A New class of Binary Open Sets in Binary Topological Space

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Abstract: In this paper, we introduced new type of binary open sets namely binary \(s_{\alpha}\)-open sets in binary topological space. Also, some of the properties have been discussed.

Keywords: \(bS_{\alpha}\)-closed set, \(bS_{\alpha}\)-open set.

I. INTRODUCTION

In 2011, S.Nithyanantha Jothi and P.Thangavelu \([2]\) introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from \(X\) to \(Y\) which is defined to be the ordered pairs \((A,B)\) where \(A \subseteq X\) and \(B \subseteq Y\). In 2000, G.B.Navalagi proposed semi-\(\alpha\) open sets in topological spaces. In 2014, S.N.Jothi and P.Thangavelu \([5]\) introduced generalized binary closed sets in binary topological spaces. Also, S.N.Jothi and P.Thangavelu \([3]\) introduced binary semiopen open sets and discussed some of their properties in binary topological spaces. In continuation, we have found \(bS_{\alpha}\)-closed set in binary topological spaces and analyzed some of their properties and also explored its relationship with other existing sets.

II. PRELIMINARIES

**Definition 2.1.** \([2]\) Let \(X\) and \(Y\) be any two nonempty sets. A binary topology is a binary structure \(M \subseteq P(X) \times P(Y)\) from \(X\) to \(Y\) which satisfies the following axioms:

(i) \((\emptyset, \emptyset) \in M; (X, Y) \in M\).

(ii) \((A_1 \cap A_2, B_1 \cap B_2) \in M\) where \(A_1, A_2, B_1, B_2 \in M\)

(iii) If \((A_\alpha, B_\alpha : \alpha \in A)\) is a family of members of \(M\), then 
\((\bigcup_{\alpha \in A} A_\alpha, \bigcup_{\alpha \in A} B_\alpha) \in M\).

If \(M\) is a binary topology from \(X\) to \(Y\), then the triplet \((X, Y, M)\) is called binary topological space and the members of \(M\) are called the binary open sets of the binary topological space \((X, Y, M)\).
**Definition 2.2.**[2] Let X and Y be any two nonempty sets and let \((A, B)\) and \((C, D)\) \(\in P(X) \times P(Y)\). If \(A \subseteq C\) and \(B \subseteq D\), then \((A, B) \subseteq (C, D)\).

**Definition 2.3.**[2] Let \((X, Y, M)\) be a binary topological space and \((A, B) \subseteq (X, Y, M)\).

\((A, B)^{\star} = \bigcup\{A_a : (A_a, B_a) \text{ is binary open and } (A_a, B_a) \subseteq (A, B)\}\)

\((A, B)^{\circ} = \bigcup\{B_a : (A_a, B_a) \text{ is binary open and } (A_a, B_a) \subseteq (A, B)\}\).

**Definition 2.4.[2]** The ordered pair \(((A, B)^{\circ}, (A, B)^{\circ})\) is called the binary interior of \((A, B)\) and it is denoted by \(b\cdot\text{int}(A, B)\).

**Definition 2.5.[2]** Let \((X, Y, M)\) be a binary topological space and \((A, B) \subseteq (X, Y, M)\).

\((A, B)^{\circ} = \bigcap\{A_a : (A_a, B_a) \text{ is binary closed and } (A_a, B_a) \supseteq (A, B)\}\)

\((A, B)^{\star} = \bigcap\{B_a : (A_a, B_a) \text{ is binary closed and } (A_a, B_a) \supseteq (A, B)\}\).

**Definition 2.6.[2]** The ordered pair \(((A, B)^{\circ}, (A, B)^{\circ})\) is called the binary closure of \((A, B)\). The binary closure of \((A, B)\) is denoted by \(b\cdot\text{cl}(A, B)\).

**Definition 2.7.[2]** A subset \((A, B)\) of a binary topological space \((X, Y, M)\) is called

- \((i)\) binary regular open if \((A, B) = b\cdot\text{int}(b\cdot\text{cl}(A, B))\) and binary regular closed if \((A, B) = b\cdot\text{cl}(b\cdot\text{int}(A, B))\).

- \((ii)\) binary semi open set if \((A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(A, B))\). The complement of binary semiopen set is called binary semi closed set.

**Definition 2.8.[3]** A subset \((A, B)\) of a binary topological space \((X, Y, M)\) is called

- \((i)\) binary pre closed if \(b\cdot\text{cl}(b\cdot\text{int}(A, B)) \subseteq (A, B)\)

- \((ii)\) binary semi pre closed (or binary \(\beta\) closed if \(b\cdot\text{cl}(b\cdot\text{int}(b\cdot\text{cl}(A, B))) \subseteq (A, B)\)

- \((iii)\) binary \(\alpha\) closed if \(b\cdot\text{int}(b\cdot\text{cl}(b\cdot\text{int}(A, B))) \subseteq (A, B)\).

**Definition 2.9.[4]** In a topological space \((X, \tau)\), the subset \(A\) of \(X\) is said to be semi-\(\alpha\)-open if there exists a \(\alpha\)-open set \(U\) in \(X\) such that \(U \subseteq A \subseteq \text{cl}(U)\). The family of all semi-\(\alpha\)-open sets of \(X\) is denoted by \(S_{\alpha}(X)\).

**Definition 2.10.[3]** Let \((X, Y, M)\) be a binary topological space. Let \((A, B) \subseteq (X, Y)\). Then \((A, B)\) is called binary semi open if there exists a binary open set \((U, V)\) such that \((U, V) \subseteq (A, B) \subseteq b\cdot\text{cl}(U, V)\).

**III. On Binary Semi-\(\alpha\)-open Sets**

**Definition 3.1.** Let \((X, Y, M)\) be a binary topological space and \((A, B) \subseteq (X, Y)\). The subset \((A, B)\) is said to be binary semi \(\alpha\)-open \((bS_{\alpha}O)\) if there exists a binary \(\alpha\)-open set \((U, V)\) in \(X\) such that \((U, V) \subseteq (A, B) \subseteq \text{cl}(U, V)\).

**Theorem 3.2.** In a binary topological space \((X, Y, M)\), if the subset \((A, B) \in b\cdot\alpha\cdot\text{int}(U, V)\) iff there exists a binary open set \((C, D)\) such that \((C, D) \subseteq (A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(C, D))\).

**IV. Proof.** Let \((A, B)\) be a binary \(\alpha\)-open set in binary topological space. Then, \((A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(A, B))\). We have \(b\cdot\text{int}(A, B) \subseteq (A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(A, B))\). Let \((C, D) = b\cdot\text{int}(A, B)\). Then there exists a open set \(b\cdot\text{int}(A, B)\) such that \((C, D) \subseteq (A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(C, D))\). Conversely, suppose there exists a binary open set \((C, D)\) such that \((C, D) \subseteq (A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(C, D))\). Since, \(b\cdot\text{int}(A, B)\) is the largest binary open set contained in \((A, B)\), then \((C, D) \subseteq b\cdot\text{int}(A, B)\) which implies \(b\cdot\text{cl}(C, D) \subseteq b\cdot\text{cl}(b\cdot\text{int}(A, B))\). Hence, \(b\cdot\text{int}(b\cdot\text{cl}(C, D)) \subseteq b\cdot\text{int}(b\cdot\text{cl}(b\cdot\text{int}(A, B)))\). But we have \((C, D) \subseteq (A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(C, D))\). Therefore, \((A, B) \subseteq b\cdot\text{int}(b\cdot\text{cl}(A, B))\). Hence \((A, B) \in b\cdot\alpha\cdot\text{int}(U, V)\).
Theorem 3.3. In a binary topological space, union of any family of binary $S_\alpha$-open set is binary $S_\alpha$-open set.

V. Proof. Let $\{(A_i, B_i)\}$ be a family of binary $S_\alpha$-open set in a binary topological space. To prove, $\bigcup_{i \in \Delta} (A_i, B_i)$ is a binary $S_\alpha$-open set. Since, $(A_i, B_i) \in bS_\alpha O(X)$, then there exists a binary $\alpha$-open set $(U_i, V_i)$ such that $(U_i, V_i) \subseteq (A_i, B_i) \subseteq bcl(U_i, V_i)$ which implies $\bigcup_{i \in \Delta} (U_i, V_i) \subseteq \bigcup_{i \in \Delta} bcl(U_i, V_i) \subseteq bcl(\bigcup_{i \in \Delta} (U_i, V_i))$. Since, arbitrary union of binary $\alpha$-open set is binary $\alpha$-open, $\bigcup_{i \in \Delta} (U_i, V_i)$ is also binary $\alpha$-open set. Hence, $\bigcup_{i \in \Delta} (A_i, B_i)$ is binary $S_\alpha$-open set.

Remark 3.4.

1. Every binary $S_\alpha$ open set is binary semi open set.
2. Every binary $S_\alpha$ open set is binary $\alpha$ open set.
3. Every binary $S_\alpha$ open set is binary pre open set.
4. Every binary $S_\alpha$ open set is binary $\beta$ open set.
5. Every binary $S_\alpha$ open set is binary $b$ open set.

Following example shows that converse of the above remarks need not be true.

Example 3.5

$X = \{a, b, c\}, Y = \{1, 2\}$

$M = \{(\emptyset, X, Y), (X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

$S_\alpha$ open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

Semi open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

$\alpha$ open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

Pre open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

$\beta$ open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, \emptyset), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

$b$ open set $= \{(\emptyset, X, Y), (\emptyset, \{2\}), (\{b\}, \emptyset), (\{b\}, \{1\}), (\{b\}, X, \emptyset), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\})\}$

the set $(b, c, Y)$ is binary semi open and binary $\alpha$ open but not binary $S_\alpha$ open set.
the set \((b,Y)\) is binary pre open , binary \(\beta\) open set and binary \(b\) open set but not binary \(S_\alpha\) open set

VI. REFERENCES


