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Fixed Point Results For Interval-Valued Intuitionistic Fuzzy Metric Space With Applications

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Abstract: In this paper, we introduced a new notion which is called interval-valued intuitionistic fuzzy metric space and examined some basic properties. The purpose of the paper is to obtain fixed point results on interval-valued intuitionistic fuzzy metric space

***keywords* -** Interval-valued fuzzy set(IVFS), Interval-valued fuzzy metric space(IVFMS), Intuitionistic fuzzy set, Intuitionistic fuzzy metric space (IFMS), Interval-valued Intuitionistic fuzzy metric space (IVIFMS).

I. Introduction

The idea of an IVIFMS has been introduced in this study. The idea of fuzzy sets was first presented by Zadeh [10] in 1965, and Since then, several authors have looked at the properties of fuzzy metric space in various ways. Intuitionistic fuzzy sets are a generalization of fuzzy sets that were first described by Atanassav [1] in 1986. Since then, the research of intuitionistic fuzzy sets has advanced significantly. To generalize fuzzy metric space, Park [6] defined IFMS through the use of Continuous τ -norm and τ -conorm. A fuzzy set that is intuitionistic in nature is determined by the degree of both membership and non-membership. In 1975, Zadeh proposed a concept of an IVFS. IVFS which is defined by an interval-valued membership function is another generalization of the fuzzy set notion. Shen, Li, and Wang [7] first proposed the idea of IVFMS with the interval-valued τ -norm (Continuous) in 2012, which was a generalization of the George and Veeramani [3] fuzzy metric space concept. Recently Sewani et.al [9] generalized Intuitionistic Fuzzy b-Metric Space. Sewani [8] constructed some new results in IFMS by applying the abstraction of fuzzy iterated contraction

II. Preliminaries

Definition 2.1 Let G is a non-empty set. The mapping $\mathfrak{R} : G \to [I]$ is referred to as an IVFS on G. if $\mathfrak{R} \in IVF(G)$,

Let $\mathfrak{R}(\rho)= [\, \mathfrak{R}^-(\rho), \mathfrak{R}^+(\rho),]$, $\mathfrak{R}^-(\rho), \leq \mathfrak{R}^+(\rho)$, for all $\rho \in G$, then the conventional fuzzy set $\mathfrak{R}^- : G \to [I]$ and $\mathfrak{R}^+: G \to [I]$ are referred to as the Lower Fuzzy Set and Upper Fuzzy Set respectively. In particular, $\mathfrak{R}$ is referred to as a degenerate fuzzy set if $\mathfrak{R}^-(\rho) = \mathfrak{R}^+(\rho)$,for any $\rho \in G$.

Definition 2.2 An iv-τnorm is a binary operation of the form ∗I : [I] x [I]→ [I] on [I]. such that the four conditions

listed below are satisfied for all σ , ς, ν ,υ∈ [I]:

(V1) Commutativity: $\sigma *_I \varsigma = \varsigma *_I \sigma$,

(V2) Associativity: $\sigma *_I [\varsigma *_I \nu] = [\sigma *_I \varsigma] *_I \nu$,

(V3) Monotonicity: $\sigma *_I \varsigma \le \nu *_I \upsilon$ whenever $\sigma \le \nu$, and $\varsigma \le \upsilon$

(V4) Boundary Condition: $\sigma *_I \bar{1} = [\sigma^-, \sigma^+] *_I [0, 1] = [\sigma^+, \bar{1}]$

Definition 2.3: An iv-τconorm is a binary operation of the form ⊛I : [I] x [I]→ [I] on [I]. such that the four conditions

listed below are satisfied σ , ς, ν ,υ∈ [I]:

(V5) Commutativity: $\sigma \circledast_I \varsigma = \varsigma \circledast_I \sigma$,

(V6) Associativity: $\sigma \circledast_I [\varsigma \circledast_I \nu] = [\sigma \circledast_I \varsigma] \circledast_I \nu$,

(V7) Monotonicity: $\sigma \circledast_I \varsigma \le \nu \circledast_I \upsilon$ whenever $\sigma \le \nu$, and $\varsigma \le \upsilon$

(V8) Boundary Condition: $\sigma *_I \bar{0} = [\sigma^-, \sigma^+] *_I [0, 1] = [\bar{0}, \sigma^-]$

Definition 2.4: Let $\{\alpha_n^-\} = \{[[\alpha^-, \alpha^+]\}$, η ∈ N + be a sequence of interval numbers in [I], $\bar{\alpha} = [\alpha^-, \alpha^+] \in [I]$,

if limη→∞ $\alpha_n^- = \alpha^-$, and limη→∞ $\alpha_n^+ = \alpha^+$, , then we say that the Ⓢequence $\{\alpha_n^-\}$ is convergent to $\bar{\alpha}$, and which is denoted by limη→∞ $\alpha_n^- = \bar{\alpha}$.

Definition 2.5: An iv-τnorm $*_I$ is continuous iff it is continuous in the initial part, i.e., for each ς ∈ [I], if limη→∞ $\bar{\sigma}_n = \bar{\sigma}$

then limη→∞ $(\bar{\sigma}_n *_I \bar{\varsigma}) = \bar{\sigma} *_I \bar{\varsigma})$, where $\{\bar{\sigma}_n\} \subseteq [I]$, $\bar{\sigma} \in [I]$.

Definition 2.6: An iv-τconorm $\circledast_I$ is continuous iff it is continuous in the initial part, i.e., for each σ ∈ [I], if limη→∞ $\bar{\sigma}_n = \bar{\sigma}$

then limη→∞ $(\bar{\sigma}_n \circledast_I \bar{\varsigma}) = \bar{\sigma} \circledast_I \bar{\varsigma})$, where $\{\bar{\varsigma}_n\} \subseteq [I]$, $\bar{\varsigma} \in [I]$.

Definition 2.7: The triple (G, k, $*_I$) is know as IVFMS, if G represent an arbitrary set., , $*_I$ is a continuous interval-valued τ -norm on [I] and is an interval-valued fuzzy set on $G^2 \times (0,\infty)$ fulfills the requirements listed below:

(C1) k(ρ, ϱ, τ) > $\bar{0}$,

(C2) k(ρ, ϱ, τ) = [$\bar{1}$ if and only if ρ = ϱ,

(C3) k(ρ, ϱ, τ) = k(ϱ, ρ, τ),

(C4) k(ρ, ϱ, τ) , $*_I$ k(ρ ,r, s) ≤ k(ρ, r, τ + s),

(C5) k(ρ, ϱ,.):(0,∞)→ [I] is continuous,

(C6) limη→∞ k(ρ, ϱ, τ) = [$\bar{1}$, where ρ, ϱ ,r ∈ G and τ ,s > 0.

Definition 2.8: A 5 tuple (G, k, ℏ, $*_I$, $\circledast_I$) is said to be an IVIFMS if G is arbitrary set, , $*_I$ is Ⅽontinuous iv-τnorm on [I], $\circledast_I$ is Ⅽontinuous iv-τconorm on [I] and k, ℏ IVFS on $G^2 \times (0,\infty)$ satisfying the

following requirements:

(R1) k(ρ, ϱ ,τ) + ℏ(ρ, ϱ, τ) ≤ $\bar{1}$,

(R2) k(ρ, ϱ, $\bar{0}$) = $\bar{0}$,

(R3) k(ρ, ϱ, τ) = $\bar{0}$ for all τ > 0 iff ρ = ϱ ,

(R4) k(ρ, ϱ, τ) = k(ϱ, ρ, τ) ,

(R5) k(ρ, ϱ, τ) , $*_I$ k(ϱ, r, s) ≤ k(ρ, r, τ + s) ,

(R6) k(ρ, ϱ,.) : (0,∞)→ [I] is left continuous ,

(R7) limη→∞ k(ρ, ϱ, τ) = $\bar{1}$, where ρ, ϱ ∈ G ,

(R8) ℏ(ρ, ϱ, $\bar{0}$) = $\bar{1}$,

(R9) ℏ(ρ, ϱ, τ) = $\bar{0}$ for all τ > 0 iff ρ = ϱ,

(R10) ℏ(ρ, ϱ, τ) = ℏ(ϱ, ρ, τ),

(R11) ℏ(ρ, ϱ, τ) $\circledast_I$ ℏ(ϱ, r, s) ≥ ℏ(ρ, r, τ + s) , (R12) $\hbar(\rho, \varrho,.) : (0,\infty)\to$ [I] is right continuous ,

(R13) $\lim_{\eta\to\infty} \hbar(\rho, \varrho, \tau) = \bar{0}$, where $\rho, \varrho \in G$.

Example1 : Let $G = \mathfrak{R}^+$ be the set of non-negative real number. k, $\hbar$ are on $G^2 \times (0,\infty)$ defined by

$k(\rho, \varrho, \tau) = \frac{\tau}{\tau+£|\rho-\varrho|}$ and $\hbar(\rho, \varrho, \tau) = \frac{£|\rho-\varrho|}{(\tau+\upsilon)+£|\rho-\varrho|}$ for all ρ, ϱ in G. Then $(G, k, \hbar, *_I, \circledast_I)$ be an IVIFMS,

where £=1.

Proof : We start to prove (R5) and (R11)

$$\Rightarrow k(\rho, \varrho, \tau) *_I k(\varrho, \sigma, \upsilon) = \frac{\tau}{\tau+£|\rho-\varrho|} *_I \frac{\upsilon}{\upsilon+£|\varrho-\sigma|}$$

$$= \frac{1}{1+\frac{£|\rho-\varrho|}{\tau}} *_I \frac{1}{1+\frac{£|\varrho-\sigma|}{\tau}}$$

$$\leq \frac{1}{1+\frac{£|\rho-\varrho|}{\tau+\upsilon}} *_I \frac{1}{1+\frac{£|\varrho-\sigma|}{\tau+\upsilon}}$$

$$\leq \frac{1}{1+\frac{£|\rho-\varrho|+£|\varrho-\sigma|}{\tau+\upsilon}}$$

$$\leq \frac{1}{1+\frac{£|\rho-\sigma|}{\tau+\upsilon}}$$

$$\leq \frac{\tau+\upsilon}{(\tau+\upsilon)+£|\rho-\sigma|} = k(\rho, \varrho, \tau+\upsilon)$$

Now Let $£|\rho-\sigma| = £|\rho-\sigma| \left\{ \frac{\tau}{\tau+£|\rho-\varrho|} \circledast_I \frac{\upsilon}{\upsilon+£|\varrho-\sigma|} \right\}$

$$\leq £|\rho-\sigma| + |\tau+\upsilon| \left\{ \frac{\tau}{\tau+£|\rho-\varrho|} \circledast_I \frac{\upsilon}{\upsilon+£|\varrho-\sigma|} \right\}$$

$$= \frac{£|\rho-\sigma|}{£|\rho-\sigma| + |\tau+\upsilon|} \leq \left\{ \frac{\tau}{\tau+£|\rho-\varrho|} \circledast_I \frac{\upsilon}{\upsilon+£|\varrho-\sigma|} \right\}$$

$\Rightarrow \hbar(\rho, \sigma, \tau+\upsilon) \leq \hbar(\rho, \varrho, \tau) \circledast_I \hbar(\varrho, \sigma, \upsilon)$. Hence (R5) and (R11) are satisfied.

Definition 2.8: A 5 tuple $(G, k, \hbar, *_I, \circledast_I)$ be an IVIFMS then a $\{p_n\}$ in G is cauchy sequence if and only if for each $\tau > 0$ and

$q > 0$, $\lim_{n\to\infty} k(p_n, p_{n+q}, \tau) = \bar{1}$, $\lim_{n\to\infty} \hbar(p_n, p_{n+q}, \tau) = \bar{0}$.

Definition 2.9: A 5 $(G, k, \hbar, *_I, \circledast_I)$ be an IVIFMS then a $\{p_n\}$ in G is convergent to $\rho \in G$ if and only if for each $\tau > 0$,

$\lim_{n\to\infty} k(p_n, p_n, \tau) = \bar{1}$, $\lim_{n\to\infty} \hbar(p_n, p_n, \tau) = \bar{0}$.

Definition 2.10: IVIFMS is said to be complete if and only if every cauchy sequence is convergent.

Definition 2.11: A 5 tuple $(G, k, \hbar, *_I, \circledast_I)$ be an IVIFMS then open ball $B(p_0, i, \tau)$ with center $p_0 \in$ G and interval number

i $\bar{0} < i < \bar{1}$ with $\tau > \bar{0}$ is defined as $B(p_0, i, \tau) = \{ \varrho \in G ; k(\rho, \varrho, \tau) < i, \hbar(\rho, \varrho, \tau) < 1-i \}$

.

III. Results

Theorem3.1 Let (G, k, $\hbar$, $*_I$, $\circledast_I$) be a complete IVIFMS to the effect that for every s-increasing sequence and absurd

$$\rho, \varrho \in G, \lim_{n\to\infty} \prod_{i=n}^{\infty} k(\rho, \varrho, \tau_i) = \bar{1}, \lim_{n\to\infty} \prod_{i=n}^{\infty} \hbar(\rho, \varrho, \tau_i) = \bar{0}$$

Let $\ell \in (0,1)$ and $\Upsilon : G \to G$ be a mapping satisfying $k(\Upsilon\rho, \Upsilon\varrho, \ell\tau) \geq k(\rho, \varrho, \tau)$ and $\hbar(\Upsilon\rho, \Upsilon\varrho, \ell\tau) \leq \hbar(\rho, \varrho, \tau)$ for all $\rho, \varrho \in G$. There is only one fixed point for G.

Proof : Let point $\rho \in G$. Let $p_n = \Upsilon^n(\rho)$, $n \in \aleph$

$$k(p_1, p_2, \tau) = k(\Upsilon(\rho), \Upsilon^2(\rho), \tau) \geq k(\rho, \Upsilon(\rho), \tfrac{\tau}{\ell}) = k(\rho, p_1, \tfrac{\tau}{\ell}) \ldots\ldots\ldots\ldots 3.1$$

$$\hbar(p_1, p_2, \tau) = \hbar(\Upsilon(\rho), \Upsilon^2(\rho), \tau) \leq \hbar(\rho, Y(\rho), \tfrac{\tau}{\ell}) = \hbar(\rho, p_1, \tfrac{\tau}{\ell}) \ldots\ldots\ldots 3.2$$

it follows that $k(p_n, p_{n+1}, \tau) \geq k(\rho, p_1, \frac{\tau}{\ell n})$

$$\hbar(p_n, p_{n+1}, \tau) \leq \hbar(\rho, p_1, \tfrac{\tau}{\ell n})$$

Let $\tau > 0$. For $m,n \in \aleph$, we assume $n < m$; if we so desire. $s_i > 0$, i=n,....,m-1, fulfilling

$s_n + s_{n+1} + \ldots + s_{m-1} \leq 1$, then there's

$$k(p_n, p_m, \tau) \geq k(p_n, p_{n+1}, s_n\tau) *_I \ldots *_I k(p_{m-1}, p_m, s_{m-1}\tau)$$

$$\geq k(\rho, p_1, \tfrac{s_n\tau}{\ell n}) *_I \ldots *_I k(\rho, p_1, s_{m-1}, \tfrac{s_{m-1}\tau}{\ell m-1}) \ldots\ldots\ldots\ldots 3.3$$

$$\hbar(p_n, p_m, \tau) \geq \hbar(p_n, p_{n+1}, s_n\tau) \circledast_I \ldots \circledast_I \hbar(p_{m-1}, p_m, s_{m-1}\tau)$$

$$\geq \hbar(\rho, p_1, \tfrac{s_n\tau}{\ell n}) \circledast_I \ldots \circledast_I \hbar(\rho, p_1, s_{m-1}, \tfrac{s_{m-1}\tau}{\ell m-1}) \ldots\ldots\ldots\ldots 3.4$$

Science $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$, taking $s_i = \frac{1}{i(i+1)}, i = n, \ldots, m-1$,

$$k(p_n, p_m, \tau) \geq k(\rho, p_1, \tfrac{\tau}{n(n+1)\ell^n}) *_I \ldots *_I k(\rho, p_1, \tfrac{\tau}{m(m-1)\ell^{m-1}}) \ldots\ldots\ldots 3.5$$

$$\hbar(p_n, p_m, \tau) \geq \hbar(\rho, p_1, \tfrac{\tau}{n(n+1)\ell^n}) \circledast_I \ldots \circledast_I \hbar(\rho, p_1, \tfrac{\tau}{m(m-1)\ell^{m-1}}) \ldots\ldots\ldots 3.6$$

we define $\tau_n = \frac{\tau}{n(n+1)\ell^n}$ indicates that $(\tau_{n+1} - \tau_n) \to \infty$, as $n \to \infty$ so τn is an sincreasing sequence and,

$$\lim_{m\to\infty} \prod_{n=m}^{\infty} k(\rho, \varrho, \tfrac{\tau}{n(n+1)\ell^n}) = \bar{1} \ldots\ldots\ldots\ldots 3.7$$

$$\lim_{m\to\infty} \prod_{n=m}^{\infty} \hbar(\rho, \varrho, \tfrac{\tau}{n(n+1)\ell^n}) = \bar{0} \ldots\ldots\ldots\ldots 3.8$$

now from equation (3.5),(3.6),(3.7) and (3.8) we get

$$\lim_{n\to\infty} k(\rho, \varrho, \tau_i) = \bar{1}, \lim_{n\to\infty} \hbar(\rho, \varrho, \tau_i) = \bar{0} \text{ for } m > n$$

Therefore p_n is cauchy sequence. while G is complete, $\varrho \in G$ such that $\lim_{n\to\infty} p_n = \varrho$.

It is our claim that ϱ is a fixed point in G.

$k(\Upsilon(\varrho),(\varrho),\tau) \geq [\lim n\to\infty\, k(\Upsilon(\varrho), \Upsilon(p_n), \frac{\tau}{2})] *_I [\lim n\to\infty\, k(p_{n+1}, \varrho, \frac{\tau}{2})]$

$\geq [\lim n\to\infty\, k(\varrho, p_n, \frac{\tau}{2\ell})] *_I [\lim n\to\infty\, k(p_{n+1}, \varrho, \frac{\tau}{2})]$

$= \bar{1} *_I \bar{1}$ 3.9

$\hbar(\Upsilon(\varrho),(\varrho),\tau) \leq [\lim n\to\infty\, \hbar(\Upsilon(\varrho), \Upsilon(p_n), \frac{\tau}{2})] \circledast_I [\lim n\to\infty\, \hbar(p_{n+1}, \varrho, \frac{\tau}{2})]$

$\leq [\lim n\to\infty\, \hbar(\varrho, p_n, \frac{\tau}{2\ell})] \circledast_I [\lim n\to\infty\, \hbar(p_{n+1}, \varrho, \frac{\tau}{2})]$

$= \bar{0} \circledast_I \bar{0}$ 3.10

Thus $k(\Upsilon(\varrho),(\varrho),\tau) = \bar{1}$ and $\hbar(\Upsilon(\varrho),(\varrho),\tau) = \bar{0}$, we obtain $\Upsilon(\varrho) = \varrho$.

We demonstrate how the fixed point is unique.

we assume $\Upsilon(\xi) = \xi$ for some $\xi \in G$.

$\bar{1} \geq k(\varrho, \xi, \tau) = k(\Upsilon(\varrho), \Upsilon(\xi), \tau)$

$\geq k(\varrho, \xi, \frac{\tau}{\ell}) = k(\Upsilon(\varrho), \Upsilon(\xi), \frac{\tau}{\ell})$

$\geq k(\varrho, \xi, \frac{\tau}{\ell^2}) = k(\Upsilon(\varrho), \Upsilon(\xi), \frac{\tau}{\ell^2})$

$\geq \lim n\to\infty\, k(\varrho, \xi, \frac{\tau}{\ell n}) = \bar{1}$

$\bar{\bar{0}} \leq \hbar(\varrho, \xi, \tau) = \hbar(\Upsilon(\varrho), \Upsilon(\xi), \tau)$

$\leq \hbar(\varrho, \xi, \frac{\tau}{\ell}) = \hbar(\Upsilon(\varrho), \Upsilon(\xi), \frac{\tau}{\ell})$

$\leq \hbar(\varrho, \xi, \frac{\tau}{\ell^2}) = \hbar(\Upsilon(\varrho), \Upsilon(\xi), \frac{\tau}{\ell^2})$

$\leq \lim n\to\infty\, \hbar(\varrho, \xi, \frac{\tau}{\ell n}) = \bar{0}$

Hence $k(\varrho, \xi, \tau) = \bar{\bar{1}}$ and $\hbar(\varrho, \xi, \tau) = \bar{0}$, and $\varrho = \xi$.

Lemma3.2 For any function that is monotonically nondecreasing, $\Re : (0,\infty) \to [0, 1]$ the following circumstance exists:

$\lim n\to\infty \prod_{i=n}^{\infty} \Re(\vartheta_0^i) = \bar{0} \Rightarrow \lim n\to\infty \prod_{i=n}^{\infty} \Re(\vartheta^i) = \bar{0}$.......(3.11)

for all $\vartheta \in (0, 1)$ where ⊛I is a reference to the infinite product Π.

Proof

case 1. $\vartheta < \vartheta^0$. For $i \in \aleph$, $\vartheta^i < \vartheta_0^i$, and $\Re$ is nondecreasing function $\Re(\vartheta^i) \leqslant \Re(\vartheta_0^i)$.

Hence $\prod_{i=n}^{\infty}\Re(\vartheta^{i}) \le \prod_{i=n}^{\infty}\Re(\vartheta_0^{i})$, n ∈ ℵ, so condition (3.11) holds.

case 2. If $\vartheta = \sqrt{\vartheta_0}$

$$\prod_{i=2m}^{\infty}\Re(\vartheta_0^{i}) = \{\prod_{i=m}^{\infty}\Re(\vartheta_0^{2i})\} \circledast_I \{\prod_{i=m}^{\infty}\Re(\vartheta_0^{2i+1})\} \ldots\ldots(3.12)$$

$$\le \{\prod_{i=m}^{\infty}\Re(\vartheta_0^{i})\} \circledast_I \{\prod_{i=2m}^{\infty}\Re(\vartheta_0^{i})\}$$

then we have $\lim_{m\to\infty} \prod_{i=2m}^{\infty}\Re(\vartheta_0^{i}) \le \bar{0} \circledast_I \bar{0} = \bar{0}$

$\lim_{m\to\infty} \prod_{i=2m+1}^{\infty}\Re(\vartheta^{i}) \le \lim_{m\to\infty} \prod_{i=2m+2}^{\infty}\Re(\vartheta_0^{i}) = \bar{0}$

Thus it follows that $\lim_{m\to\infty}\prod_{i=m}^{\infty}\Re(\vartheta^{i}) = \bar{0}$ for $\vartheta = \sqrt{\vartheta_0}$.

Since $\Re$ is nondecreasing For an arbitrary $\vartheta > \vartheta0$, there exit m ∈ ℵ such that $\vartheta > \vartheta_0^{1\backslash 2m}$ and repeat the above procedure m-times to obtain $\lim_{m\to\infty}\prod_{i=m}^{\infty}\Re(\vartheta^{i}) = \bar{0}$.

Lemma3.3 For any monotonically nondecreasing function $\wp : (0,\infty) \to [0, 1]$ the following condition holds

$$\lim_{n\to\infty}\prod_{i=n}^{\infty}\wp(\vartheta_0^{i}) = \bar{1} \Rightarrow \lim_{n\to\infty}\prod_{i=n}^{\infty}\wp(\vartheta^{i}) = \bar{1} \ldots\ldots(3.13)$$

for all $\vartheta \in (0, 1)$ where $*_I$ is a reference to the infinite produc Π.

Proof

case 1. $\vartheta < \vartheta_0$. For i ∈ ℵ, $\vartheta^{i} < \vartheta_0^{i}$, and $\wp$ is nondecreasing function $\wp(\vartheta^{i}) \leqslant \wp(\vartheta_0^{i})$.

Hence $\prod_{i=n}^{\infty}\wp(\vartheta^{i}) \le \prod_{i=n}^{\infty}\wp(\vartheta_0^{i})$, n ∈ ℵ, so condition (3.31) holds.

case 2. If $\vartheta = \sqrt{\vartheta_0}$

$$\prod_{i=2m}^{\infty}\wp(\vartheta_0^{i}) = \{\prod_{i=m}^{\infty}\wp(\vartheta_0^{2i})\} *_I \{\prod_{i=m}^{\infty}\wp(\vartheta_0^{2i+1})\} \ldots\ldots(3.12)$$

$$\le \{\prod_{i=m}^{\infty}\wp(\vartheta_0^{i})\} *_I \{\prod_{i=2m}^{\infty}\wp(\vartheta_0^{i})\}$$

then we have $\lim_{m\to\infty} \prod_{i=2m}^{\infty}\wp(\vartheta_0^{i}) \le \bar{1} *_I \bar{1} = \bar{1}$

$\lim_{m\to\infty} \prod_{i=2m+1}^{\infty}\wp(\vartheta^{i}) \le \lim_{m\to\infty} \prod_{i=2m+2}^{\infty}\wp(\vartheta_0^{i}) = \bar{1}$

Thus it follows that $\lim_{m\to\infty}\prod_{i=m}^{\infty}\wp(\vartheta^{i}) = \bar{1}$ for $\vartheta = \sqrt{\vartheta_0}$.

Since $\Re$ is nondecreasing For an arbitrary $\vartheta > \vartheta0$, there exit m ∈ ℵ such that $\vartheta > \vartheta_0^{1\backslash 2m}$ and repeat the above procedure m-times to obtain $\lim_{m\to\infty}\prod_{i=m}^{\infty}\wp(\vartheta^{i}) = \bar{1}$.

Lemma3.4 We define $p_n = \Upsilon^{n}(p_0)$, n ∈ ℵ. Then p_n is cauchy sequence.

Proof Let $\Re(\rho) = \hbar(p_0, \Upsilon(p_0), \frac{1}{\rho})$ and $\wp(\rho) = k(p_0, \Upsilon(p_0), \frac{1}{\rho})$ for ρ > 0.

Then $\Re(\rho)$, $\wp(\rho)$ is a nondecreasing (nonincreasing) mapping from (0,∞) into [0, 1].
By applying Lemma 3.2 and 3.3 to $1 > \vartheta > \ell$, we obtain

$\lim_{n\to\infty} \prod_{i=n}^{\infty} k(p_0, \Upsilon(p_0), \frac{1}{\frac{\ell^i}{\vartheta}}) = \overline{1}$ ………3.15

$\lim_{n\to\infty} \prod_{i=n}^{\infty} \hbar(p_0, \Upsilon(p_0), \frac{1}{\frac{\ell^i}{\vartheta}}) = \overline{0}$ ………3.16

Since $\vartheta < 1$, $\sum_{n=1}^{\infty} \vartheta^n < \infty$, for any $\epsilon_0 > 0$ there exit n_0 such that

$\sum_{n=1}^{\infty} \vartheta^n < \epsilon_0$. $\epsilon_0 > 0$, if $m > n > n_0$, and $\tau > \epsilon_0$.

$k(p_n, p_m, \tau) \geq k(p_n, p_m, \epsilon_0) \geq \prod_{i=n}^{m-1} k(p_i, p_{i-1}, \vartheta^i) \geq \prod_{i=n}^{\infty} k(p_0, \Upsilon(p_0), \frac{1}{\frac{\ell^i}{\vartheta^i}})$

$\hbar(p_n, p_m, \tau) \leq \hbar(p_n, p_m, \epsilon_0) \leq \prod_{i=n}^{m-1} \hbar(p_i, p_{i-1}, \vartheta^i) \leq \prod_{i=n}^{\infty} \hbar(p_0, \Upsilon(p_0), \frac{1}{\frac{\ell^i}{\vartheta^i}})$

from equation 3.14 and 3.15 we have

$\lim_{n\to\infty} k(p_n, p_m, \tau) = \overline{1}$ and $\lim_{n\to\infty} \hbar(p_n, p_m, \tau) = \overline{0}$, for m > n. So p_n is cauchy sequence.

Theorem3.5 Let $(G, k, \hbar, *_I, \circledast_I)$ be a complete IVIFMS such that for some $\vartheta_0 \in (0, 1)$ and $p_0 \in G$,

$$\lim_{n\to\infty} \prod_{i=n}^{\infty} k(p_0, \Upsilon(p_0), \frac{1}{\vartheta_0^i}) = \overline{1}$$

$$\lim_{n\to\infty} \prod_{i=n}^{\infty} \hbar(p_0, \Upsilon(p_0), \frac{1}{\vartheta_0^i}) = \overline{0}$$

Let $\ell \in (0,1)$ and $\Upsilon : G \to G$ be a mapping satisfying

$k(\Upsilon\rho, \Upsilon\varrho, \ell\tau) \geq k(\rho, \varrho, \tau)$ and $\hbar(\Upsilon\rho, \Upsilon\varrho, \ell\tau) \leq \hbar(\rho, \varrho, \tau)$ for all $\rho, \varrho \in G$.

There is only one fixed point for G.
Proof . Identical to the procedure used to prove theorem 3.1.

IV Application

Let $(G, k, \hbar, *_I, \circledast_I)$ be a complete with IVIFMS $\lim_{n\to\infty} k(p_n, p_m, \tau) = \overline{1}$

and $\lim_{n\to\infty} \hbar(p_n, p_m, \tau) = \overline{0}$, and Υ be an intuitionistic fuzzy contraction.

There is only one fixed point for Υ.

Proof Let point P∈ G. Let $P_n = \Upsilon^n(P)$, $n \in \aleph$ we have

$k(P_n, P_{n+1}, \tau) = k(\Upsilon_{n-1}, \Upsilon_n, \tau) \geq k(P_{n-1}, P_n, \tau) \geq k(P_{n-2}, P_n, \frac{\tau}{\ell}) \geq k(P_{n-2}, P_n, \frac{\tau}{\ell^{n-1}}) \geq k(P_0, P_1, \frac{\tau}{\ell^{n-1}})$

$k(P_n, P_{n+1}, \tau) \geq k(P_n, P_{n+1}, \frac{\tau}{2}) *_I k(P_{n+1}, P_{n+P}, \frac{\tau}{2})$

$\geq k(P_n, P_{n+1}, \frac{\tau}{2}) *_I k(P_{n+1}, P_{n+2}, \frac{\tau}{2^2}) *_I k(P_{n+2}, P_{n+p}, \frac{\tau}{2^2}) *_I \ldots\ldots *_I k(P_{n+p-1}, P_{n+p}, \frac{\tau}{2^{p-1}})$

$\geq k(P_0, P_1, \frac{\tau}{2}) *_I k(P_1, P_2, \frac{\tau}{2^2}) *_I k(P_{n+2}, P_{n+p}, \frac{\tau}{2^2}) *_I \ldots\ldots *_I k(P_{p-1}, P_p, \frac{\tau}{2^{p-1}})$

$= k_n(\frac{\tau}{2}) *_I \ k_{n+1}(\frac{\tau}{2^2}) *_I \ldots\ldots\ldots *_I \ k_{n+p-1}(\frac{\tau}{2^{p-1}})$

$\rightarrow 0 < \ell < 1$ and $\lim_{n\to\infty} k(p, \varrho, \tau) = \overline{1}$ for all $p, \varrho \in G$.

$\lim_{n\to\infty} k(P_n, P_{n+1}, \tau) = \overline{1} *_I \overline{1} *_I \ldots\ldots *_I \overline{1} = \overline{1}$

$\hbar(P_n, P_{n+1}, \tau) = \hbar(\Upsilon_{n-1}, \Upsilon_n, \tau) \geq \hbar(P_{n-1}, P_n, \tau) \geq \hbar(P_{n-2}, P_n, \frac{\tau}{\ell}) \geq \hbar(P_{n-2}, P_n, \frac{\tau}{\ell^{n-1}}) \geq \hbar(P_0, P_1, \frac{\tau}{\ell^{n-1}})$

$\hbar(P_n, P_{n+1}, \tau) \geq \hbar(P_n, P_{n+1}, \frac{\tau}{2}) *_I \hbar(P_{n+1}, P_{n+P}, \frac{\tau}{2})$

$\geq \hbar(P_n, P_{n+1}, \frac{\tau}{2}) *_I \hbar(P_{n+1}, P_{n+2}, \frac{\tau}{2^2}) *_I \hbar(P_{n+2}, P_{n+p}, \frac{\tau}{2^2}) *_I \ldots\ldots *_I \hbar(P_{n+p-1}, P_{n+p}, \frac{\tau}{2^{p-1}})$

$\geq \hbar(P_0, P_1, \frac{\tau}{2}) *_I \hbar(P_1, P_2, \frac{\tau}{2^2}) *_I \hbar(P_{n+2}, P_{n+p}, \frac{\tau}{2^2}) *_I \ldots\ldots *_I \hbar(P_{p-1}, P_p, \frac{\tau}{2^{p-1}})$

$= \hbar_n(\frac{\tau}{2}) *_I \ \hbar_{n+1}(\frac{\tau}{2^2}) *_I \ldots\ldots\ldots *_I \ \hbar(\frac{\tau}{2^{p-1}})$

$\rightarrow 0 < \ell < 1$ and $\lim_{n\to\infty} \hbar(p, \varrho, \tau) = \overline{0}$ for all $p, \varrho \in G$.

$\lim_{n\to\infty} \hbar(P_n, P_{n+1}, \tau) = \overline{0} \circledast_I \overline{0} \circledast_I \ldots\ldots \circledast_I \overline{0} = \overline{0}$

Hence P_n is cauchy sequence. while G is complete, $\varrho \in G$ such that $\lim_{n\to\infty} P_n = \varrho$.

Now, we will show that ϱ is a fixed point of Υ.

$k(\Upsilon(P), P, \tau) \geq k(\Upsilon(P), P_n, \frac{\tau}{2}) *_I k(P, \Upsilon(P_n), \frac{\tau}{2}) \geq k(P, P_n, \frac{\tau}{2\ell}) *_I k(P, P_{n+1}, \frac{\tau}{2\ell})$

Since Υ is intuitionistic fuzzy contraction and $\Upsilon(P_n = P_{n+1}$ as $n \rightarrow \infty$

$= \overline{1} *_I \overline{1} *_I \ldots\ldots *_I \overline{1} = \overline{1}$

$\hbar(\Upsilon(P), P, \tau) \geq \hbar(\Upsilon(P), P_n, \frac{\tau}{2}) *_I \hbar(P, \Upsilon(P_n), \frac{\tau}{2}) \geq \hbar(P, P_n, \frac{\tau}{2\ell}) *_I \hbar(P, P_{n+1}, \frac{\tau}{2\ell})$

Since Υ is intuitionistic fuzzy contraction and $\Upsilon(P_n = P_{n+1}$ as $n \rightarrow \infty$

$= \overline{0} \circledast_I \overline{0} \circledast_I \ldots\ldots \circledast_I \overline{0} = \overline{0}$

Then $\Upsilon(P)$, = P, hence ϱ is only one fixed point for Υ.

V Conclusion

The notions of continuous τ-norm and continuous τ-conorm and some important properties of continuous Interval- valued τ-norm and continuous interval-valued τ-conorm, are established. During this the continuity of an interval-valued τ-norm and interval-valued τ-conorm, was defined by means of the convergence of sequence of interval numbers. Moreover, we have proved some results for IVIFMS with the help of fixed point concepts.

Our results are useful for theoretical mathematics and computer science.

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