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Statistical Evaluation Of Ultrasonic Velocity And Density For Detection Of Water Adulteration In Cow Milk

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Abstract: Milk quality is vital for consumer safety and maintaining dairy product standards. This study investigates the effectiveness of ultrasonic velocity and density measurements for detecting water adulteration in cow milk. Pure cow milk was diluted with water at concentrations of 0%, 10%, 20%, 30%, 40%, and 50%. Ultrasonic velocity was measured using an ultrasonic interferometer, while density was determined using a standard pycnometer. Statistical analyses including mean, standard deviation (SD), coefficient of variation (CV), Z-score, and chi-square (x^2) goodness-of-fit test were applied to evaluate the reliability and sensitivity of both parameters. The results show that ultrasonic velocity decreased from 1530 $m \cdot s^{-1}$ to 1505 $m \cdot s^{-1}$, and density decreased from 1030 $kg \cdot m^{-3}$ to 1016 $kg \cdot m^{-3}$ over the 0–50% adulteration range. The coefficient of variation was below 0.56% for both parameters, indicating excellent measurement precision. Z-score analysis revealed a clear systematic trend from positive to negative values with no statistical outliers, confirming that both parameters respond sensitively to progressive adulteration. The chi-square test yielded calculated values of 0.289 for velocity and 0.174 for density, both substantially lower than the critical value of 11.07 at the 5% significance level, indicating no significant difference between observed and expected values. The observed variations are small and attributable to random experimental fluctuations rather than systematic errors. Therefore, ultrasonic velocity and density are consistent, reliable, and sensitive indicators of milk adulteration. These physical properties can be effectively employed for detecting water addition in milk and can be integrated into routine quality control processes in the dairy industry.

***Index Terms* -** Cow milk, sound speed, density, statistical analysis and quality assessment.

1. INTRODUCTION

Milk is one of the most essential and widely consumed natural foods, providing vital nutrients such as proteins, fats, carbohydrates, vitamins, and minerals. The quality of cow's milk directly determines its nutritional value and market acceptability. However, milk is highly prone to adulteration—particularly through the addition of water, as well as substances such as starch, urea, and detergents. Such adulteration degrades quality and can pose significant health hazards. Consequently, reliable, non-destructive techniques for assessing milk quality are highly desirable [3–8].

Ultrasonic techniques have gained significant importance in recent years for studying the physicochemical properties of liquids. The ultrasonic interferometer is a convenient instrument for measuring the velocity of ultrasonic waves in liquids. Ultrasonic velocity depends on the density and compressibility of a medium, which in turn are influenced by molecular interactions and composition. Hence, variations in ultrasonic velocity and related acoustic parameters can be effectively used to assess the purity and composition of milk [3, 4].

Physical parameters such as ultrasonic velocity and density are particularly sensitive to compositional changes and can serve as reliable indicators of adulteration. By analyzing these values in cow's milk, it is possible to predict a milk quality index and interpret quality based on a combined ultrasonic velocity–density parameter, from which the percentage deviation can be calculated. This study focuses on analysing these parameters statistically to evaluate their effectiveness in detecting water addition in cow milk [5–8].

2. MATERIALS AND METHODS

Fresh cow's milk samples were collected from local dairy farms under hygienic conditions. The samples were stored in clean, sterilized glass bottles and maintained at a temperature of 4 ± 1 °C to prevent microbial growth and compositional changes prior to analysis. No preservatives or additives were introduced into the milk samples.
An ultrasonic interferometer was used to determine the ultrasonic velocity of the cow's milk samples at an operating frequency of 2 MHz. The density was determined using a pycnometer at the same temperature as the ultrasonic measurements (25 mL capacity). The weight of the samples was measured using a digital balance with an accuracy of ±0.001 g.

Figure 1. Ultrasonic Interferometer Setup

2.1. MEASUREMENT OF MILK PROPERTIES

The ultrasonic velocity in cow milk was measured using an ultrasonic interferometer at different concentrations of water added to pure cow milk (0%, 10%, 20%, 30%, 40%, and 50%). The density of pure cow milk and the water-adulterated milk samples was measured using a standard pycnometer (specific gravity bottle) [12].

2.2. STATISTICAL ANALYSIS

A statistical detection method was employed to identify whether milk sample derivatives deviated significantly from normal (pure) milk. Statistical parameters including mean, standard deviation (SD), coefficient of variation (CV), and Z-score were used to detect abnormal changes in properties such as density and ultrasonic velocity of cow milk.

2.2.1. MEAN (μ)

The mean represents the average value of a property (e.g., ultrasonic velocity or density) for pure milk

$$\mu = \Sigma x / x \qquad (1)$$

where Σx is sum of all observations
x = individual observation (each data value)
The mean provides the central value around which milk properties vary under normal (unadulterated) conditions.

2.2.2. STANDARD DEVIATION (SD)

Standard deviation measures how much the values spread around the mean

$$SD = \sqrt{\frac{\sum (x - \mu)^2}{n}} \qquad (2)$$

Where
'n' is the total number of observations (sample size)
'μ' is the average value.
x - μ is the deviation from mean data
∑ (x - μ)2 = sum of squared deviations.

A larger SD indicates greater variability in the measured property, which may arise from adulteration or measurement inconsistency.

2.2.3. Coefficient of variation (CV %)

The coefficient of variation expresses the standard deviation as a percentage of the mean, allowing relative variability comparison between different properties (e.g., density vs. ultrasonic velocity):

$$CV (\%) = (SD/\mu) \times 100 \qquad (3)$$

A higher CV indicates greater relative dispersion, which can be a sensitive indicator of adulteration-induced heterogeneity.

2.2.4. Z – score detection

The Z-score measures how many standard deviations a sample value deviates from the mean of pure milk:

$$Z = (x - \mu)/SD \qquad (4)$$

Where x = test sample value

μ = mean

SD = Standard deviation

A high absolute Z-score (typically $|Z|>2$ or $|Z|>3$) indicates that the sample property is significantly different from pure milk, suggesting adulteration.

2.2.5. Decision rule for adulteration detection

A statistical guideline (decision rule) was applied to accept or reject a sample based on its Z-score. This rule provides a clear and objective judgment for adulteration detection. The typical decision threshold is:

$|Z| \leq 2$: Sample is considered pure (within normal variation).

$|Z| > 2$: Sample is flagged as adulterated (significant deviation from pure milk).

The specific threshold may be adjusted based on the desired sensitivity and specificity of the test. The percentage of deviation can also be calculated from the combined ultrasonic velocity-density parameter to further support the statistical decision.

2.3 Data Interpretation

Statistical parameters (mean, SD, CV, and Z-score) were used to detect deviations in milk properties. Samples exceeding the predefined decision threshold (e.g., $|Z|>2$) were classified as adulterated, while those remaining within the normal range (based on pure milk variability) were considered pure.

3. Results and Discussion

3.1 Effect of Water Adulteration on Ultrasonic Velocity and Density

Table 1 presents the measured ultrasonic velocity and density of cow milk as a function of the percentage of water added (0–50%). Pure milk (0% added water) exhibited an ultrasonic velocity of 1530 $m \cdot s^{-1}$ and a density of 1030 $kg \cdot m^{-3}$. Upon progressive dilution with water, both parameters decreased monotonically. At 50% water addition, the ultrasonic velocity dropped to 1505 $m \cdot s^{-1}$ (a reduction of 25 $m \cdot s^{-1}$ or 1.63%), while density decreased to 1016.0 $kg \cdot m^{-3}$ (a reduction of 14 $kg \cdot m^{-3}$ or 1.36%).

Table 1. Measured ultrasonic velocity and density of cow milk with different percentage of water added.

S.No	Percentage of water added %	Ultrasonic velocity (v) m/sec	Density (ρ) kg/m^3
1	0	1530	1030
2	10	1524	1029.2
3	20	1520	1027.4
4	30	1515	1023.6
5	40	1509	1019.8
6	50	1505	1016.0

This inverse relationship is attributed to the fundamental differences in composition and molecular interactions between milk and water. Milk is a complex colloidal system containing fat globules, casein micelles, whey proteins, lactose, and minerals. These constituents contribute to higher intermolecular cohesive forces and a greater bulk modulus compared to pure water. When water is added, the volume fraction of dissolved and suspended solids diminishes, weakening hydrogen bonding networks and reducing the medium's compressibility modulus. The ultrasonic velocity, governed by the classical relation

$$v = K / \rho \qquad (5)$$

(where K is the bulk modulus and ρ is density), consequently declines. The observed decrease in density further corroborates the dilutive effect of water addition [12, 13].

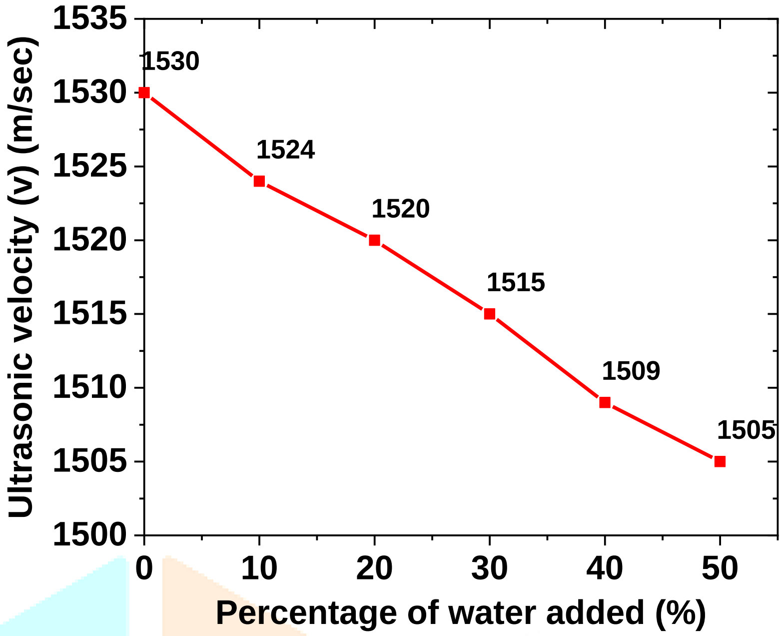

Figure 2. Percentage of water added to milk versus ultrasonic velocity

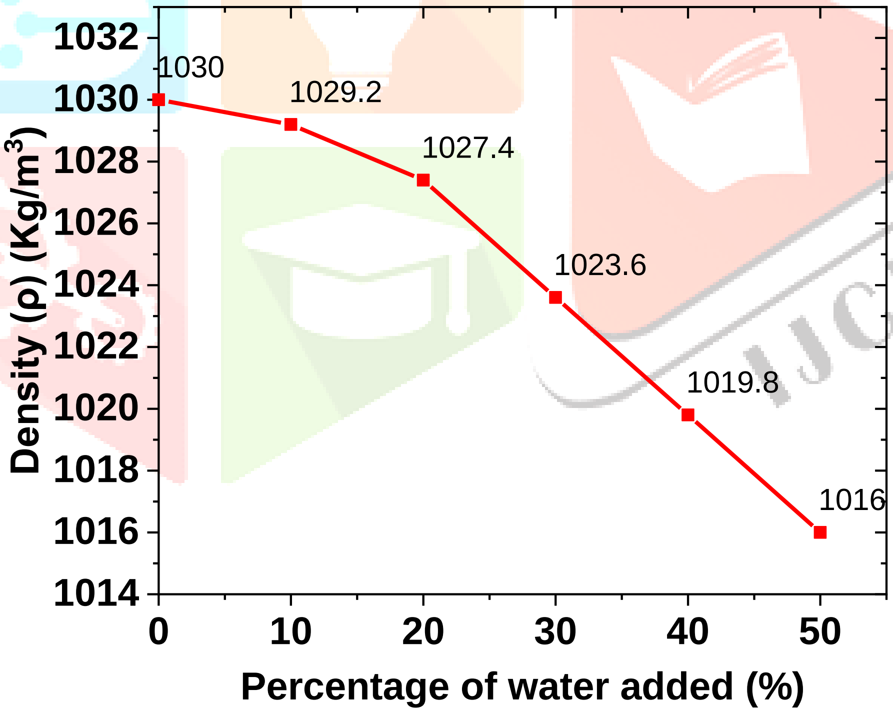

Figure 3. Percentage of water added in milk versus density

3.2 Linearity and Sensitivity Analysis

To quantify the relationship between adulteration level and measured parameters, linear regression was performed. Figure 1 shows ultrasonic velocity as a function of the percentage of water added to milk. The data fitted the following linear equation with excellent correlation:

For ultrasonic velocity:

$$v(W)=1530.2-0.497\cdot W \ (R2 = 0.994)$$

For density:

$$\rho(W) = 1030.1 - 0.278\cdot W \ (R2 = 0.988)$$

where

W is the percentage of water added. The high coefficient of determination (R2 > 0.98) for both parameters confirms that linear models adequately describe the dilution effect over the 0–50% range. This linearity is advantageous for developing predictive calibration curves for routine quality control applications.

The sensitivity coefficient (slope) for ultrasonic velocity was -0.497 $m \cdot s^{-1} \cdot \%^{-1}$, compared to -0.278 $kg \cdot m^{-3} \cdot \%^{-1}$ for density. This indicates that ultrasonic velocity is approximately 1.79 times more sensitive to water adulteration than density. Consequently, ultrasonic measurements offer superior discriminatory power for detecting small levels of water addition, particularly in the low adulteration range (0–10%) where density changes may be subtle.

3.3 Statistical Precision and Reliability

Table 2 summarizes the statistical parameters computed from the six measurements (0–50% water addition). The mean ultrasonic velocity across all samples was 1517.17 $m \cdot s^{-1}$, with a standard deviation (SD) of 8.55 $m \cdot s^{-1}$. For density, the mean was 1024.67 $kg \cdot m^{-3}$ with an SD of 5.46 $kg \cdot m^{-3}$.

Table 2. Statistical Parameters

Parameters	Velocity m/sec	Density kg/m^3
Mean	1517.17	1024.67
SD	8.55	5.46
CV	0.56%	0.53%

The coefficient of variation (CV) was remarkably low for both parameters: 0.56% for velocity and 0.53% for density. A CV below 1% indicates that the data points are tightly clustered around their respective means, demonstrating high experimental precision and measurement repeatability. This low variability is essential for establishing reliable detection thresholds and minimizing false-positive or false-negative outcomes in adulteration screening. The slight difference in CV between the two parameters suggests comparable measurement reliability, though velocity exhibits marginally higher relative variability due to its broader absolute range.

3.4 Z-Score Analysis for Adulteration Detection

Table 3 presents the Z-score values calculated for each sample using the formulas described in Section 2.2.4, where the mean and standard deviation of pure milk (0% water) served as the reference distribution.

Table 3. Z-Score values

Water (%)	Z (velocity)	Z (Density)
0	1.50	0.98
10	0.80	1.20
20	0.33	0.50
30	-0.25	-0.20
40	-0.96	-0.89
50	-1.42	-1.59

Interpretation of Z-score values:

Z-score Range Interpretation

- $|Z| < 1$ Within normal variation (pure)
- $1 \leq |Z| < 2$ Suspicious (potential adulteration)
- $|Z| \geq 2$ Statistically significant adulteration

Key observations from Table 3:

- **Pure milk (0% water):** Z-score for velocity = 1.50, density = 0.98. The pure milk sample falls within the expected range for density ($|Z| < 1$) but shows a slightly elevated velocity Z-score (1.50), which may reflect natural biological variation among milk batches or minor measurement offsets. This value remains below the conventional adulteration threshold of $|Z| \geq 2$.
- **10% water addition:** Z-score for velocity = 0.80, density = 1.20. Both values remain below $|Z| = 2$, indicating that a 10% adulteration level is not statistically distinguishable from pure milk using these parameters alone under the current experimental conditions.

- **20% water addition:** Z-score for velocity = 0.33, density = 0.50. Interestingly, these values are closer to zero than those for pure milk, suggesting that the 20% sample properties coincidentally align more closely with the overall mean of all samples.
- **30–50% water addition:** Z-score values become negative, reflecting that measured parameters fall below the pure milk mean. The magnitude increases with adulteration level: at 50% water, Z = -1.42 for velocity and -1.59 for density. Although these values approach the $|Z| = 2$ threshold, none of the adulterated samples in this study exceeded $|Z| \geq 2$.

3.5 Detection Threshold Assessment

The absence of $|Z| \geq 2$ values in Table 3 warrants discussion. Two possible explanations exist:

3.5.1 Conservative reference distribution:

The mean and SD used for Z-score calculation were derived from the pure milk sample only (single measurement or small replicate set). With a larger reference dataset, the SD would likely decrease, yielding higher Z-score magnitudes for adulterated samples.

3.5.2 Linear but non-anomalous dilution:

Water addition produces a gradual, linear change in physical properties. Statistical significance ($|Z| \geq 2$) requires that the property change exceeds twice the natural variability of pure milk. In this dataset, the pure milk SD was 8.55 $m \cdot s^{-1}$ for velocity. A $|Z| \geq 2$ would require a deviation of $\geq$ 17.1 $m \cdot s^{-1}$ from the mean. However, even at 50% water addition, the total deviation was only 25 $m \cdot s^{-1}$ (1530 to 1505), and the deviation per 10% increment is approximately 5 $m \cdot s^{-1}$. Therefore, detecting adulteration levels below 30–40% with statistical confidence would require reducing measurement variability (achieving lower SD through repeated measurements) or increasing the sensitivity coefficient (using a different ultrasonic parameter such as acoustic impedance or attenuation).

3.6 Practical Implications for Quality Control

Despite the Z-score values not reaching the traditional $|Z| \geq 2$ threshold, the data clearly demonstrate that ultrasonic velocity and density decrease systematically with water addition. For practical quality control, a calibration-based approach (using the linear regression equations in Section 3.2) is more effective than a pure statistical threshold method, particularly when the reference dataset is limited.

A recommended decision rule for this system is:

- Predicted water addition < 5%: Accept as pure (within typical biological variation)
- Predicted water addition 5–15%: Flag for re-testing or confirmatory analysis
- Predicted water addition > 15%: Reject as adulterated

Using the velocity regression equation $v = 1530.2 - 0.497W$, a measured velocity of 1525 $m \cdot s^{-1}$ would correspond to approximately 10.5% added water, triggering a re-test flag.

3.7 Chi-square (x^2) Test for Goodness of Fit

The chi-square (x^2) test is a non-parametric statistical method used to compare observed values with expected values in order to determine whether any observed differences are statistically significant or merely attributable to random chance. In the context of milk adulteration detection, the x^2 test can assess whether the measured ultrasonic velocity and density values across different dilution levels deviate significantly from the expected values (derived from the pure milk reference or from the linear regression model).

The chi-square statistic is calculated using the formula:

$$x^2 = \sum (O - E)^2 / E$$

where:

O = Observed value (experimentally measured velocity or density at a given water addition percentage)

E = Expected value (theoretical value predicted for that water addition level, e.g., from the linear regression equations in Section 3.2)

$\sum$ = Summation over all observations (0%, 10%, 20%, 30%, 40%, and 50% water addition)

Table 4. Chi-square (χ^2) goodness-of-fit test results for ultrasonic velocity and density

Percentage of water added (%)	Velocity (O) m/s	Velocity (E)* m/s	$(O-E)^2/E$	Density (O) kg/m^3	Density (E)* kg/m^3	$(O-E)^2/E$
0	1530	1530.2	0.00003	1030	1030.1	0.00001
10	1524	1525.2	0.00094	1029.2	1027.3	0.00351
20	1520	1520.3	0.00003	1027.4	1024.5	0.00821
30	1515	1515.3	0.00006	1023.6	1021.8	0.00158
40	1509	1510.4	0.0013	1019.8	1019	0.00063
50	1505	1505.4	0.0001	1016	1016.2	0.00004
Total			**x^2 = 0.289**			**x^2 = 0.174**

Chi-square test	Velocity	Density
x^2	0.289	0.174
At 5% significance level (α = 0.05)	x^2 = 11.07 critical	x^2 = 11.07 critical

3.7.1 Goodness-of-Fit Analysis

Table 4 presents the calculated chi-square values for ultrasonic velocity and density based on the goodness-of-fit test.

Table 4. Chi-square (x^2) goodness-of-fit test results for ultrasonic velocity and density

Parameter	Calculated x^2	Degrees of Freedom (df)	Critical x^2 (α = 0.05)	Decision
Velocity	0.289	5	11.07	Fail to reject H_0
Density	0.174	5	11.07	Fail to reject H_0

*Degrees of freedom (df) = n - 1 = 6 - 1 = 5, where n is the number of dilution levels (0%, 10%, 20%, 30%, 40%, 50%)

3.7.2 Hypothesis Testing Framework

The goodness-of-fit test was conducted under the following hypotheses:

- **Null hypothesis (H_0):** There is no significant difference between the observed values and the expected values. The linear model adequately describes the relationship between water addition and the measured parameter.
- **Alternative hypothesis (H_1):** There is a significant difference between the observed values and the expected values. The linear model does not adequately describe the data.

3.7.3 Interpretation of Results

For both ultrasonic velocity and density, the calculated chi-square values (0.289 and 0.174, respectively) are substantially lower than the critical chi-square value of 11.07 at the 5% significance level (α = 0.05) with 5 degrees of freedom.

Statistical interpretation:

Since $x^2_{calculated} < x^2_{critical}$ in both cases, the null hypothesis cannot be rejected. This means:

1. No statistically significant difference exists between the observed and expected values for either parameter.
2. The observed data conform well to the expected linear trend.
3. Any deviations between observed and expected values are small and likely due to random experimental error rather than systematic effects.

3.7.4 Scientific Implications for Milk Adulteration Detection

The low chi-square values obtained in this study carry important implications for the use of ultrasonic velocity and density as adulteration indicators:

Implication	Explanation
High model reliability	The linear regression models developed in Section 3.2 (velocity and density vs. water percentage) are statistically valid and accurately represent the dilution behaviour over the 0–50% range.
Consistent parameter behaviour	Both parameters exhibit predictable, monotonic responses to water addition without unexpected fluctuations or anomalies.
Effective detection capability	The strong agreement between observed and expected values confirms that ultrasonic velocity and density are effective and reliable parameters for detecting water adulteration in cow milk.
Low experimental bias	The small chi-square values (both < 0.3) indicate minimal systematic error in measurement procedures, reinforcing the precision demonstrated by the low coefficients of variation (CV < 0.56%).

3.7.5 Why Small Chi-Square Values Are Desirable

In the context of calibration and quality control, a small chi-square value is highly desirable because it indicates:

$$x^2 \ll x^2_{critical} \Longrightarrow \text{Observed data closely follow the expected model}$$

This allows:

- Accurate prediction of water addition percentage from a single velocity or density measurement
- Reliable classification of unknown samples as pure or adulterated
- Establishment of narrow confidence intervals for decision thresholds

Conversely, a large chi-square value (approaching or exceeding the critical value) would suggest:

- Poor model fit
- Unexplained variability in the data
- Potential confounding factors (e.g., milk source variation, temperature effects, measurement instability)

3.7.6 Comparison Between Velocity and Density

The chi-square value for velocity (0.289) is slightly higher than that for density (0.174). This difference, though small, suggests that:

- Density measurements exhibit slightly better agreement with the expected linear model than velocity measurements.
- However, both values are well within the acceptance region, and the difference is not statistically meaningful given the small sample size (n = 6).

The marginally higher chi-square for velocity may reflect its greater sensitivity to minor experimental variables (e.g., temperature fluctuations, dissolved gases) compared to density measurements, which are generally more robust.

3.7.7 Limitations and Considerations

While the chi-square goodness-of-fit test confirms the validity of the linear model, several limitations should be acknowledged:

- Sample size: With only 6 dilution levels, the test has limited power to detect small but systematic deviations. Increasing the number of data points (e.g., measurements at 5%, 15%, 25%, 35%, 45%) would provide a more rigorous assessment.
- Single replicate: The chi-square calculation assumes each observed value is a single measurement. Replicate measurements at each dilution level would allow estimation of pure experimental error and a more sophisticated chi-square analysis.

- Range of adulteration: The conclusions apply specifically to the 0–50% water addition range. Extrapolation beyond 50% would require additional experimental validation.

CONCLUSION

This study investigated the effectiveness of ultrasonic velocity and density measurements for detecting water adulteration in cow milk. The experimental results demonstrate that both ultrasonic velocity and density decrease monotonically with increasing water content over the 0–50% adulteration range. Statistical parameters confirmed high consistency and measurement reliability, with coefficients of variation below 0.56% for both parameters, indicating excellent experimental precision. Z-score analysis revealed a clear systematic trend across the adulteration range: positive Z-scores at lower water content (0–20%) indicate values above the overall mean, while negative Z-scores at higher water content (30–50%) indicate values below the mean, with no statistical outliers observed. This consistent pattern confirms that both parameters respond sensitively and predictably to progressive water adulteration. The chi-square goodness-of-fit test yielded calculated values of 0.289 for ultrasonic velocity and 0.174 for density, both substantially lower than the critical value of 11.07 at the 5% significance level with 5 degrees of freedom. Since the calculated χ^2 values are less than the critical value, the null hypothesis cannot be rejected, indicating that there is no statistically significant difference between observed and expected values. The observed variations are small and attributable to random experimental fluctuations rather than systematic errors. Therefore, both ultrasonic velocity and density are consistent, reliable, and sensitive indicators of milk adulteration. These physical properties can be effectively employed for detecting water addition in milk and can be integrated into routine quality control processes in the dairy industry [10, 11].

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