IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

A New Class Of $(1,2)b$-Connectedness And Compactness In Bitopological Spaces

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Abstract: The purpose of this paper is to introduce the notion of $(1,2)b$-connectedness and $(1,2)b$ –Compactness and their properties by using $(1,2)b$-open and $(1,2)b$ -closed sets for bitopological spaces. We will investigate several results in $(1,2)b$-connectedness and $(1,2)b$ –Compactness for subsets in bitopological spaces.

Keywords: **$(1,2)b$-openset, $(1,2)b$-closed set, $(1,2)b$-Connectedness, $(1,2)b$ Compactness.**

1. Introduction

The notion of bitopological spaces was first introduced by Kelly [5]. Let (X,τ_1,τ_2) be a bitopological space. For a subset A of X, the interior (resp., closure) of A with respect to τ_i will be denoted by $int_i(A)$ (resp. $Cl_i(A)$) for $i=1,2$. The notion of b-open set was introduced in [1]. In particular, we will discuss the relationship related to b-connectedness between the topological spaces and bitopological space. That is, In addition, we introduce the result which states that a bitopological space (X,τ_1,τ_2) is $(1,2)b$-connected if and only if X and ϕ are the only subsets of X which are $(1,2)b$-clopen sets. Moreover, we have proved some results in compactness also. Altogether, several results $(1,2)b$-connectedness and compactness in a bitopological space have been discussed.

2. Preliminaries

Throughout this paper, $i,j\in\{1,2\}$, $i\neq j$ and (X,τ_1,τ_2) and (Y,τ_1,τ_2) stand for bitopological spaces with no separation axioms assumed unless otherwise mentioned.

Definition 2.1

Let (X,τ_1,τ_2) be a bitopological space. A subset A of X is called $(1,2)b$-open if $A\subseteq int_1(cl_2(A))\cup cl_2(int_1(A))$ and the complement $X-A$ of a $(1,2)b$- open set A is called $(1,2)b$-closed.

Example 2.2

Let $X=\{a,b,c\}$, $\tau_1=\{X,\emptyset,\{a\},\{b\}.\{a,b\}\}$, $\tau_2=\{X,\emptyset,\{b\},\{c\},\{b.c\}\}$.
The Collection of $(1,2)b$-open sets of X =$\{X,\emptyset,\{a\}.\{b\},\{a,b\},\{b,c\}\}$.

Definition 2.3

Let (X,τ_1,τ_2) be a bitopological space. A subset A of X is called $(1,2)b$-closure if the intersection of all $(1,2)b$-closed sets containing A. Which is denoted by $bCl_2(A)$.

Remark 2.4

$bCl_2(A)$ is the smallest $(1,2)b$ -closed set containing A.

Definition 2.5

Let (X,τ_1,τ_2) be a bitopological space. A subset A of X is called $(1,2)b$-interior if the union of all $(1,2)b$-open sets contained in A. Which is denoted by $bInt_1(A)$.

Remark 2.6

$bInt_1(A)$ is the largest $(1,2)b$-open set contained in A.

Definition 2.7

The family of all $(1,2)b$-open (resp.$(1,2)b$-closed) sets in a bitopological space (X, τ_1, τ_2) will be denoted by $BO(X)$ (resp. $BCl(X)$).

Proposition 2.8

Let (X, τ_1, τ_2) be a bitopological space.

(i) The union of any family of $(1,2)b$ -open sets is a $(1,2)b$ -open set.

(ii) The intersection of a $(1,2)b$ -open set is a $(1,2)b$ -open set.

Lemma 2.10

Let (X, τ_1, τ_2) be a bitopological space. The $(1,2)b$-closure of a subset A of X, denoted by $bCl_2(A)$ is the set of all $x \in X$ such that $U \cap A \neq \emptyset$ for every $U \in BO(X, x)$, where $BO(X, x) = \{U: x \in U \in BO(X, \tau)\}$.

Definition 2.11

Let (X, τ_1, τ_2) be a bitopological space. The $(1,2)b$ -boundary of a set A of a space X is defined by $(1,2)b - bd(A) = bCl_2(A) \cap bCl_2(X - A)$.

3.CONNECTED SPACE

Definition 3.1

Let (X, τ_1, τ_2) be a bitopological space. A subset A of X is called $(1,2)b$-Connected if X cannot be expressed as the union of two nonempty disjoint sets A and B such $(1,2)b$ -open sets of X.

Remark 3.2

If a bitopological space (X, τ_1, τ_2) is $(1,2)b$-connected, then the topological spaces (X, τ_1) and (X, τ_2) are $(1,2)b$-connected.

Definition 3.3

A topological space (X, τ_1, τ_2) is said to be $(1,2)b$-disconnected if $X = A \cup B$,where A and B are any two non-empty $(1,2)b$-separated sets. Thus, X is $(1,2)b$-disconnected if

(i) $X = A \cap B$

(ii) $\left(A \cap bcl_2(B)\right) \cup \left(B \cap bcl_2(A)\right) = \emptyset$

Theorem 3.4

Let C be a $(1,2)b$-connected subset of a topological space (X, τ_1, τ_2) . Let $X = A \cup B$, where A and B are $(1,2)b$-separated subsets of X.Then either or $C \subset A$ or $C \subset B$.

Proof

Let C be a $(1,2)b$-connected subset of X and $X = A \cup B$. $\Rightarrow C \subset X = A \cup B$.

$$\Rightarrow C \subset A \cup B.$$

$$C = C \cap (A \cup B)$$

$$C = (C \cap A) \cup (C \cap B)\ldots\ldots\ldots\ldots\ldots(1)$$

Since A and B are $(1,2)b$-separated sets. Hence and $A \cap bcl_2(B) = \emptyset$ and $B \cap bcl_2(A) = \emptyset$.

Since is $(1,2)b$-connected. Hence, $C \cap A = \emptyset$ or $C \cap B = \emptyset$ [From (1)] . Therefore, $C \subset B$ or $C \subset A$.

Theorem: 3.5

Let C be a $(1,2)b$-connected subset of a space (X, τ_1, τ_2) and Let D be a subset such that $\subset D \subset bcl_2(C)$. Then D is $(1,2)b$-connected.

Proof

To prove: D is $(1,2)b$-connected. Assume that D is $(1,2)b$-disconnected. Let $D = A \cup B$. Where A and B are $(1,2)b$-separated subsets of . Then $A \neq \emptyset$ and $\neq \emptyset$. Since C is $(1,2)b$-connected and $C \subset D$. $\Rightarrow C \subset A \cup B$. By theorem 3.4, Either $C \subset A$ (or) $C \subset B$. Suppose $\subset A$. Then $bcl_2(C) \subset bcl_2(A)$. $\Rightarrow bcl_2(C) \cap B \subset bcl_2(A) \cap B = \emptyset$, Since A and B are $(1,2)b$-separated. But $B \subset D$ and $D \subset bcl_2(C)$.

$$\Rightarrow B \cap D \subset B \cap bcl_2(C).$$

Hence $\emptyset \subset B = B \cap D \subset B \cap bcl_2(C) = \emptyset$. Therefore, $\emptyset \subset B \subset \emptyset$. $\Rightarrow B = \emptyset$. Which is a contradiction to our assumption. Hence D must be $(1,2)b$-connected.

4.COMPACT SPACE

Definition 4.1

A collection $\{A_\alpha\}_{\alpha\epsilon I}$ of subsets of a bitopological space (X, τ_1, τ_2) is said to be a $(1,2)$ b- open cover for X if $\bigcup_{\alpha\in I} A_\alpha = X$ and A_α are $(1,2)$ b-open sets in X for each$\alpha \in I$.

Definition 4.2

A bitopological space (X, τ_1, τ_2) is said to be $(1,2)$ b-compact if every $(1,2)$ b-open cover $\{A_\alpha\}$ of X contains a finite sub collection that also covers.

Theorem 4.3

Let Y be a subspace of X .Then Y is *b-compact if and only if every covering of Y by sets $(1,2)$ b-open in X contains a finite sub collection covering .

Proof

Let Y be a subspace of X .Suppose that Y is $(1,2)b$-compact. To prove: Every covering of Y by sets $(1,2)b$-open in X contains a finite sub collection covering . Let $V = (A_\alpha)$ be a covering of Y by sets $(1,2)b$-open in . $\Longrightarrow A_\alpha$ is $(1,2)b$-open in X . $\Longrightarrow A_\alpha \cap Y$is $(1,2)b$-open in Y. Therefore, The collection $V_1 = \{A_\alpha \cap Y \in BO(X)\}$ is covering of Y by sets $(1,2)b$-open in Y .Since Y is $(1,2)b$-compact. There exists a finite sub collection $A_{\alpha_1} \cap Y, A_{\alpha_2} \cap Y, \ldots\ldots\ldots.., A_{\alpha_n} \cap Y$ from V_1 such that

$$(A_{\alpha_1} \cap Y) \cup (A_{\alpha_2} \cap Y) \cup \ldots\ldots\ldots..\cup (A_{\alpha_n} \cap Y) = Y$$

$$\Rightarrow (A_{\alpha_1} \cup A_{\alpha_2} \cup \ldots\ldots\ldots\ldots.\cup A_{\alpha_n}) \cap Y = Y$$

$$\Rightarrow Y \subset A_{\alpha_1} \cup A_{\alpha_2} \cup \ldots, \ldots\ldots\ldots \cup A_{\alpha_n}$$

Therefore, $\{A_{\alpha_1}, A_{\alpha_2}, \ldots\ldots\ldots.., A_{\alpha_n}\}$ Covers Y. $\{A_{\alpha_1}, A_{\alpha_2}, \ldots\ldots\ldots.., A_{\alpha_n}\}$ is a finite sub collection of V that covers .

Conversly, Suppose that every covering of Y by sets $(1,2)b$ -open in X contains a finite sub collection covering . To prove: Y is $(1,2)b$ -compact Let $\{B_\alpha\}$ be a covering of Y by sets $(1,2)b$ -open in . B_α is $(1,2)b$-open in Y . $\Rightarrow$ there exist a $(1,2)b$-open set A_α in X such that $B_\alpha = A_\alpha \cap Y$. Therefore $B_\alpha \subset A_\alpha$for each α. Since $\{B_\alpha\}$ is a cover for . $\Rightarrow Y = \cup B_\alpha \subset \cup A_\alpha$.

Therefore, $\{A_\alpha\}$ is a cover for Y by sets $(1,2)b$-open in X . By our assumption there exists a finite sub collection $A_{\alpha_1}, A_{\alpha_2}, \ldots\ldots\ldots.., A_{\alpha_n}$ covering Y .

(ie) $Y \subset A_{\alpha_1} \cup A_{\alpha_2} \cup \ldots, \ldots\ldots\ldots \cup A_{\alpha_n}$

$$\Rightarrow (A_{\alpha_1} \cup A_{\alpha_2} \cup \ldots\ldots\ldots\ldots.\cup A_{\alpha_n}) \cap Y = Y$$

(ie) $B_{\alpha_1} \cup B_{\alpha_2} \cup \ldots\ldots.\cup B_{\alpha_n} = Y$. Since $B_{\alpha_1} = A_{\alpha_1} \cap Y$

Therefore,$\{B_{\alpha_1} \cup B_{\alpha_2} \cup \ldots\ldots.\cup B_{\alpha_n}\}$ is a finite sub collection of $\{B_\alpha\}$ that covers . Therefore, Every $(1,2)b$-open covering of Y has a finite sub collection covering . Therefore Y is $(1,2)b$-compact.

Preposition 4.3

Every $(1,2)b$-closed subspace of a $(1,2)b$-compact space is $(1,2)b$-compact.

Proof

Let X be a $(1,2)b$-compact space and Y be a $(1,2)b$-closed subspace of X . To prove: Y is $(1,2)b$-compact. Let $\{B_\alpha\}$ be a $(1,2)b$-open cover for Y . B_α is $(1,2)b$-open

in Y. $\Rightarrow$ there exists a $(1,2)b$-open set A_α in X. such that $B_\alpha = A_\alpha \cap Y$. $B_\alpha \subset A_\alpha$ for all α.

$\Rightarrow \cup B_\alpha \subset \cup A_\alpha$. $\{B_\alpha\}$ is a $(1,2)b$-open cover for Y.

$$Y = \cup B_\alpha \subset \cup A_\alpha$$

Therefore, $V = \{A_\alpha\}$ is a $(1,2)b$-open cover for Y with $(1,2)b$-open sets in . Since Y is $(1,2)b$-closed in . Therefore, $X - Y$ is $(1,2)b$-open in X . Consider $V_1 = V \cup \{(X - Y)\}$. V_1 is a $(1,2)b$-open cover for X. Since X is $(1,2)b$-compact. There exists a finite sub collection $V_2 = \{A_{\alpha_1}, A_{\alpha_2}, \dots \dots \dots .., A_{\alpha_n}\}$ from V_1 that covers X .Let $V_3 = V_2 - \{(X - Y)\}$. That is $X - Y \in V_2$ discard it. Now V_3 covers Y. V_2 is finite $\Rightarrow V_3$ is finite. $V_3 = \{A_{\alpha_1}, A_{\alpha_2}, \dots \dots \dots .., A_{\alpha_n}\}$ is a finite sub collection that covers Y .

$$A_{\alpha_1} \cup A_{\alpha_2} \cup \dots \dots \dots . \cup A_{\alpha_n} \supset Y$$

$$(A_{\alpha_1} \cup A_{\alpha_2} \cup \dots \dots \dots . \cup A_{\alpha_n}) \cap Y = Y$$

$$B_{\alpha_1} \cup B_{\alpha_2} \cup \dots \dots . \cup B_{\alpha_n} = Y$$

where $B_{\alpha_1} = A_{\alpha_1} \cap Y$. Therefore, $\{B_{\alpha_1}, B_{\alpha_2}, \dots \dots, B_{\alpha_n}\}$ is a finite sub collection that covers Y. Therefore, Every $(1,2)b$-open covering of Y has a finite sub collection covering Y .

5.References

[1] Kelly .J. C., “Bitopological Spaces,” Proceedings of the London Mathematical Society, vol. 3-13, no. 1, pp. 71–89, 1963.

[2] Andrijevi'c .D., “On b-open sets”, Mat. Bech. pp.59-64,48 (1996), .

[3] El-Atik. A.A., Abu Donia.H.M., Salama. A.S., “On b-connectedness and b-disconnectedness and their applications”, Journal of the Egyptian Mathematical Society, pp.63–67 (2013) 21,.

[4] Caldas.M and Jafari.S, “On some applications of b-open sets in topological spaces”, Kochi, J.Math., 11-19,2(2007).

[5] Edward.A Samuel and Balan.D, “δ-interior *and* δ-closure in bitopological spaces”, thesis-Bharath University, Chennai, Tamilnadu,India, 2015.

[6] ChandrasekharaRao.K and Subha.V, “A note on the Associated bitopological space (X, τ_1, τ_2) ,” International Journal of MathematicalAnalysis, vol. 7, pp. 323–330, 2003.