operation on them is denoted by $(\mathcal{F}, M) \wedge (\mathcal{S}, N) = (T, M \times N)$ and is defined by,

$$(T, M \times N) = \{ ((v_1, v_2), (x, \alpha_{T(v_1, v_2)}(x), \beta_{T(v_1, v_2)}(x))) : x \in X, (v_1, v_2) \in M \times N \}$$

where

$$(\alpha_{v_1, v_2})(x) = \min \{ \alpha_{\mathcal{F}(v_1)}(x), \alpha_{\mathcal{S}(v_2)}(x) \}$$

1 2

$$(\beta_{v_1, v_2})(x) = \max \{ \beta_{\mathcal{F}(v_1)}(x), \beta_{\mathcal{S}(v_2)}(x) \}$$

- b) Given a parameter set V , two Quin-Terranean fuzzy soft sets $(\mathcal{F}, M)$ and (S, N) over the universe set X and $M, N \subseteq V$. Then the "or" operation on them, which is represented by $(\mathcal{F}, M) \vee (S, N) = (T, M \times N)$ and is defined by,

$$(T, M \times N) = \{((v_1, v_2), (\alpha_{T(v_1, v_2)}(x), \beta_{T(v_1, v_2)}(x))) : x \in X, (v_1, v_2) \in M \times N\}$$

where

$$\alpha_{T(v_1, v_2)}(x) = \max\{\alpha_{\mathcal{F}(v_1)}(x), \alpha_{S(v_2)}(x)\}$$

$$\beta_{T(v_1, v_2)}(x) = \min\{\beta_{\mathcal{F}(v_1)}(x), \beta_{S(v_2)}(x)\}.$$

Definition 2.13

Let X and Y be non-empty sets. Let M and N be Quin-Terranean fuzzy subsets of X and Y , respectively, and let $f: X \rightarrow Y$ be a function. The image of M with respect to f , represented by $[M]$, with membership and non-membership functions that are defined by,

sup

$$\alpha_{[M]}(y) = \sup_{z \in f^{-1}(y)} \alpha_M(z), \text{ if } f^{-1}(y) \neq \emptyset$$

$$\beta_{[M]}(y) = \inf_{z \in f^{-1}(y)} \beta_M(z), \text{ if } f^{-1}(y) \neq \emptyset$$

$$0, \text{ otherwise}$$

0 otherwise

The pre-image of N with respect to f , represented by $f^{-1}[N]$, has membership and non membership functions that are defined by $\alpha_{f^{-1}[N]}(x) = \alpha_N(f(x))$ and $\beta_{f^{-1}[N]}(x) = \beta_N(f(x))$, respectively. In this study they proved that demonstrated that

$$\alpha_{f^{-1}[M]}(x) + \beta_{f^{-1}[\bar{M}]}(x) \leq 1 \quad \text{for } x \in X$$

Quin-Terranean fuzzy images and pre-images.

Definition 2.14

Let $X \neq \emptyset$ be an universal set then the collection τ of Quin-Terranean fuzzy soft set over X Represented by $\tau \in QTFSS(X)$, is said to be a Quin-Terranean fuzzy soft topology on X if,

- (1) $0_{(X)}, 1_{(X)} \in \tau$,
- (2) Arbitrary union of any Quin-Terranean fuzzy soft sets in τ belong to τ .
- (3) the intersection of any two Quin-Terranean fuzzy soft sets in τ belong to τ .

A Quin-Terranean fuzzy soft topological space over X is denoted by the triple $(X, \tau, \bar{\tau})$. Every element of τ is called as a Quin-Terranean fuzzy soft open set in X .

Definition 2.15

- a) Let X be an universal set, V be the parameter set and $\tau_X = \{0_{(X)}, 1_{(X)}\}$. Then $(X, \tau_X, \bar{\tau}_X)$ is referred to as a Quin-Terranean fuzzy soft indiscrete topology on X and $(X, \tau_X, \bar{\tau}_X) \in QTFSS(X, V)$ is a Quin-Terranean fuzzy soft indiscrete space over X .
- b) Let X be an universal set, V be the parameter set and the collection of all Quin-Terranean fuzzy soft sets that can be defined over X are denoted by τ . Then $(X, \tau, \bar{\tau})$ is referred to as a Quin-Terranean fuzzy soft discrete topology on X and $(X, \tau, \bar{\tau}) \in QTFSS(X, V)$ is a Quin-Terranean fuzzy soft discrete space over X .

Definition 2.16

Suppose that $(X, \tau, \bar{\tau}) \in QTFSS(X, V)$ is a Quin-Terranean fuzzy soft topological space over X , and that $(T, V) \in QTFSS(X, V)$. The union of all Quin-Terranean fuzzy soft open sets that are contained in (T, V) is the Quin-Terranean fuzzy soft interior of (T, V) , representing $Q_{int}(T, V)$.

3. Quin-Terranean Fuzzy Soft Continuity

Let X and Y be non-empty initial universes with parameter sets V, V' and $M \subseteq V, N \subseteq V'$.

Definition 3.1

Let $(X, \tau_1, \bar{\tau}_1)$ and $(Y, \tau_2, \bar{\tau}_2) \in QTFSS(X, V)$ and $(Y, \tau_2, \bar{\tau}_2) \in QTFSS(Y, V')$ be two Quin-Terranean fuzzy soft topological spaces, and let $\theta: X \rightarrow Y, \rho: V \rightarrow V'$ be mappings. Then, a mapping $f = (\theta, \rho): QTFSS(X, V) \rightarrow QTFSS(Y, V')$ is defined as for $(T, M) \in QTFSS(X, V)$, $f(T, M) \in QTFSS(Y, V')$ is the image of (T, M) under f is a Quin-Terranean fuzzy soft set in $QTFSS(Y, V')$ provided by

sup

$$(T)(v')(y) = \begin{cases} \{v \in \rho^{-1}(v') \cap M, \in \theta \theta^{-1}(y)\} & \text{if } \theta \theta^{-1}(\bar{y}) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

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$$(T) \quad \begin{cases} \bar{v}'(y) = \{v \in \rho^{-1}(v') \cap M, \in \theta \theta^{-1}(\bar{y})\} & \text{if } \theta \theta^{-1}(\bar{y}) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$



- (2) $f(1(X,V)) \subseteq 1(Y,V')$,
 (3) $f((T,M) \cup \mathcal{E}(S,N)) = f(T,M) \cup \mathcal{E}f(S,N)$,
 (4) $f((T,M) \cap \mathcal{E}(S,N)) \subseteq f(T,M) \cap \mathcal{E}f(S,N)$,
 (5) $f((T,M) \cup R(S,N)) = f(T,M) \cup Rf(S,N)$,
 (6) $f((T,M) \cap R(S,N)) \subseteq f(T,M) \cap Rf(S,N)$,
 (7) If $(T,M) \subseteq (S,N)$ then $f(T,M) \subseteq f(S,N)$.

Proof:

We just provide proofs of (3),(4), and(7).The others can be proved similarly.

- (3) We consider, $(T,M) \cup \mathcal{E}(S,N) = (B, (M \cup N))$ and



$f((T,M) \cup \mathcal{E}(S,N)) = (\theta\theta(T), \rho(M)) \cup \mathcal{E}(\theta\theta(S), \rho(N)) = (A, \rho(M) \cup \rho(N))$. Then

$f((T,M) \cup \mathcal{E}(S,N)) = (\theta\theta(B), \rho(M \cup N)) = (\theta\theta(B), \rho(M) \cup \rho(N))$. For any $y \in Y$ and $v' \in \rho(M)$

$\cup \rho(N)$, if $\theta\theta^{-1}(y) \neq \emptyset$, then $\alpha A(v')(\bar{y}) = \alpha\theta(B)(v')(\bar{y}) = 0$ and $\beta A(v')(\bar{y}) = \beta\theta(B)(\bar{v}')(\bar{y}) = 1$.

Otherwise,

- i. If $v' \in \rho(M) - \rho(N)$, then $A(v') = \theta\theta(T)(v')$. However, $v' \in \rho(M) - \rho(N)$ implies that there isn't any $v \in N$ such that $\rho(v) = v'$. In other words, for any $v \in \rho^{-1}(v') \cap (M \cup N)$ we have $v \in \rho^{-1}(v') \cap (M - N)$. Therefore, according to Definition 4.1, we have

$$\sup_{\alpha\theta(B)(v')(\bar{y})} = \sup_{v \in \rho^{-1}(v') \cap (M \cup N), x \in \theta\theta^{-1}(\bar{y})} \alpha B(v)(x_x)$$

$$= \sup_{v \in \rho^{-1}(v') \cap (M - N), x \in \theta\theta^{-1}(\bar{y})} \alpha B(v)(x_x) -$$

$$\sup_{v \in \rho^{-1}(v') \cap (M - N), x \in \theta\theta^{-1}(\bar{y})} \alpha T(v)(x_x)$$

$$= \alpha\theta(T)(v')(\bar{y})$$

- ii. Similar to (i), if $v' \in \rho(N) - \rho(M)$, then we have $\alpha\theta(B)(v')(\bar{y}) = \alpha A(v')(\bar{y})$,

$$\beta\theta(B)(v')(\bar{y}) = \beta A(v')(\bar{y}).$$

- iii. If $v' \in \rho(M) \cap \rho(N)$, then

$$\sup_{\alpha\theta(B)(v')(\bar{y})} = \sup_{v \in \rho^{-1}(v') \cap (M \cup N), x \in \theta\theta^{-1}(\bar{y})} \alpha B(v)(x_x) -$$

$$= \sup_{v \in (\rho^{-1}(v') \cap M) \cup (\rho^{-1}(v') \cap N), x \in \theta\theta^{-1}(\bar{y})} \alpha B(v)(x_x) -$$

$$= \sup_{v \in (\rho^{-1}(v') \cap M) \cup (\rho^{-1}(v') \cap N), x \in \theta\theta^{-1}(\bar{y})} \alpha T(v)(x_x)$$

$$\sup_{v \in (\rho^{-1}(v') \cap M) \cup (\rho^{-1}(v') \cap N), x \in \theta\theta^{-1}(\bar{y})} \max\{\alpha T(v)(x_x), \alpha S(v)(x_x)\} -$$

$$\sup_{v \in (\rho^{-1}(v') \cap N) \cup (\rho^{-1}(v') \cap M), x \in \theta\theta^{-1}(\bar{y})} \alpha S(v)(x_x) -$$

$$= \max\left\{\sup_{v \in (\rho^{-1}(v') \cap M), x \in \theta\theta^{-1}(\bar{y})} \alpha T(v)(x_x), \sup_{v \in (\rho^{-1}(v') \cap N), x \in \theta\theta^{-1}(\bar{y})} \alpha S(v)(x_x)\right\}$$

$$= \max\{\alpha\theta(T)(v')(\bar{y}), \alpha\theta(S)(v')(\bar{y})\} = \alpha A(v')(\bar{y})$$

Similarly, we get $\beta\theta(B)(v')(\bar{y}) = \beta A(v')(\bar{y})$. Consequently, $((T,M) \cup \mathcal{E}(S,N)) = f(T,M) \cup \mathcal{E}f(S,N)$.

- (4) Let $((T,M) \cap \mathcal{E}(S,N)) = (T, M \cup N)$ and $f((T,M) \cap \mathcal{E}(S,N)) = (\theta\theta(T), \rho(M)) \cap \mathcal{E}(\theta\theta(S), \rho(N)) = (A, \rho(M) \cup \rho(N))$. Consequently, $f((T,M) \cap \mathcal{E}(S,N)) = (\theta\theta(B), \rho(M \cup N)) = (\theta\theta(B), \rho(M) \cup \rho(N))$. Assuming that $\theta\theta^{-1}(y) \neq \emptyset$, for any $y \in Y$ and $v' \in \rho(M) \cup \rho(N)$ then,

$\alpha A(v')(yy) = \alpha \theta(B)(v')(yy) = 0$ and $\beta A(v')(y) = \beta \theta(B)(v')(y) = 1$. Otherwise,



- i. If $v' \in \rho(M) - \rho(N)$ then, $A(v') = \theta\theta(T)(v')$. Conversely, $v' \in \rho(M) - \rho(N)$, implies that there doesn't exist $v \in N$ such that $\rho(v) = v'$. In other words, for any $v \in \rho^{-1}(v') \cap (M \cup N)$, we have $v \in \rho^{-1}(v') \cap (M - N)$. Therefore, according to Definition 4.1, we have

$$\begin{aligned} \alpha_{\theta(B)}(v')(y) &= \sup_{v \in \rho^{-1}(v') \cap (M \cap N), x \in \theta\theta^{-1}(\bar{y})} \alpha_B(v)(\bar{x}) \\ &= \sup_{v \in \rho^{-1}(v') \cap (M - N), x \in \theta\theta^{-1}(\bar{y})} \alpha_B(v)(\bar{x}) \\ &= \sup_{v \in \rho^{-1}(v') \cap (M - N), x \in \theta\theta^{-1}(\bar{y})} \alpha_T(v)(\bar{x}) \\ &\leq \min\{\sup_{v \in \rho^{-1}(v') \cap (M), x \in \theta\theta^{-1}(\bar{y})} \alpha_T(v)(\bar{x}), \\ &\quad \sup_{v \in \rho^{-1}(v') \cap (N), x \in \theta\theta^{-1}(\bar{y})} \alpha_S(v)(\bar{x})\} \\ &= \min\{\alpha_{\theta(T)}(v')(y), \alpha_{\theta(S)}(v')(y)\} \\ &= \alpha_A(v')(y) \end{aligned}$$

Similarly, we get $\beta_{\theta(B)}(v')(yy) \geq \beta_A(v')(y)$

Similar to (i), if $v' \in \rho(N) - \rho(M)$, then

we have $\alpha_{\theta(B)}(v')(y) = \alpha_A(v')(y)$,

$\beta_{\theta(B)}(v')(y) = \beta_A(v')(y)$.

- ii. If $v' \in \rho(M) \cap \rho(N)$, then

$$\begin{aligned} \alpha_{\theta(B)}(v')(y) &= \sup_{v \in \rho^{-1}(v') \cap (M \cap N), x \in \theta\theta^{-1}(\bar{y})} \alpha_B(v)(\bar{x}) \\ &= \sup_{v \in \rho^{-1}(v') \cap (M \cap N), x \in \theta\theta^{-1}(\bar{y})} \max\{\alpha_T(v)(\bar{x}), \alpha_S(v)(\bar{x})\} \\ &\leq \max\{\sup_{v \in \rho^{-1}(v') \cap (M), \bar{x} \in \theta\theta^{-1}(\bar{y})} \alpha_T(v)(\bar{x}), \\ &\quad \sup_{v \in \rho^{-1}(v') \cap (N), x \in \theta\theta^{-1}(\bar{y})} \alpha_S(v)(\bar{x})\} \\ &= \max\{\alpha_{\theta(T)}(v')(y), \alpha_{\theta(S)}(v')(y)\} \end{aligned}$$

$$= \alpha A(v')(y)$$

Similarly, we get $\beta\theta(B)(v')(yy) = \beta A(v')(\bar{y})$. Therefore $f((T, M) \cap \mathcal{E}(S, N)) \subseteq f(T, M) \cap \mathcal{E}(S, N)$.

(7) Let us assume that, $(T, M) \subseteq (S, N)$. Then, $M \subseteq N$ and for any $v \in M$ and $x \in X$ we get $(M) \subseteq (N)$. We have,



$$\begin{aligned}
& \sup_{v \in \rho^{-1}(v') \cap (M \setminus N), x \in \theta^{-1}(y)} \alpha_T(v)(x), & \text{if } \theta^{-1}(y) \neq \emptyset, \\
& 0, & \text{otherwise} \\
& = \alpha_{\theta(S)}(v')(y) & \text{if } \theta^{-1}(y) \neq \emptyset \\
& & \text{otherwise}
\end{aligned}$$

Similarly, we have, $\beta_{\theta(T)}(v')(y) \geq \beta_{\theta(S)}(v')(y)$. Therefore, $f(T, M) \subseteq f(S, N)$.

Proposition 3.4

Assume that $f: \mathcal{QTFSS}(X, V) \rightarrow \mathcal{QTFSS}(Y, V')$ is a Quin-Terranean fuzzy soft mapping. Then for Quin-Terranean fuzzy soft sets (T, M) and (S, N) in $\mathcal{QTFSS}(X, V)$, we obtain

- (1) $f^{-1}(0(X, V)) = 0(Y, V')$,
- (2) $f^{-1}(1(X, V)) \subseteq 1(Y, V')$,
- (3) $f^{-1}((T, M) \cup \mathcal{E}(S, N)) = f^{-1}(T, M) \cup \mathcal{E}f^{-1}(S, N)$,
- (4) $f^{-1}((T, M) \cap \mathcal{E}(S, N)) \subseteq f^{-1}(T, M) \cap \mathcal{E}f^{-1}(S, N)$,
- (5) If $(T, M) \subseteq (S, N)$ then $f^{-1}(T, M) \subseteq f^{-1}(S, N)$,
- (6) $(T, M) \subseteq f^{-1}(f(T, M))$, then $f^{-1}(f(S, N)) = (S, N) \cap \mathcal{E}f(1X)$ Proof : Simple and direct.

Definition 3.5

Let (X, τ_1, V) and (Y, τ_2, V') be two Quin-Terranean fuzzy soft topological spaces, a Quin-Terranean fuzzy soft mapping $f = (\theta, \rho): \mathcal{QTFSS}(X, V) \rightarrow \mathcal{QTFSS}(Y, V')$ is called a "Quin-Terranean fuzzy soft continuous" if $f^{-1}(S, N) \in \tau_1$ for all $(S, N) \in \tau_2$.

Example 3.6

Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $V = \{v_1, v_2\}$ and $V' = \{v'_1, v'_2\}$. $\tau_1 = \{0_X, 1_X, (T_1, V), (T_2, V)\}$ where $(T_1, V), (T_2, V)$ Quin-Terranean fuzzy soft sets over X , defined as follows,

$$(T_1, V) = \{(v_1, \{(x_1, 0.78, 0.80), (x_2, 0.80, 0.70)\})\}$$

$$(T_2, V) = \{(v_1, \{(x_1, 0.65, 0.85), (x_2, 0.80, 0.90)\})\}$$

Then τ_1 is a Quin-Terranean fuzzy soft topology over X and hence (X, τ_1, V) is a Quin-Terranean fuzzy soft topological space.

$\tau_2 = \{0_{Y'}, 1_{Y'}, (S_1, V'), (S_2, V')\}$ where $(S_1, V'), (S_2, V')$ Quin-Terranean fuzzy soft sets over Y , defined as follows,

$$\bar{(S1,V)} \quad \begin{aligned} & (v'_1, \{(-y_1, 0.80, 0.70), (-y_2, 0.90, 0.80)\}) \\ & \{(v'_2, \{(y_1, 0.78, 0.80), (y_2, 0.80, 0.70)\})\} \end{aligned}$$



$$\begin{aligned} &= (v'_1, \{(y_1, 0.80, 0.70), (y_2, 0.65, 0.85)\}) \\ (S_2, V) &= \{(v'_2, \{(y_1, 0.65, 0.85), (y_2, 0.80, 0.90)\})\} \end{aligned}$$

Then τ_2 is a Quin-Terranean fuzzy soft topology over X and hence (Y, τ_2) is a Quin-Terranean fuzzy soft topological space. If $f = (\theta, \rho): QTFSS(X, V) \rightarrow QTFSS(Y, V')$ defined as follow:

$$\bar{f}(x) = y_2 \quad \rho(v_1) = v'_1$$

$$\bar{f}(x) = y_1 \quad \rho(v_2) = v'_2$$

Then it is easy to verify that $f^{-1}(S, V') \in \tau_1$ for all $(S, V') \in \tau_2$. Thus $f = (\theta, \rho)$ is a Quin-Terranean fuzzy soft continuous mapping.

Theorem 3.7

Let (X, τ_1) and (Y, τ_2, V') be two Quin-Terranean fuzzy soft topological spaces, $f = (\theta, \rho): QTFSS(X, V) \rightarrow QTFSS(Y, V')$ is a Quin-Terranean fuzzy soft continuous mapping. If $(S, V') \in QTFSS(Y, V')$ is a Quin-Terranean fuzzy soft closed set, then $f^{-1}(S, V') \in QTFSS(X, V)$ is a Quin-Terranean fuzzy soft closed set.

Proof

Assume that $f: QTFSS(X, V) \rightarrow QTFSS(Y, V')$ is a Quin-Terranean fuzzy soft continuous mapping on the Quin-Terranean fuzzy soft topological space (X, τ_1, V) and let (S, V') be any Quin-Terranean fuzzy soft closed set in Y . Then $f^{-1}((S, V')^c) = (f^{-1}(S, V'))^c$ and $(S, V')^c$ is a Quin-Terranean fuzzy soft open set, the following results are obtained. Consequently, $(f^{-1}(S, V'))^c$ is a Quin-Terranean fuzzy soft open set in X which implies, $f^{-1}(S, V')$ is a Quin-Terranean fuzzy soft closed set in X .

Conversely, let's assume that $f^{-1}(S, V')$ is a Quin-Terranean fuzzy soft closed set in X whenever (S, V') is a Quin-Terranean fuzzy soft closed set in Y . In Y , for every Quin-Terranean fuzzy soft open set (T, V') , $f^{-1}((T, V')^c) = (f^{-1}(T, V'))^c$. According to the hypothesis, $f^{-1}((T, V')^c)$ is a Quin-Terranean fuzzy soft closed set in X . Therefore $f^{-1}(T, V')$ is a Quin-Terranean fuzzy soft continuous mapping on a Quin-Terranean fuzzy soft topological space (X, τ_1, V) .

Theorem 4.2

Let (X, τ_1, V) and (Y, τ_2, V') be two Quin-Terranean fuzzy soft topological spaces and

$f = (\theta, \rho) : QTFSS(X, V) \rightarrow QTFSS(Y, V')$ be a Quin-Terranean fuzzy soft continuous mapping. Then f is a Quin-Terranean fuzzy soft continuous mapping on Quin-Terranean fuzzy soft topological space (X, τ_1, V) if and only if $f^{-1}(Q_{tint}(S, V')) \subseteq Q_{tint}(f^{-1}(T, V'))$ for each $(S, V') \in QTFSS(Y, V')$.

Proof

Let $(S, V') \in QTFSS(Y, V')$ and f be a Quin-Terranean fuzzy soft continuous mapping. Accordingly, $f^{-1}(Q_{tint}(S, V')) \in \tau_1$ and from $Q_{tint}(S, V') \subseteq (S, V')$ we have $f^{-1}(Q_{tint}(S, V')) \subseteq f^{-1}(S, V')$.



$(S, V') \subseteq f^{-1}(S, V')$. Since $Q_{\text{tint}}(f^{-1}(S, V'))$ is the largest Quin-Terranean fuzzy soft open set that contains $f^{-1}(S, V')$, $f^{-1}(Q_{\text{tint}}(S, V')) \subseteq Q_{\text{tint}}(f^{-1}(S, V'))$.

Conversely, assuming that for every $(S, V') \in Q\mathcal{TFSS}(Y, V')$, $f^{-1}(Q_{\text{tint}}(S, V')) \subseteq$

$Q_{\text{tint}}(f^{-1}(S, V'))$. Assuming $(S, V') \in \tau_2$, we obtain $f^{-1}(S, V') = f^{-1}(Q_{\text{tint}}(S, V')) \subseteq Q_{\text{tint}}(f^{-1}(S, V')) \subseteq f^{-1}(S, V')$. Therefore, $f^{-1}(S, V') \in \tau_1$. This implies that f is a Quin-Terranean fuzzy soft continuous mapping.

Theorem 3.8

Let (X, τ_1) and (Y, τ_2, V') be two Quin-Terranean fuzzy soft topological spaces and let $f : Q\mathcal{TFSS}(X) \rightarrow Q\mathcal{TFSS}(Y, V')$ be a Quin-Terranean fuzzy soft continuous mapping. Then, $f = (\theta, \rho)$ is a Quin-Terranean fuzzy soft continuous mapping on a Quin-Terranean fuzzy soft topological space (X, τ_1, V) if and only if $f(Q\mathcal{TCI}(T, V)) \subseteq Q\mathcal{TCI}(f(T, V))$ for $(T, V) \in Q\mathcal{TFSS}(X, V)$.

Proof

Let f be a Quin-Terranean fuzzy soft continuous mapping on Quin-Terranean fuzzy soft topological space (X, τ_1, V) and $(T, V) \in Q\mathcal{TFSS}(X, V)$. Since $Q\mathcal{TCI}((T, V))$ is a Quin-Terranean fuzzy soft closed set in Y , $f^{-1}(Q\mathcal{TCI}(f(T, V)))$ is a Quin-Terranean fuzzy soft closed set in X . Hence, $(f^{-1}(Q\mathcal{TCI}(f(T, V)))) = f^{-1}(Q\mathcal{TCI}(f(T, V)))$ and $f(T, V) \subseteq Q\mathcal{TCI}(f(T, V))$.

Consequently, we obtain $(T, V) \subseteq f^{-1}(f(T, V)) \subseteq f^{-1}(Q\mathcal{TCI}(f(T, V)))$ then we have, $(T, V) \subseteq Q\mathcal{TCI}(f^{-1}(Q\mathcal{TCI}(f(T, V)))) \subseteq f^{-1}(Q\mathcal{TCI}(f(T, V)))$. Accordingly, $f(Q\mathcal{TCI}(T, V)) \subseteq Q\mathcal{TCI}(f(T, V))$.

Conversely, assume that $(Q\mathcal{TCI}(T, V)) \subseteq Q\mathcal{TCI}(f(T, V))$ for every $(T, V) \in Q\mathcal{TFSS}(X, V)$ and (S, V') is any Quin-Terranean fuzzy soft closed set in Y . Hence, $Q\mathcal{TCI}(S, V') = (S, V')$. From the result of the hypothesis, $f(Q\mathcal{TCI}(f^{-1}(S, V'))) \subseteq Q\mathcal{TCI}(f(f^{-1}(S, V'))) \subseteq Q\mathcal{TCI}(S, V') = (S, V')$. Thus, $Q\mathcal{TCI}(f^{-1}(S, V')) = f^{-1}(S, V')$ and $f^{-1}(S, V') \subseteq Q\mathcal{TCI}(f^{-1}(S, V'))$. This means that $Q\mathcal{TCI}(f^{-1}(S, V')) = f^{-1}(S, V')$ and hence, $f^{-1}(S, V')$ is a Quin-Terranean fuzzy soft closed set in X . Consequently, f is a Quin-Terranean fuzzy soft continuous mapping on a Quin-Terranean fuzzy soft topological space (X, τ_1, V) .

4. CONCLUSION

The concept of Quin-Terranean fuzzy set which is extension of fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set and soft set theories are all crucial mathematical instruments for handling uncertainties. Presented in this study are Quin-Terranean fuzzy soft continuous spaces. Next, we looked into the concepts of Quin-Terranean fuzzy soft closure and Quin-Terranean fuzzy soft interior. We also define the pre-image and image of Quin-Terranean fuzzy soft sets.

5. REFERENCES

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