Bhaskara II :- A Pioneer In Mathematics And Astronomy

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ABSTRACT :-

Bhaskara II, also known as Bhaskaracharya, was a renowed Indian mathematician and astronomer who lived in 12th century. He considered one of the greatest mathematicians of medieval India, whose Contributions to mathematics and astronomy have these a significant impact on the development of those fields. His works, particularly siddhanta Siromani and Lilvati, Continue to be appreciated for their depth and clarity.

Bhaskara II known for his work on quadratic equations, Diophantine equations and the pell equation. He also developed the concept of the Charavela method, an iterative algorithm for solving pell's equation.

Key words :- Astronomy, Medieval, Significant, impact, Iterative, Algorithm.

Introduction :-

He was born in saka 1036 (A.D. 1114) and he wrote Siddhantasiromani in Saka 1072 (A.D. 1150) at the age of 36. Bhaskaracarya has given some information about his family background at the end of the chapter on problems in Goladhyaya. From this, it follows that he belonged to the Sandilya lineage and that he lived in Vijjalavida. In the second stanza of the following verses, he talks about his art of writing with great confidence:
He studied all sciences under the guidance of his father Mahesvara. Mahesvara was a great astrologer. According to late Mr. S.B. Dixit, Mahesvara was born in A.D. 1078. He wrote two books Karana-Grantha and Jataka-Tīka-Grantha. As Bhaskaracarya studied under the guidance of such a competent teacher, he became an intelligent in all over Subjects.

Bhaskara - II is a famous mathematician of ancient India. He is not only a famous Mathematician, but also an excellent Astronomer, and a Scientist during 1114-1185 A.D. Born on:- Biddur Bijapur, Karnataka state, India, and Death on Ujjain, India. Bhaskara - II is also called as Bhaskaracharya which mean "Bhaskara, the Teacher" or Bhaskara or Acharya . His father name was Mahesvara, a famous astrologer, who taught him Mathematics. This happened frequently in Indian society with generations of a family being excellent mathematicians and also acting as teachers to other family members. He was the leading mathematician of 12th century, who wrote the first work with full and systematic use of the decimal number system. He became head of the astronomical observatory at Ujjain, which was the leading mathematical centre of ancient India at that time. He was the lineal successor of the noted Indian outstanding Mathematicians:- Varahamihira "505-587" A.D. and Brahmagupta who worked there and built up this strong school of mathematical astronomy.

In different ways Bhaskracharya represented the peak of mathematical knowledge in the 12th century. He reached an understanding of the number systems and solving equations which was not achieved in Europe for several centuries.As an astronomer Bhaskara wrote about planetary positions and eclipses.In his mathematical works, particularly, Lilavati "The Beautifiul" arithmetic and Bijaganita "seed counting"

algebra, he not only used the decimal system but also compiled earlier problems from Brahmagupta and others.

He also filled many of the gaps in Brahmagupta's work, especially in obtaining a general solutions of different types of equations. He anticipated the modern convention of signs (minus by minus makes plus [(-)x(-) =+] minus by plus makes minus [(-) × (+) = (-)) and evidently was the first to gain some understanding of the meaning of division by zero, for he specifically stated that the value of is an infinite quantity, though his understanding seems to have been limited, for he also stated false that × 0 = a. 3-0 Bhaskara - II mentioned in his Beejaganitha that, ∞± k = ∞.

Bhaskara - II used letter to represent unknown quantities, much as in modern algebra, and solved indeterminate equation of 1" and 2nd degrees. He investigated regular polygons up to those having 384 sides, thus obtaining a good approximated value of = 3.141666. In other of his works, notably Siddhantasiromani "Head Jewel of Accuracy" and Karanakutuhal "Calculation of Astronomical Wonders", he wrote on his astronomical observations of planetary positions, conjunctions, eclipses, cosmography, geography, and the mathematical techniques and astronomical equipment used in these studies. He wrote siddhanta Shiromani at the age of 36 in 1150 A.D., and this work consisting of about 1450 verses has divided into four parts:- Each and every part of the book consists of huge number of verses, and so each of them can be considered as separate book.

1) Lilatvati has 278 verses.

2) Bijaganit has 213 verses.

3) Goladhyaya has 451 verses, and

4) Grahaganit has 501 verses.

Bhaskara II's most significant work, the Siddhanta Shiromani, is a comprehensive treatise on mathematics and astronomy. It is divided into four parts:

1. Lilavati:- This section is dedicated to arithmetic, covering topics such as fractions, progressions, and the solution of linear and quadratic equations. It also includes discussions on zero, infinity, and negative numbers.

2. Bijaganita:- This section focuses on algebra, exploring topics such as indeterminate equations, quadratic quations, and the solution of simultaneous equations. Bhaskara II introduced Innovative techniques for solving algebraic problems, including the use of symbols to represent unknown quantities.

3. Grahaganita:- This section deals with planetary motions and astronomical calculations. Bhaskara II presented accurate calculations for planetary positions and eclipses, demonstrating a deep understanding of celestial mechanics.

4. Goladhyaya:- This section covers spherical astronomy, including discussions on the celestial sphere, eclipses, and the construction of astronomical instruments.

Key Contributions:-

Bhaskara II's contributions to mathematics and astronomy are numerous and far-reaching. Some of his most notable achievements include here as follows.

1. **Decimal Number System:-**

   Bhaskara II was one of the earliest mathematicians to systematically use the decimal number system, including the use of a decimal point. This was a significant advancement over the numeral systems used in other parts of the world at that time.

2. Arithmetic:-

Bhaskara's arithmetic text Lilavati covers the topics of definitions, arithmetical terms, interest computation, arithmetical and geometrical progressions, plane geometry, solid geometry, the shadow of the gnomon, methods to solve indeterminate equations, and combinations. Lilavati is divided into 13 chapters and covers many branches of mathematics, arithmetic, algebra, geometry, and a little trigonometry and measurement. Very specifically the contents include:

1. Definitions.

2. Properties of zero including division, and rules of operations with zero.

3. Further extensive numerical work, including use of negative numbers and surds.

4. Estimation of π.

5. Arithmetical terms, methods of multiplication, and squaring.

6. Inverse rule of three, and rules of 3, 5, 7, 9, and 11.

7. Problems involving interest and interest computation.

8. Indeterminate equations Kuttaka, integer solutions first and second order.

His contributions to this topic are particularly important, since the rules he gives are the same as those given by the renaissance European mathematicians of the 17th century, yet his work was of the 12th century. Bhaskara's method of solving was an improvement of the methods found in the work of Aryabhata and subsequent mathematicians.

His work is outstanding for its systematisation, improved methods and the new topics that he introduced. Furthermore, the Lilavati contained excellent problems and it is thought that Bhaskara's intention may have been that a student of 'Lilavati' should concern himself with the mechanical application of the method.

3. Algebra :-

His Bijaganita was a work in twelve chapters. It was the first text to recognize that a positive number has two square roots a positive and negative square root. His work Bijaganita is effectively a treatise on algebra and contains the following topics:-

1. Positive and negative numbers.

2. The 'unknown' (includes determining unknown quantities).

3. Determining unknown quantities.

4. Surds includes evaluating surds and their square roots.

5. Kuttaka for solving indeterminate equations and Diophantine equations.

6. Simple equations indeterminate of second, third and fourth degree .

7. Simple equations with more than one unknown.

8. Indeterminate quadratic equations as the type $ax^2 + b = y^2$.

9. Solutions of indeterminate equations of the second, third and fourth degree.

10. Quadratic equations.

11. Quadratic equations with more than one unknown.

12. Operations with products of several unknowns.

Bhaskara derived a cyclic, chakravala method for solving indeterminate quadratic equations of the form $ax^2 + bx + c = y$. Bhaskara's method for finding the solutions of the problem $Nx^2 + 1 = y^2$ the so-called "Pell's equation is very importance in Number Theory.

4. Trigonometry :-

The Siddhanta Shiromani demonstrates Bhaskara's knowledge of trigonometry, including the sine table and relationships between different trigonometric functions. He also developed spherical trigonometry, along with other interesting trigonometrical results. In particular Bhaskara seemed more interested in trigonometry for its own sake than his predecessors who saw it only as a tool for calculation. Among the many interesting results given by Bhaskara, results found in his works include computation of sines of angles of 18 and 36 degrees, and the now well known formulae for sin(a + b) and sin(a - b).

5. calculus :-

His work, the Siddhanta Shiromani, is an astronomical treatise and contains many theories not found in earlier works. Preliminary concepts of infinitesimal calculus and mathematical analysis, along with a number of results in trigonometry, differential calculus and integral calculus that are found in the work are of particular interest.

Evidence suggests. He was acquainted with some ideas of differential calculus Bhaskara also goes deeper into the differential calculus and suggests the differential coefficient vanishes at an extremum value of the function, indicating knowledge of the concept of infinitesimals

1. There is evidence of an early form of Roile's theorem in his work. The modem formutation of Rolle's theorem states that if f(a)(b)=0, then f(z) = 0 for some z with a <z<b

2. In this astronomical work he gave one procedure that looks like a precursor to infinitesimal methods. In terms is if zy then sin(y)-sin(x) = (y-x) cos(y) that is a derivative of sine although he did not develop the notion on derivative

3. Bhaskara uses this result to work out the position angle of the ecliptic, a quantity required for accurately predicting the time of an eclipse.

4. In computing the instantaneous motion of a planet, the time interval between successive positions of the planets was no greater than a truti, or a so of a second, and his measure of velocity was expressed in this infinitesimal unit of time.

5. He was aware that when a variable attains the maximum value, its differential vanishes.

6. He also showed that when a planet is at its farthest from the earth, or at its closest, the equation of the centre's measure of how far ra a planet planet is from the position in which it is predicted to be, by assuming it is to move uniformly vanishes. He therefore concluded that for some intermediate position the differential of the equation of the centre is equal to zero. In this result, there are traces of the general mean value theorem, one of the most important theorems in analysis, which today is usually derived from Rolle's theorem. The mean value formula for inverse interpolation of the sine was later founded by Parameshvara in the 15th century in the Lilavati Bhasya, a commentary on Bhaskara's Lilavati.

Madhava and the Kerala School mathematicians from the 14th century to the 16th century expanded on Bhaskara's work and further advanced the development of calculus in India.

6. Astronomy :-

As Using an astronomical model developed by Brahmagupta in the 7th century, Bhaskara accurately defined many astronomical quantities, including, for example, the length of the sidereal year, the time that is required for the Earth to orbit the Sun, as approximately 365.2588 days which is the same as in Suryasiddhanta, The modern accepted measurement is 365.25636 days,a difference of 3.5 minutes 29. His mathematical astronomy text Siddhanta Shiromani is written in two parts: the first part on mathematical astronomy and the second part on the sphere. The twelve chapters of the first part cover topics such as:

1. Mean longitudes of the planets.

2. True longitudes of the planets.

3. The three problems of diurnal rotation. Diurnal motion refers to the apparent daily motion of stars around the Earth, or more precisely around the two celestial poles. It is caused by the Earth's rotation on its axis, so every star apparently moves on a circle that is called the diurnal circle.

4. Syzyges.

5. Lunar eclipses.

6. Solar eclipses

7. Latitudes of the planets.

8. Sunrise equation,

9. The Moon's crescent.

10. Conjunctions of the planets with each other.

11. Conjunctions of the planets with the fixed stars.

12. The paths of the Sun and Moon.

The second part contains thirteen chapters on the sphere. It covers topics such as:

1. Praise of study of the sphere.
2. Nature of the sphere.
3. Cosmography and geography.
4. Planetary mean motion.
5. Eccentric epicyclic model of the planets.
6. The armillary sphere.
7. Spherical trigonometry.
8. Ellipse calculations.
9. First visibilities of the planets.
10. Calculating the lunar crescent.
11. Astronomical instruments.
12. The seasons.
13. Problems of astronomical calculations

Legacy :-

Bhaskara II's work had a profound impact on the development of mathematics and astronomy in India and beyond. His ideas were transmitted to other parts of the world through translations and commentaries, influencing the work of later mathematicians and astronomers. His contributions to the decimal number system, algebra, and calculus are still studied and appreciated in modern Day.

Conclusion :-

Bhaskara II's legacy as a brilliant mathematician and astronomer endures. His work continues to inspire and challenge mathematicians and scientists, demonstrating the remarkable achievements of Indian mathematics during the medieval period. By studying his contributions, we can gain a deeper understanding of the history of mathematics and the intellectual brilliance of ancient Indian civilization.

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