# INTEGRAL INVOLVING EXTENDED JACOBI POLYNOMIAL, I-FUNCTION AND GENERAL CLASS OF POLYNOMIALS

Dr. PAWAN AGRAWAL
*Department of Mathematics, Raj Rishi College,Alwar (Rajasthan)*

***Abstract*—In this paper we establish finite integral which are believed to be new .Our integral involve the product of the extended Jacobi polynomials,I-function and general class of polynomials, on account of the general nature of the function and polynomials occurring in the integral our findings provide interesting extensions of a large number of results.**

**I. INTRODUCTION :** TO UNIFY THE CLASSICAL ORTHOGONAL POLYNOMIALS VIZ.JACOBI, HERMITE AND LAGUERRE FUJIWARA[2] DEFINED A CLASS OF GENERALIZED CLASSICAL POLYNOMIALS BY MEANS OF FOLLOWING RODRIGUES FORMULA:

$$Rn(x)=\frac{(-1)^n\ k^n}{n!(x-p)^{\beta}\,(q-x)^{\alpha}}\frac{d^n}{dx^n}[(x-p)^{\beta+n}\ (q-x)^{\alpha+n}], p<x<q, \alpha>-1, \beta>-1 \quad (1.1)$$

Denote these polynomials by Fn (β, α; x) and call them extended Jacobi polynomials Thakare [9] obtained the following form of Rn(x)=Fn(β, α; x)

$$Fn(\beta,\alpha;x)=\frac{(-1)^n\ K^n (q-x)^n (1+\beta)_n}{n!}\,2F1\left[\begin{matrix}-n,-n-\alpha; & \frac{p-x}{q-x}\\ 1+\beta; & \end{matrix}\right] p<x<q \quad (1.2)$$

Fujiwara [2] proved that when p=-1,q=1 and k=$\frac{1}{2}$

$$Fn(\beta,\alpha;x)=P_n^{(\alpha,\beta)}(x) \text{Where } P_n^{(\alpha,\beta)}(x)=\frac{(1+\beta)_n}{n!}\left(\frac{x-1}{2}\right)^n 2F1\left[\begin{matrix}-n,-n-\alpha; & \frac{x+1}{x-1}\\ 1+\beta; & \end{matrix}\right] \text{is Jacobi polynomial [3]} \quad (1.3)$$

Saxena [5] introduced the I-function defined as:

$$I^{m,n}_{p,q,R}[z]=I^{m,n}_{p_i,q_i:R}\left[z\ \Big|\ \begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right]=\frac{1}{2\pi i}\int_L \phi(\xi)z^{\xi}d\xi \quad (1.4)$$

where

$$\phi(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{R}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right\}}$$

For R=1, the I-Function reduces to well –known Fox's H-function [6]

A general class of polynomials [7, p. 1, eq. (1)]

$$S_N^M[x]=\sum_{k=0}^{[N/M]}\frac{(-N)_{Mk}A_{N,k}x^k}{k!},\quad (N=0,1,2,\ldots..) \quad (1.5)$$

where M is an arbitrary positive integer and the coefficient $A_{N,k}(N,k\geq 0)$ are arbitrary nstants real or complex. On suitably specializing the coefficient $A_{N,k}$ $S_N^M[x]$ yields a number of known polynomials as its special cases. These include, among others, Laguerre polynomials, Hermite polynomials and several others [8. pp. 1 58-161].

**PRELIMINARIES**

In this paper we need the following results :

(i) [1] p.10, eq.(13) Viz$\int_b^a (t-b)^{x-1}(a-t)^{y-1}\,dt=(a-b)^{x+y-1}B(x,y), Re(x)>0, Re(y)>0, b<a$ (2.1)

Where B(x,y) is beta function .

(ii) The Hyper Geometric function [3]$_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_nz^n}{(c)_n\ n!}$ (2.2)

(iii) Vandermonde's theorem [3]

$${}_2F_1(-n,b;c;1)=\frac{(c-b)_n}{(c)_n}, c\neq 0,-1,-2,\ldots; \quad (2.3)$$

(iv) The following results:

$$(a)_n=\frac{(-1)^n\Gamma(1-a)}{\Gamma(1-a-n)} \quad (2.4)$$

and $$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)} \quad (2.5)$$

**Main Integral :**

$$\int_p^q (x-p)^t (q-x)^\alpha F_n(\beta,\alpha;x)$$

$$I^{m,n}_{p_i q_{i},R}\left[z(x-p)^h \middle| \begin{matrix} (a_j,\alpha_j)_{1,n}; & (a_{ji},\alpha_{ji})_{m+1,r} \\ (b_{j,\beta},\beta_j)_{1,m}; & (b_{ji},\beta_{ji})_{m+1,qi} \end{matrix}\right]$$

$$S^{M_1}_{N_1}[e(x-p)^v]dx$$

$$\frac{k^n}{n!}\Gamma(n+\alpha+1)\sum_{k_1=0}^{[N_1/M_1]}\frac{(-N_1)_{M_1k_1}A_{N_1k_1}}{k_1!}e^{k_1}(q-p)^{t+\alpha+n+1+vk_1}$$

$$I^{m,n+2}_{p_i+2,q_{i},+2,R}\left[z(q-p)^h \middle| \begin{matrix} (-t-vk_1,h);\ (\beta-t,h)(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{m+1,r} \\ (b_{j,},\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,qi};(\beta-t+n,h);(-1-n-t-\alpha-vk_{1,},h) \end{matrix}\right] \quad (3.1)$$

Provided that Re $(\alpha)>-1, Re\ (t+h(\frac{b_j}{\beta_j}))>-1, h>0$, j=1,…, m

To establish (3.1) replace I-function by its Mellin-Barnes contour integral form (1.4) and get following form of integral $(say\Delta)$

$$\Delta=\int_p^q (x-p)^t(q-x)^\alpha F_n(\beta,\alpha,x)\, S^{M_1}_{N_1}[e(x-p)^v] \quad (3.2)$$

$$\left\{\frac{1}{2\pi i}\int_L \phi(\xi)z^\xi(x-p)^{h\xi}dz\right\}dx$$

Now interchange the order of integration which is justified due to absolute convergence of integral involved in the process, we get :

$$\Delta=\frac{1}{2\pi i}\int_L \phi(\xi)z^\xi\left[\left[\int_p^q (x-p)^{t+h\xi}(q-x)^\alpha\right.\right.$$

$$F_n(\beta,\alpha,x)\, S^{M_1}_{N_1}[e(x-p)^v]dx\Big]d\xi \quad (3.3)$$

Now we put value of Fn($(\beta,\alpha,x)$ from (1.2) and value of $S^{M_1}_{N_1}[e(x-p)^v]$ from (1.5) in (3.3) and interchange the order of integration and summation and using the result (2.1) to (2.5) and after little simplification it becomes:

$$\Delta=\sum_{k_1=0}^{[N_1/M_1]}\frac{(-N_1)_{M_1k_1}A_{N_1k_1}}{k_1!}e^{k_1}\frac{k^n}{n!}\Gamma(n+\alpha+1)$$

$$(q-p)^{t+\alpha+h+1+vk_1}\frac{1}{2\pi i}\int_L\frac{\Gamma(1-(-t)+h\xi+vk_1)\Gamma(1-(\beta-t)+h\xi)}{\Gamma(1-(n+\beta-t)+h\xi)\Gamma(1-(-1-t-\alpha-h-vk_1)+h\,\xi)}\phi(\xi)[z(q-p)^h]^\xi d\xi \quad (3.4)$$

Now using definition of I-function, we obtain reqd. result (3.1)

**Special Cases:**

In the main result if we take $N_1$ =0 (the polynomial $S_0^{M_1}$ will reduce to Ao,o Which can be taken to be unity without loss of generality), we arrive at a result given by S. C. Sharma [4, eq. (3.1)].

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