IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A NOTE ON THERMAL TREATMENT IN AN INVERSE TRANSIENT THIN CIRCULAR PLATE

Badrinath E Ghonge

Assistant Professor
Department of Mathematics,
S R College Ghansawangi, Dist. Jalna, (MS), India

Abstract: This work deals with determination of unknown temperature and thermal deflection of thin circular plate on lower surface with the stated conditions. The inverse heat conduction equation is solved by using finite Hankel transform and Laplace transform simultaneously and the results for unknown temperature and thermal deflection are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Math-cad software and illustrated graphically by using Origin software.

***Index Terms* - Inverse Transient, Thermal deflection, Laplace-Hankel transform, Thin circular plate.**

I. INTRODUCTION

The inverse problems of thermoelasticity consist of determination of temperature distribution and thermal deflection of solids when the conditions of temperature and deflection are known at the some points of the solid under consideration. Grysa and Cialkowski [1], Grysa and Koalowski [2] studied one-dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. In [3] Dai et. al studied Based on the thermoviscoelastic theory and the classic plate theory, thermoviscoelastic behavior of a circular plate made from high strength low alloy (HSLA) steel material is investigated. The entire problem is solved by utilizing the finite difference method, Newmark method and iterative method. Khobragade et al.[4] and [5] discuss an inverse steady state and transient thermoelastic problem of thin circular plate and annular disc in Marchi-Fasulo transform domain. Deshmukh et al. [6] investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by quasi-static approach. Ghonge and Ghadle [7]-[10] investigated an inverse problems of thermoelastic behavior in different solids by integral transform methods. Further Ghonge and Ghadle [11]-[14] derived the analytical solution to deflection of thermoelastic circular plates for different conditions by using Marchi-Fasulo, Marchi-Zgrablich and Laplace integral transform.

In this work, the temperature, unknown temperature on lower surface and quasi-static thermal deflection due to unknown temperature $g(t,r)$ are discuss. The inverse heat conduction equation is solved by using finite Hankel transform and Laplace transform simultaneously and the results for unknown temperature and thermal deflection are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Math-cad software and illustrated graphically by using Origin software.

II. MATHEMATICAL FORMULATION

A thin wall work piece under lathe machine is modeled a circular plate occupying the space $D: 0 \le r \le a, 0 \le \theta \le 2\pi, -h \le z \le h$ in terms of cylindrical coordinates. As the work piece is rotated with outer edge is clamped and machined by tool moving along the horizontal radius. The zero heat flux is applied on the upper surface of plate and known temperature $f(t,r)$ is provided at some fix interior $z=\xi$ and plate is insulated at curved surface. Under this more realistic prescribed conditions, the unknown temperature on lower surface and quasi-static thermal deflection due to unknown temperature $g(t,r)$ are required to determine. The differential equation satisfying the deflection function as in [15,16] is given as

$$\nabla^4 w = \frac{-1}{(1-\nu)D} \nabla^2 M_T \tag{1}$$

Where, the operator ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{2}$$

M_T is the thermal moment of the plate defined as

$$M_T = \alpha E \int_{-h}^{h} zT(r,z,t)dz \tag{3}$$

and D is the flexural rigidity of the plate denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4}$$

α, E and ν are the coefficients of the linear thermal expansion, the Young's modulus and the Poisson's ration of the plate material respectively.

Since the edge of the circular plate is fixed and clamped;

$$w = \frac{\partial w}{\partial r} = 0 \qquad at \;\; r = a \tag{5}$$

$$w = 0 \qquad at \;\; t = 0 \tag{6}$$

The temperature of the circular plate satisfying the heat conduction equation as in [15, 16] as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad in\; 0 \le r \le a,\; -h \le z \le h, t > 0 \tag{7}$$

with initial condition

$$T(t,r,z) = 0 \qquad at\; t = 0,\; 0 \le r \le a,\; -h \le z \le h \tag{8}$$

the boundary condition's

$$T(t,r,z) = 0 \qquad at\; r = a,\;\; -h \le z \le h, t \ge 0 \tag{9}$$

$$\frac{\partial T}{\partial z} = 0 \qquad z = h,\; 0 \le r \le a \tag{10}$$

and interior condition

$$T(t,r,z) = f(t,r) \qquad known\, at\;\; z = \xi,\; -h \le \xi \le h,\; 0 \le r \le a, t \ge 0 \tag{11}$$

$$T(t,r,z) = g(t,r) \qquad unknown\; at\;\; z = -h,\; 0 \le r \le a, t \ge 0 \tag{12}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces, k is the thermal diffusivity of the material of the circular plate. The equations (1) to (12) constitute the mathematical formulation of the inverse transient thermoelastic deflection problem of circular plate.

III. Analysis of the Problem

Applying the finite Hankel transform to the equations (7)-(11), then making use of Laplace transform in Hankel domain as in [17], the equation (7) can be converted in second order differential equation in transform domain, solving this for temperature in domain by making use of boundary conditions in transform domain. Then taking inverse Laplace transform and making use of inversion theorem. Finally inverse Hankel transform gives as temperature distributions function as below

$$\begin{aligned} T(t,r,z) = {} & \frac{2k\pi}{a^2(h-\xi)^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{J_1^2(\lambda_n a)}\sum_{m=0}^{\infty}(2m+1)(-1)^m \cos\left(\frac{(2m+1)\pi(z-h)}{2(h-\xi)}\right) \\ & \times \int_0^t \overline{f}(t',\lambda_n).e^{-k\left[\lambda_n^2 + \frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right](t-t')} dt' \end{aligned} \tag{13}$$

Now using value of temperature distribution in equation (10), one we obtain the unknown temperature as below

$$\begin{aligned} g(t,r) = {} & \frac{2k\pi}{a^2(h-\xi)^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{J_1^2(\lambda_n a)}\sum_{m=0}^{\infty}(2m+1)(-1)^m \cos\left(\frac{(2m+1)\pi h}{(h-\xi)}\right) \\ & \times \int_0^t \overline{f}(t',\lambda_n).e^{-k\left[\lambda_n^2 + \frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right](t-t')} dt' \end{aligned} \tag{14}$$

Using equation (13) in equation (3), we obtain

$$M_T = \frac{2\alpha Ek\pi}{a^2(h-\xi)^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{J_1^2(\lambda_n a)}\sum_{m=0}^{\infty}(2m+1)(-1)^m \times\left[\frac{m\pi h}{2(h+\xi)}\sin\left(\frac{m\pi}{(h-\xi)}\right)+\frac{m^2\pi^2}{4(h-\xi)^2}\cos\left(\frac{m\pi}{(h-\xi)}\right)-\frac{(2m+1)^2 m^2}{4(h-\xi)^2}\right] \times\int_0^t \overline{f}(t',\lambda_n).e^{-k\left[\lambda_n^2+\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right](t-t')}dt' \tag{15}$$

Assume the solution of (1) satisfying the (6) as

$$w(t,r)=\sum_{n=1}^{\infty}C_n(t)\left[J_0(\lambda_n r)-J_0(\lambda_n a)\right] \tag{16}$$

Using the (15), (16) and the result $\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]J_0(\lambda_n r)=-\lambda_n^2 J_0(\lambda_n r)$ in (1), once we obtain the expression for $C_n(t)$ as

$$C_n(t)=\frac{2\alpha Ek\pi}{D(1-\nu)a^2(h+\xi)^2}\frac{1}{\lambda_n^2 J_1^2(\lambda_n a)}\sum_{m=0}^{\infty}(2m+1)(-1)^m \times\left[\frac{(2m+1)\pi h}{2(h-\xi)}\sin\left(\frac{(2m+1)\pi}{(h-\xi)}\right)+\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\cos\left(\frac{(2m+1)\pi}{(h-\xi)}\right)-\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right] \times\int_0^t \overline{f}(t',\lambda_n).e^{-k\left[\lambda_n^2+\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right](t-t')}dt' \tag{17}$$

Substituting the equation (17) in the equation (16), once obtain the expression for thermal deflection function as

$$w(r,t)=\frac{2\alpha Ek\pi}{D(1-\nu)a^2(h-\xi)^2}\sum_{n=1}^{\infty}\frac{\left[J_0(\lambda_n r)-J_0(\lambda_n a)\right]}{\lambda_n^2 J_1^2(\lambda_n a)}\sum_{m=0}^{\infty}(2m+1)(-1)^m \times\left[\frac{(2m+1)\pi h}{2(h-\xi)}\sin\left(\frac{(2m+1)\pi}{(h-\xi)}\right)+\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\cos\left(\frac{(2m+1)\pi}{(h-\xi)}\right)-\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right] \times\int_0^t \overline{f}(t',\lambda_n).e^{-k\left[\lambda_n^2+\frac{(2m+1)^2\pi^2}{4(h-\xi)^2}\right](t-t')}dt' \tag{18}$$

IV. PARTICULAR CASE AND NUMERICAL OUTCOMES

For formulation of particular case and examine the numerical calculation of analytical behaviour of a circular plate, we consider the following functions and parameters:

$f(t,r)=(h-\xi)\left(1-e^{-t}\right)(r-a)e^h$ and $t=1$ sec.

Dimensions

Radius of a circular plate $a=1$ m. Thickness of a circular plate $2h=0.2$ m.

Material properties

The numerical calculation has been carried out for an aluminum (pure) circular plate with material properties as,

Thermal diffusivity $k=84.18\times10^{-6}\,(m^2s^{-1})$

Density $\rho=2707\ Jkg/m^3$

Poisson ratio $\nu=0.35$

Coefficient of linear thermal expansion $\alpha=22.2\times10^{-6}\,1/K$

Lame constant $\mu=26.67$

Roots of transcendental equation

$\lambda_1=2:4048;\ \lambda_2=\ 5:5201;\ \lambda_3=\ 28:6537;\ \lambda_4=\ 11:7915;\ \lambda_5=\ 14:9309;\lambda_6\ =\ 18:0711;\ \lambda_7=\ 21:2116;$
$\lambda_8=\ 24:3525;\ \lambda_9=\ 27:4935\ and\ \lambda_{10}=30.6346$

are the positive roots of the transcendental equation $J_0(\lambda_n r)=0$.

We set for our convenience, $X=10^7$ and $Y=10^5 h\alpha E/D(1-\nu)$ which assume to be constants.

The numerical calculation has been carried out with the help of computational mathematical software Mathcad-2000 and graphs are plotted by using Origin software.

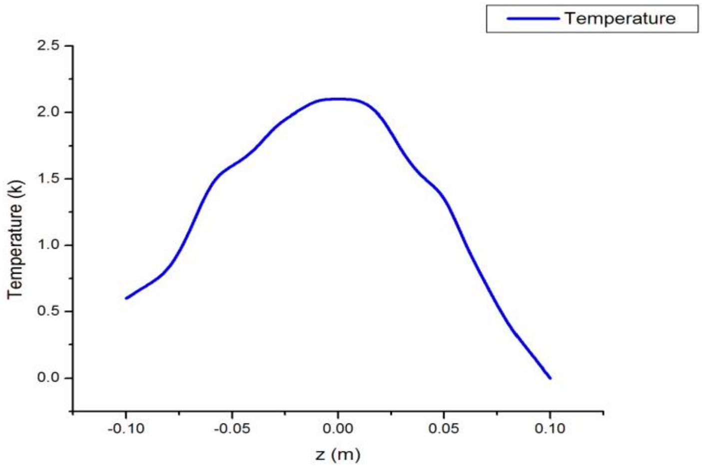

Fig. 1: Temperature distribution in thin circular plate

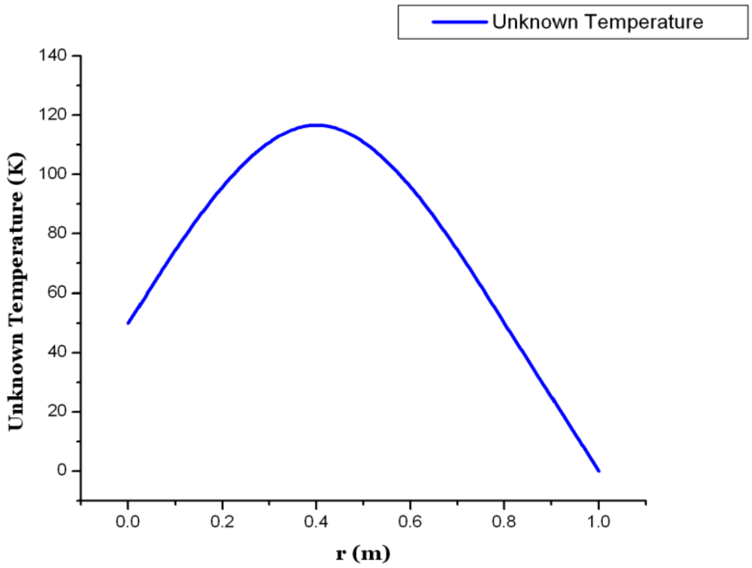

Fig. 2: Unknown Temperature distribution in thin circular plate

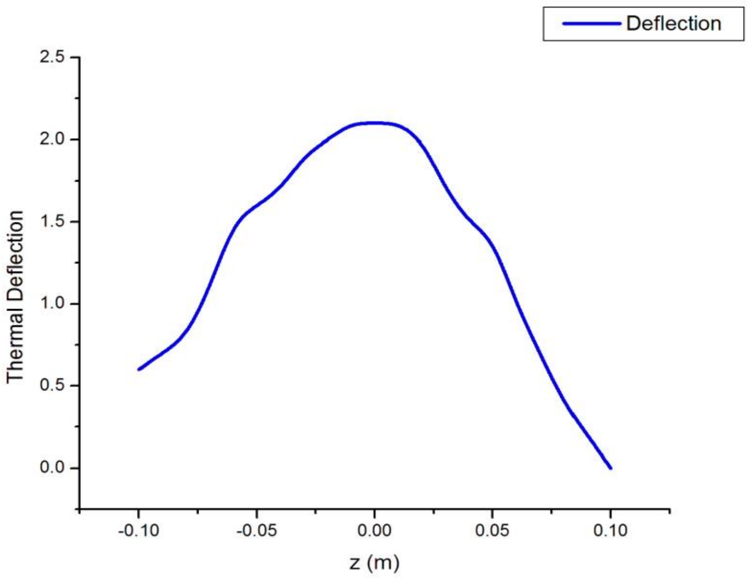

Fig. 3: Deflection distribution in thin circular plate

Figure 1, show that the temperature goes on increasing from upper surface up to the center and then slowly goes on decreasing towards lower surface of the thin circular plate.

Figure 2, show that the unknown temperature at center increases up to $r = 0.4$ and then vanishes on outer surface of the thin circular plate.

Figure 3, show that the quasi-static thermal deflection which increases from upper surface up to $z=0$ and then analytically goes on decreases towards lower surface of the thin circular plate.

V. CONCLUSION

This article investigates the temperature, unknown temperature at lower surface and quasi-static thermal deflection due to unknown temperature $g(t,r)$. First, the mathematical model is constructed, and then the series solutions are obtained by using integral transform methods. As a special case and numerical results the functions and parameters are consider and the temperature, unknown temperature and quasi-static thermal deflection on upper surface determine by using MathCAD software and illustrated graphically by using Origin software. This type of inverse problems has the many applications in engineering such as main shaft of a lathe machine and aircraft structure. The results obtained here are mainly useful in the determination of state of strain in a circular plate.

REFERENCES

[1] Grysa, K. and Cialkowski M. J. 1980. On a certain inverse problem of temperature and thermal stress fields, Acta Mechanics, 36: 169-185.

[2] Grysa, K. and Koalowski, Z. 1982. One dimensional problem of temperature and heat flux at the surfaces of a thermoelastic slab, Nuclear Engg., 74: 1-14.

[3] Dai, H.L., Yan, X. and Jiang, H.J. 2013. Thermoviscoelastic behavior in a circular HSLA steel plate," J. Therm. Stress., 36 (10): 1112–1130. DOI: 10.1080/01495739.2013.818895.

[4] Khobragade and P. C. Wankhede, An inverse steady state problem of thin circular plate in Marchi-Fasulo transform domain, The Journal of Indian Academy of Mathematics, 25(2).

[5] Khobragade, K. W., Varghese, V. and Khobragade, N. W. 2009 An inverse transient thermoelastic problem of a thin annular disc, Applied Mathematics E-Notes, 6: 17-25.

[6] Deshmukh, K. C., Warbhe, S. D. and Kulkarni, V. S. 2009. Quasi-static thermal deflection of thin clamped circular plate due to heat generation, Journal of Thermal Stresses, 32: 877-886.

[7] Ghonge, B. E. and Ghadle, K. P. 2010. An inverse transient thermoelastic problem of solid sphere, Bulletin of Pure & Applied Sciences-Mathematics and Statistics, 29 (1): 13-21. DOI: 10.5958/2320-3226.

[8] Ghonge, B. E. and Ghadle, K. P. 2011. An Inverse Transient Thermoelastic Problem of Hollow Sphere, Advances in Applied Research 3 (1): 76-82.

[9] Ghonge, B. E. and Ghadle, K. P. 2011. Inverse transient thermoelastic behavior of annular disc by using Marchi-Zgrablich and Laplace transform technique, International Journal of Physics and Mathematical Sciences, 1(1): 68-75.

[10] Ghonge, B. E. and Ghadle, K. P. 2012. Study of exact solution of deflection of thermoelastic circular plate by using Marchi-Fasulo integral transform, Global Journal of Mathematical Science. 4(2): 115-122.

[11] Ghonge, B. E. and Ghadle, K. P. 2011. An analytical approach to deflection of inverse transient thermoelastic thin circular plate-I, Global Journal of Computational Science and Mathematics, 1(2): 87-94.

[12] Ghonge, B. E. and Ghadle, K. P. 2012. Deflection of Transient Thermoelastic Circular Plate by Marchi-Zgrablich and Laplace Transform Technique, Theoretical and Applied Mechanics Letter, 2(2): 021004. DOI:10.1063/2.1202104

[13] Ghonge, B. E. and Ghadle, K. P. 2012. Two dimensional unsteady state non-homogeneous thermoelastic problem of circular plate, International Mathematical Forum, 7(33): 1645-1653.

[14] Ghonge, B. E. 2018. An Inverse Transient Thermoelastic Behavior of Circular Plate by Using Marchi-Fasulo and Laplace Integral Transform, International Journal of Engineering Science Invention, 7(2): 1-5.

[15] Eslami, M. R., Hetnarski, R. B., Ignaczak, J., Noda, N. Sumi, N. and Tanigawa, Y. 2013. Theory of Elasticity and Thermal Stresses. Netherlands: Springer, 2013.

[16] Hahn, D. W. and Ozisik, M. N. 2012. Heat Conduction, 3rd ed. Hoboken, NJ: John Wiley & Sons.

[17] Sneddon, I. N. 1974. The Use of Integral Transforms, McGraw-Hill Company, New York.