Madhava Of Sangamagrama: A Pioneer Mathematical Anlysis

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Abstract:-

In the tapestry of mathematical history, the name of madhava of Sangamagrama Stands out as a brilliant Luminary. This 14th-century Indian mathematician and astronomer, often referred to as the " founder of mathematical analysis" made groundbreaking contributions to calculus trigonometry and infinite series centuries before their formalization in Europe. His work, developed within the Kerala school of Astronomy and mathematics, laid the foundation for many of the Key concepts that underpin modern Calculus.

Keywords :- Tapestry, Brilliant, Luminary, Analysis, Underpin, Concepts.

Introduction:-

Madhava of Sangamagrama, a brilliant Indian mathematician and astronomer, is widely regarded as the founder of the Kerala School of Astronomy and Mathematics. He born around 1340 CE in the village of Sangamagrama in present-day Kerala, India, Madhava's groundbreaking contributions to the fields of calculus, trigonometry, and infinite series have left an enduring legacy.

The Kerala School of Astronomy and Mathematics, which flourished in southern India during the Late Middle Ages, was a remarkable period of mathematical and astronomical discovery. At the heart of this intellectual movement was Madhava of Sangamagrama, a brilliant mathematician and astronomer whose contributions to the field of calculus and infinite series were centuries ahead of their time.

Little is known about his early life and education, but it is clear that he was deeply interested in mathematics and astronomy from a young age. He was likely influenced by the rich mathematical tradition of India, which had produced notable mathematicians like Aryabhata, Brahmagupta, and Bhaskara II contributions to Mathematics

Madhava's most significant contributions to mathematics were in the areas of infinite series and calculus. He developed power series expansions for a wide range of trigonometric functions, including sine, cosine, tangent, and arctangent. These series were remarkably accurate and efficient, allowing for the calculation of trigonometric values to a high degree of precision.

Contribution:-

If we consider mathematies as a progression from finite processes of algebra to considerations of the infinite, then the first steps towards this transition typically come with infinite series expansions. It is this transition to he infinite series that is attributed to Madhava. In Europe, the first such series were developed by James Gregory in 1667. Madhava's work is notable for the series, but what is truly remarkable is his estimate of an error term .This implies that he understood very well the limit nature of the infinite series.

Thus, Madhava may have invented the ideas underlying infinite series expansions of functions, power series, trigonometric series, and rational approximations of infinite series.

However, as stated above, which results are precisely Madhava's and which are those of his successors is difficult to determine. The following represents a summary of results that have been attributed to Madhava by various scholars.

Infinite series

Among his many contributions, he discovered infinite series for the trigonometric functions of sine, cosine, tangent, and many methods for calculating the circumference of a circle. One of Madhava's series is known from the text Yuktibhasa, which contains the derivation and proof of the power series for inverse tangent, discovered by Madhava In the text. Jyesthadeva describes the series in the following manner.

“The first term is the product of the given sine and radius of the desired are divided by the cosine of the arc. The succeeding terms are obtained by a process of iteration when the first term is repeatedly multiplied by the square of the sine and divided by the square of the cosine. All the terms are then divided by the odd numbers 1. 3. 5. The arc is obtained by adding and subtracting respectively the terms of odd rank and those of even rank. It is laid down that the sine of the arc or that of its complement whichever is the smaller should be taken here as the given sine. Otherwise the terms obtained by this above iteration will not tend to the vanishing magnitude.”

This yields:

$$r\theta = \frac{r\sin\theta}{\cos\theta} - (1/3)\,r\,\frac{(\sin\theta)^3}{(\cos\theta)^3} + (1/5)\,r\,\frac{(\sin\theta)^5}{(\cos\theta)^5} - (1/7)\,r\,\frac{(\sin\theta)^7}{(\cos\theta)^7} + \cdots$$

or equivalently:

$$\theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} - \frac{\tan^7\theta}{7} + \cdots$$

This series is Gregory's series named after James Gregory, who rediscovered it three centuries after Madhava. Even if we consider this particular series as the work of Jyesthadeva, it would pre-date Gregory by a century, and certainly other infinite series of a similar nature had been worked out by Madhava. Today, it is referred to as the Madhava-Gregory-Leibniz series .

Trigonometry

Madhava composed an accurate table of sines. Madhava's values are accurate to the seventh decimal place. Marking a quarter circle at twenty-four equal intervals, he gave the lengths of the half-chord (sines) corresponding to each of them. It is believed that he may have computed these values based on the series expansions:

sin q= $q-q^3/3! + q^5/5! - q^7/7! + \ldots\ldots$

cos q= $1- q^2/2! + q^4/4! - q^6/6! + \ldots\ldots$

The value of π:-

Madhava's work on the value of the mathematical constant π is cited in the Mahajyänayana prakara "Methods for the great sines". While some scholars such as Sarma feel that this book may have been composed by Madhava himself, it is more likely the work of a 16th-century successor. This text attributes most of the expansions to Madhava, and gives the following infinite series expansion of r, now known as the Madhava-Leibniz series.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1},$$

which he obtained from the power-series expansion of the arc-tangent function. However, what is most impressive is that he also gave a correction term R_n, for the error after computing the sum up to in n terms, namely:

$R_n = (-1)^n/(4n)$, or

$R_n = (-1)^n.n\ (4n^2+1)$, or

$R_n = (-1)^n\ .\ (n^2+1)/(4n^3+5n)$,

where the third correction leads to highly accurate computations of π

It has long been speculated how Madhava found these correction terms . They are the first three convergents of a finite continued fraction, which, when combined with the original Madhava's series evaluated to n terms, yields about 3n/2 correct digits:

$$\frac{\pi}{4} \approx 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^{n-1}}{2n-1} + \cfrac{(-1)^n}{4n + \cfrac{1^2}{n + \cfrac{2^2}{4n + \cfrac{3^2}{n + \cfrac{4^2}{\cdots + \cfrac{\cdots}{\cdots + \cfrac{n^2}{n[4-3(n \bmod 2)]}}}}}}}.$$

The absolute value of the correction term in next higher order is

$|R_n| = (4n^3+13n)/(16n^4+56n^2+9)$

He also gave a more rapidly converging series by transforming the original infinite series of π , obtaining the infinite series

$$\pi = \sqrt{12}\ \left(1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \cdots\right)$$

By using the first 21 terms to compute an approximation of π, he obtains a value correct to 11 decimal places (3.14159265359) The value of 3.1415926535898, correct to 13 decimals, is sometimes attributed to Madhava, but may be due to one of his followers. These were the most accurate approximations of π given since the 5th century .

The text Sadratnamala appears to give the astonishingly accurate value of π = 3.14159265358979324.

Madhava also carried out investigations into other series for arc lengths and the associated approximations to rational fractions of π.

Calculus

Madhava developed the power series expansion for some trigonometry functions which were further developed by his successors at the Kerala school of astronomy and mathematics. Madhava also extended some results found in earlier works, including those of Bhaskara II . However, they did not combine many differing ideas under the two unifying themes of the derivative and the integral, show the connection between the two, or turn calculus into the powerful problem-solving tool we have today.

Madhava's works

K. V Sarma has identified Madhava as the author of the following works:

1. Golavada
2. Madhyamanayanaprakara
3. Mahajyanayanaprakara
4. Lagnaprakarana
5. Venvaroha
6. Sphutacandrapti
7. Aganita-grahacara
8. Chandravakyani

Legacy and Impact :-

Madhenva of Sangamagrama's work, though largely unknown outside India for Centuries, has had a profound impact on the development of mathematics. His pioneering discovering in Calculus and Infinite series anticipated many of the ideas that were later formalized by European mathematicians Such as Newton and Leibniz.

The Kerala school, founded by Madhava, represents a remarkable period of mathematical and astronomical innovation in India. Its achievements, particularly in the field of Calculus, demonstrate the depth and sophisticatian of Indian mathematics during the medieval period.

Conclusion:-

Madhava of Sangamagrarma was a brilliant mathematician and astronomer whose contributions to the field of mathematics were centuries ahead of their time. His discoveries in infinite series and calculus were revolutionary and had a profound impact on the development of mathematics. The Kerala School of Astronomy and Mathematics, of which Madhava was the founder, was a remarkable period of mathematical and astronomical discovery. Madhava's Legacy Continues to inspire mathematicians and scientists all over the world.

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