



# Heat Transfer Due To Permeable Stretching Wall In Presence Of Transverse Magnetic Field With Heat Generation / Absorption.

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## Abstract

Exact similarity solution for the viscous flow due to stretching surface in the presence of magnetic field is derived. Adopting the similarity transformation, governing nonlinear partial differential equations of the problem are transformed to nonlinear ordinary differential equations. Then the numerical solution of the problem is derived using Quasilinearization of Newton's method. Two cases of heat transfer are considered. The sheet (a) with prescribed surface temperature and (b) the prescribed wall heat flux. Both the cases are further extended to study the heat transfer due to suction and injection. A simple relation for the two cases of heat transfer is obtained.

**Keywords:** Magnetic field, Heat source, Heat absorption, Heat flux, Quasilinearization of Newton's method.

## **1. Introduction:**

Flow problem with obvious relevance to polymer extrusion is an interesting area of present-day research. In a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by unidirectional orientation to the extrudate, the by improving its mechanical properties and the quality of the final product greatly depends on the rate of cooking. Crane [1] studied the two-dimensional boundary layer flow caused by the stretching of the sheet which moves in its own plane at a velocity that varies linearly with the distance from the slit. This problem was extended to heat and mass transfer with suction or blowing by Gupta and Gupta [2] who studied the temperature and concentration distributions for isothermal case. Dutta, Roy and Gupta [3] analyzed the temperature distribution in the flow over a stretching sheet with uniform heat flux. Grubka and Bobba [4] studied the heat transfer characteristics of a continuous stretching surface with variable temperature. Further study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid due to the stretching of the sheet is of considerable interest in modern metallurgical and metal-working process. To be more specific, it

may be mentioned that many metallurgical processes involve cooling of the continuous strips or filaments by drawing them through a quiescent fluid and that in process of the drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing and thinning of copper wires. In all these cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final products of the desired characteristics might be achieved. Pavlov [5] presented an exact similarity solution of the boundary-layer equation for the steady two-dimensional flow of a surface in a uniform transverse magnetic field. In this analysis he has neglected the induced magnetic field under the assumption of small magnetic Reynolds number. Anderson [6] has demonstrated that the similarity solution derived by Pavlov [5] is not only a boundary layer equation but also represents an exact solution of the complete Navier-Stokes equation. Chakrabarti and Gupta [7] extended the above analysis to Pavlov to study the temperature distribution for isothermal boundary when a uniform suction is applied at the surface. It would be of interest to study the effects of power-law variations of temperature and heat flux distribution on the heat transfer characteristics of stretching sheet in presence of a uniform transverse magnetic field subject to suction and blowing. B.S Dandapat, S.N. Singh, R.P. Singh [9] has studied the heat transfer on a stretching sheet in presence of a transverse magnetic field with suction and blowing for different types of thermal boundary conditions on the surface.

Here we propose to study the heat transfer due to a permeable stretching wall in presence of transverse magnetic field with heat generation / absorption.

## 2. Mathematical formulation.

Consider the flow of an incompressible electrically conducting fluid (with electrical conductivity  $\sigma$  and thermal diffusivity  $\alpha$ ) due to the stretching of a permeable flat sheet. It is assumed that the speed of a point on the sheet is proportional to its distance from the slit at  $x=0, y=0$ . Further we assume that the sheet lies in the  $x-z$  plane and is stretched along the  $x$  axis. A uniform transverse magnetic field  $B_0$  acts parallel to the  $y$  axis and the conducting fluid occupies the half space  $y > 0$ . We have also assumed that the sheet is subjected to either (a) prescribed surface temperature or (b) prescribed wall heat flux. The steady velocity field  $[u(x, y), v(x, y), 0]$  that developed due to the stretching of the sheet with velocity  $Ax$  satisfies the boundary-layer equation of mass, momentum and thermal energy.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots\dots\dots (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \left( \frac{\sigma B_0^2}{\rho} \right) u \quad \dots\dots\dots (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + Q(T_\infty - T) \quad \dots\dots\dots (3)$$

where the induced magnetic field is neglected by assuming the flow for small magnetic Reynolds number, as justified by Shercliff [8]. It is also assumed that the external electrical field due to polarization of charges is negligible. Further we assume that  $\nu$ , the kinematic viscosity of the ambient fluid, is constant and the gravity force gives rise to a hydrostatic pressure variation in the liquid. In order to justify

the boundary layer approximation, length scale in the primary flow direction should be significantly larger than the length scale in the cross stream direction. In fact, the flow takes place within a thin layer of thickness  $(\nu/A)^{1/2}$  due to the stretching of the sheet, the scale ratio  $x/(\nu/A)^{1/2} \gg 1$ . Further it is possible to define a local Reynolds number  $Re_x = Ux/\nu = Ax^2/\nu$  which initially equals the square of the above scale ratio. Thus, just as in aerodynamic boundary layer theory, cross-stream diffusion of momentum and thermal energy can only be neglected at a high Reynolds number. The corresponding boundary conditions are:

$$u(x, 0) = Ax, \quad v(x, 0) = V_w(x), \quad u(x, \infty) = 0 \quad \dots\dots\dots(4)$$

$$\text{either } T_w(x, 0) = T_1(x), \quad T(x, \infty) = T_\infty \quad \dots\dots\dots(5)$$

$$\text{or } -k \frac{\partial T}{\partial y} = q_w(x) \text{ for } y = 0, \quad T(x, \infty) = T_\infty \quad \dots\dots\dots(6)$$

where  $V_w$  denotes the lateral mass flux of velocity which occurs due to suction or injection.  $T_w$ ,  $T_\infty$  and  $q_w$  denote the temperature at the wall, temperature at a large distance from the wall and heat flux at the wall, respectively. Assuming the functional structure of the similarity solutions in the form

$$u(x, y) = Ax f'(\eta) \quad \dots\dots\dots(7)$$

$$v(x, y) = -(\nu A)^{1/2} f(\eta) \quad \dots\dots\dots(8)$$

$$\eta = (A/\nu)^{1/2} y \quad \dots\dots\dots(9)$$

and substituting eqn (7) to (9) in the system of equations eqn (1) to (3), it can be shown that similarity solution of the above set of boundary equations exists and reduces to

$$f''' + ff'' - (f')^2 - mf' = 0 \quad \dots\dots\dots(10)$$

Subject to the boundary conditions

$$f(0) = f_w, \quad f'(0) = 1, \quad f'(\infty) = 0 \quad \dots\dots\dots(11)$$

Where  $m \equiv \sigma \beta_0^2 / A \rho$  and  $f_w \equiv -V_w / (\nu A)^{1/2}$  denote the magnetic parameter, suction/injection

parameter, respectively and prime (') denotes derivative with respect to

The similarity variable  $\eta$ . in eqn (11),  $f_w = 0$  corresponds to an impermeable wall,

$f_w > 0$  and  $f_w < 0$  denote respectively the suction and injection of the fluid through the permeable wall.

**Heat transfer:**

In this section we are interested to study the heat transfer for two different heating processes.

**Prescribed surface temperature:**

We assign a general functional structure in eqn (5) to prescribe the temperature at the boundary as

$$T_w = T_1(x) = T_\infty + Cx^r \quad \text{for } y = 0 \quad \dots\dots\dots(12)$$

$$\text{and } T = T_\infty \text{ as } y \rightarrow \infty, \quad \dots\dots\dots(13)$$

where  $r$  is the temperature parameter and  $T_\infty$  denote temperature at a large distance from the wall and it is constant. For  $r = 0$ , the thermal boundary will be isothermal.

Introducing

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \text{ in equation (3.2.3) and using eqn (9) we get}$$

$$\theta'' + p_r f \theta' + p_r [\beta - r f'] \theta = 0 \quad \dots\dots\dots (14)$$

Where  $p_r = \frac{\nu}{\alpha}$  is the Prandtl number. The corresponding boundary conditions (5) reduces to

$$\theta(0) = 1, \quad \theta(\infty) = 0. \quad \dots\dots\dots (15)$$

**Prescribed wall heat flux:**

In this case the thermal boundary conditions will be

$$-k \frac{\partial T}{\partial y} = q_w = Dx^s \quad \text{for } y = 0 \quad \dots\dots\dots(16)$$

And

$$T = T_\infty \quad \text{as } y \rightarrow \infty \quad \dots\dots\dots(17)$$

where  $s$  is the heat flux parameter. For  $s = 0$  the stretching sheet is under uniform heat flux. We assume the similar solution as

$$T = T_\infty + \frac{Dx^s}{K} \sqrt{\frac{\nu}{A}} g(\eta) \quad \dots\dots\dots (18)$$

Where  $\eta = (A/\nu)^{1/2} y$  using eqn (1) and (2) in (3) we get

$$g'' + p_r f g' + p_r [\beta - s f'] g = 0 \quad \dots\dots\dots (19)$$

The corresponding boundary conditions (2) and (3) reduces to

$$g'(0) = 1, \quad g(\infty) = 0. \quad \dots\dots\dots (20)$$

**3. Method of solution**

Eqs. (10), (14) and (9) with boundary conditions (11), (15) and (19) is solved using Quasilinearization of Newton's method.

**4. Results and discussion**

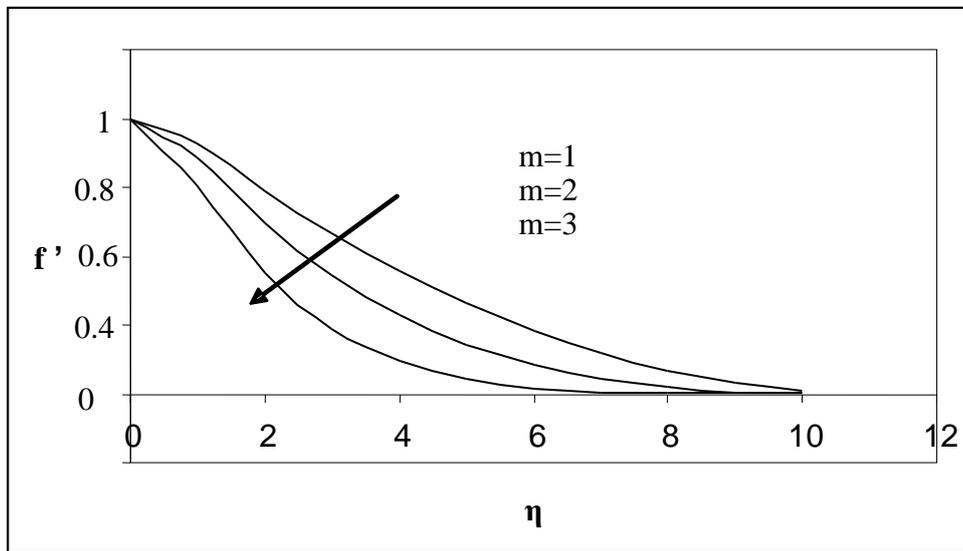
To get insight into the problem, a few figures are drawn by evaluating Eqs (1) for different values of the parameters. It is clear from the Figure (1) that when magnetic parameter increases either for suction or injection  $f'(\eta)$  decreases for all values of  $\eta$ . Further Figure (2) represents the variation of  $f'(\eta)$  with respect to  $\eta$  for different values of suction, injection and magnetic parameters. It is clear that as suction/ injection parameter increases,  $f'(\eta)$  increases. This is due to the fact that the increase of magnetic parameter implies the increase of Lorentz force which puts greater resistance to the flow, and as a result the flow decelerates. Figures (3) and (4) shows the temperature variation with  $\eta$  for different values of Prandtl number and the temperature parameter. Figure (3) shows that with increasing Prandtl number the

temperature profile decreases in PST case and Figure (4) shows that with increasing temperature parameter temperature profile decreases in PST case. For  $r > 0$  heat flows from the stretching surface to the ambient fluid and for  $r < 0$  the wall temperature gradient is positive and heat flows into the stretching surface from the ambient fluid. And from the figure (5) it is observed that as the magnetic parameter increase the temperature profile in PST case increases, and from the figure (6) we can notice the variation of temperature in PST case with similarity variable  $\eta$  for different values of source / sink parameter  $\beta$ . The presence of heat source  $\beta > 0$  in the boundary layer generates the energy, which causes the temperature of the fluid to increase, whereas, the presence of heat sink  $\beta < 0$  in the boundary layer leads to absorption of energy, which cause a decrease in the temperature of the fluid.

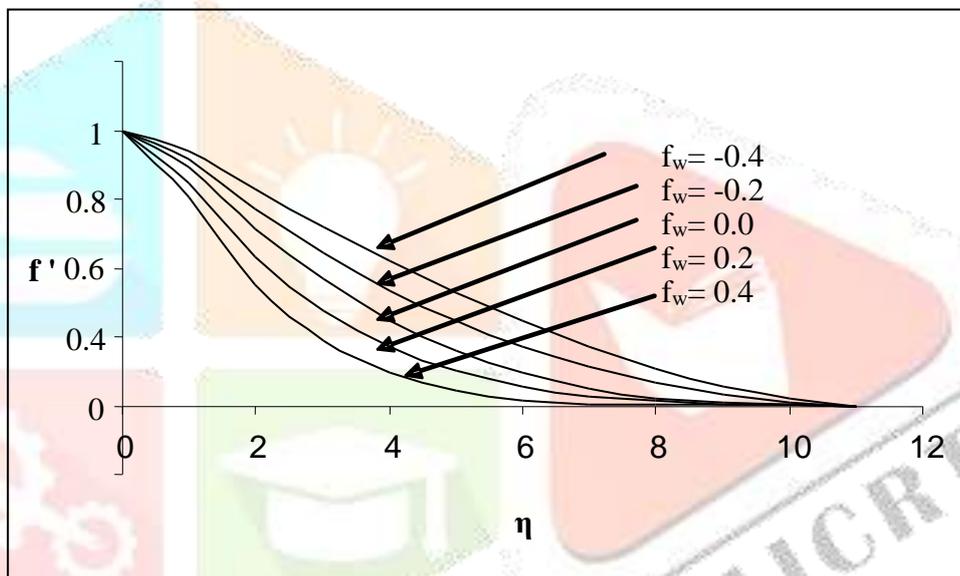
Figures (6) – (10) show the effect of Prandtl number, magnetic parameter, heat flux parameter, and heat source or sink respectively on temperature in PHF case. Through these figures we can clearly observe that the effect of these parameters on temperature in PHF is same as that of these parameter effects in PST case.

## 5. Conclusion:

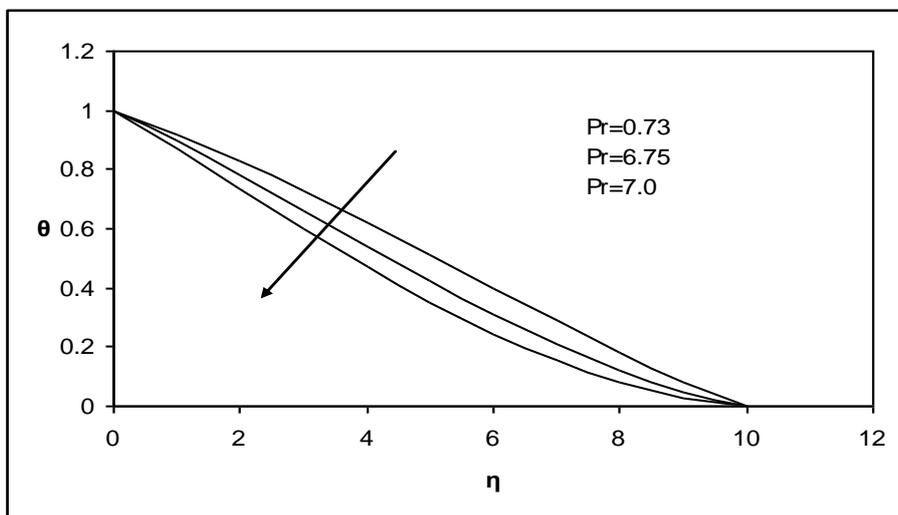
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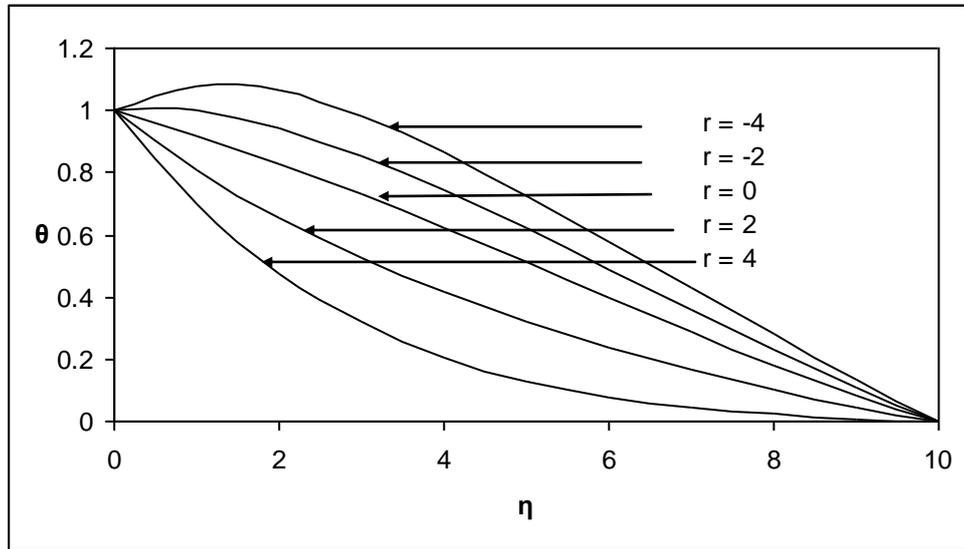
Fig(1) Effect of magnetic parameter on velocity field.



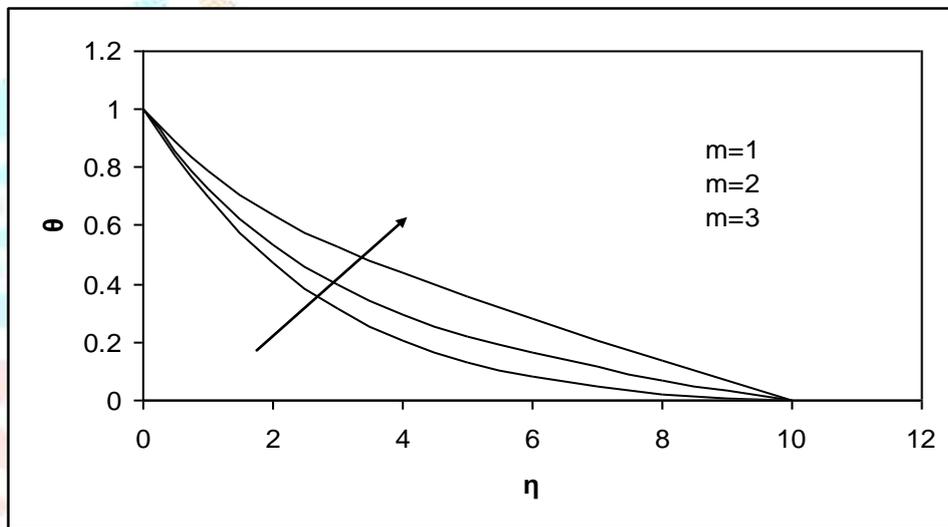
Fig(2) Effect of suction/injection parameter on velocity field.



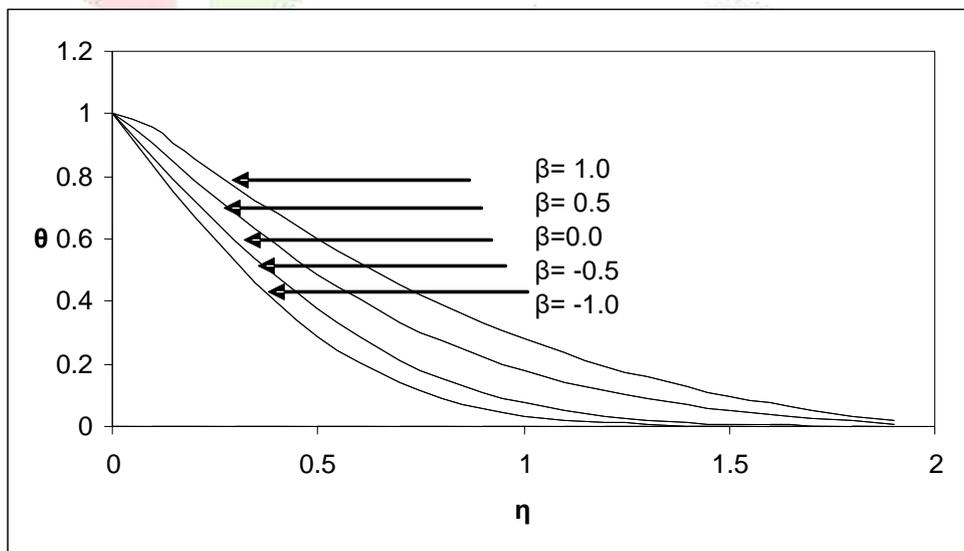
Fig(3) Effect of Prandtl number on temperature profile in PST case.



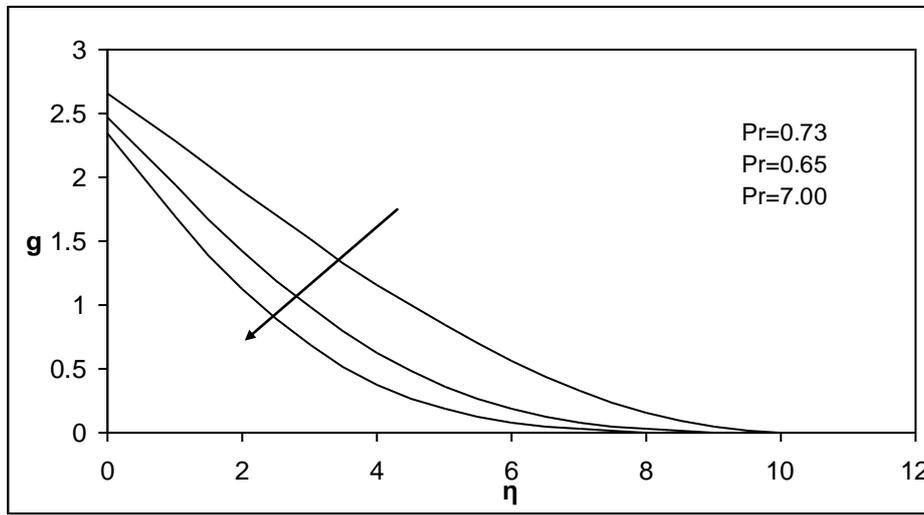
Fig(4) Effect of temperature parameter on temperature profile in PST case.



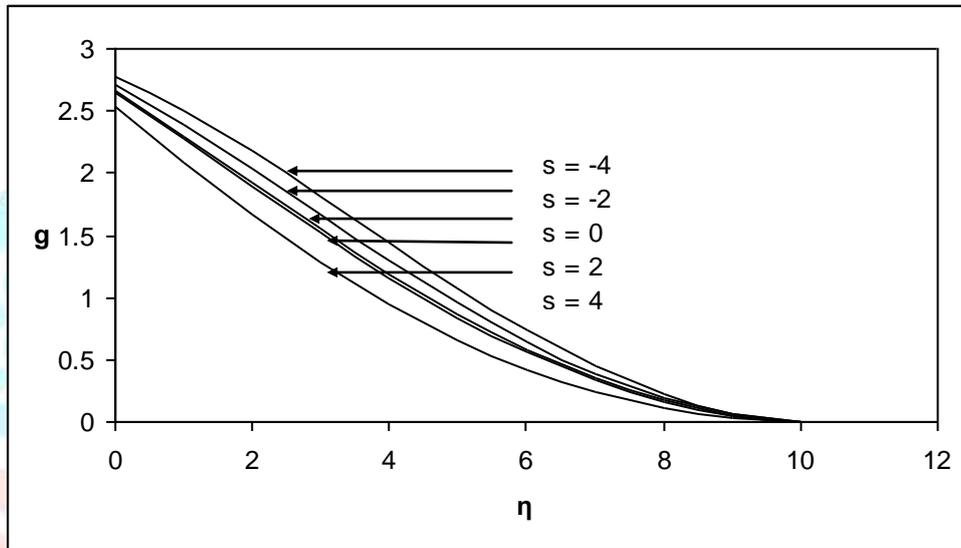
Fig(5) Effect of magnetic parameter on temperature profile in PST case.



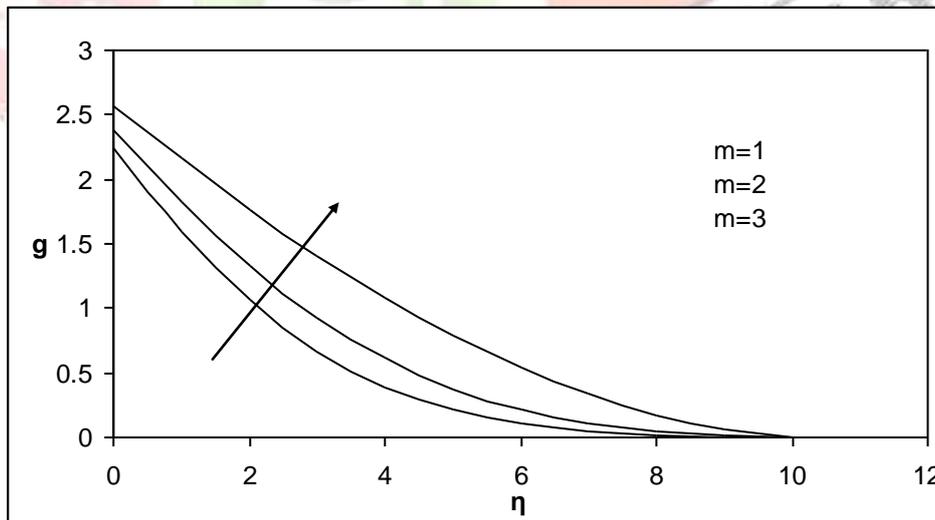
Fig(6) Effect of source/sink parameter on temperature profile in PST case.



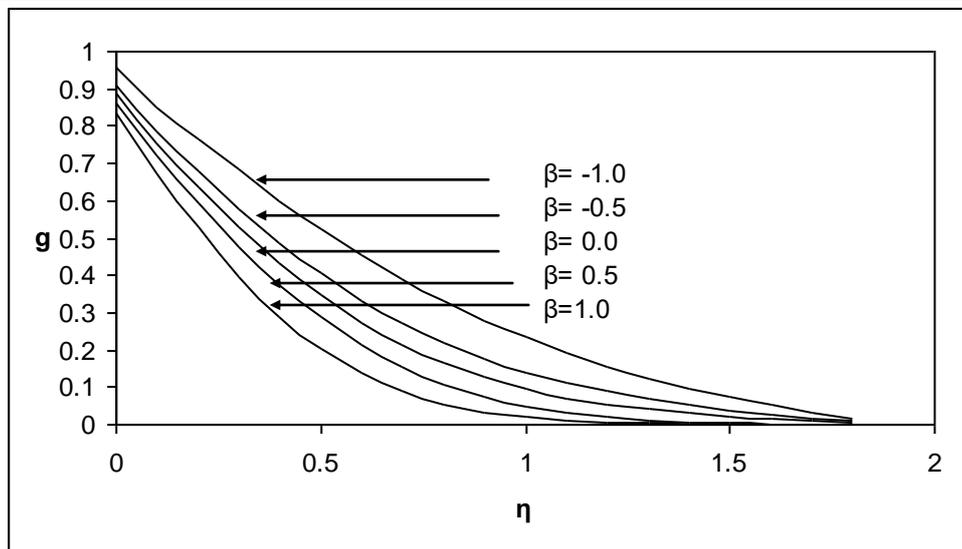
Fig(7) Effect of Prandtl number on temperature profile in PHF case.



Fig(8) Effect of heat flux parameter on temperature profile in PHF case.



Fig(9) Effect of magnetic parameter on temperature profile in PHF case.



Fig(10) Effect of source/sink parameter on temperature profile in PHF case.

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