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LOGICAL OPERATIONS FOR NEUTROSOPHIC VAGUE HYPERSOFT MATRICES'S WITH THEIR PROPERTIES

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Abstract:
The challenge of managing uncertainty, imprecision, and indeterminacy in multi-attribute decision-making processes continues to pose difficulties in mathematical modeling. Neutrosophic Vague Sets and Hypersoft Sets each provide strong frameworks for addressing indeterminate data across multi-dimensional disjoint attribute sub-domains; however, their matrix representations currently lack a formal algebraic structure to facilitate complex logical evaluations. This study aims to fill this void by formally introducing logical operations for Neutrosophic Vague Hypersoft Matrices (NVHSMs) and systematically establishing their essential properties. The main goal of this research is to construct and investigate fundamental logical operators—namely the AND, OR, NOT, Union, and Intersection matrices—within the NVHSM framework. The approach is based on precise mathematical definitions of these operators, followed by a detailed theoretical investigation of their algebraic properties. The theoretical validity and structural coherence are actively illustrated and confirmed through sequential numerical examples. The key findings demonstrate that these proposed logical operations effectively maintain the integrity of neutrosophic vague data while precisely aggregating complex information from hypersoft attribute sets. The primary contribution of this research is the development of an accurate matrix-based computational tool that improves the processing of indeterminate information. As a result, this study has important practical implications for advanced multi-criteria decision-making (MCDM) systems. The proposed NVHSM logical operations create a foundational computational framework anticipated to enhance outcomes in intricate scenarios such as personnel selection, resource distribution, and broader management approaches, addressing the computational challenges of prior models when managing overlapping uncertainties.

Keywords: Neutrosophic Vague Soft Set, Vague Hypersoft Set, Neutrosophic Vague Hypersoft Set, NVHSM, AND, OR, NOT, Union, Intersection.

Introduction

Decision-making under uncertainty has become an important area of research in mathematics, computer science, engineering, and management. To deal with uncertain and imprecise information, Zadeh [1] introduced the concept of Fuzzy Sets **(**FSs**)** in 1965, which provides a mathematical framework for representing vagueness through membership values. Later, Atanassov [2] extended this theory by proposing Intuitionistic Fuzzy Sets **(**IFSs**)**, where both membership and non-membership degrees are considered, allowing a more flexible representation of uncertainty.

To overcome the limitations of intuitionistic fuzzy sets, Yager [3] introduced Pythagorean Fuzzy Sets **(**PFSs**)**, in which the squared sum of membership and non-membership degrees is less than or equal to one. This model offers greater flexibility in expressing uncertain information. Subsequently, Yager [4] demonstrated the effectiveness of Pythagorean fuzzy membership grades in solving multicriteria decision-making (MCDM) problems, highlighting their practical significance.

The development of Pythagorean fuzzy theory has led to various decision-making approaches. Zhang and Xu [5] extended the classical TOPSIS method to the Pythagorean fuzzy environment, enabling more reliable multiple criteria decision-making.

Therefore, they have attracted significant attention in recent years and continue to play an important role in the development of advanced multiple attribute decision-making methods. Deli and Broumi [10] introduced Neutrosophic Soft Matrices and their applications in decision-making. Mondal and Roy [11] investigated the properties of intuitionistic fuzzy soft matrices, whereas Saikia et al. [12] applied generalized fuzzy soft matrices to practical decision-making problems. These developments demonstrate the growing importance of fuzzy, neutrosophic, and soft matrix models in addressing uncertainty. Motivated by these advancements, the present study focuses on extending matrix-based frameworks for efficient decision-making under complex uncertain environments.

Literature Review: Decision-making under uncertainty has become a critical area of research across mathematics, computer science, engineering, and management. The foundational mathematical framework for managing imprecision was established when Zadeh introduced the concept of Fuzzy Sets (FSs) in 1965, representing vagueness through simple membership values. While revolutionary, this model was limited by its inability to capture hesitation. To address this, Atanassov extended the theory by proposing Intuitionistic Fuzzy Sets (IFSs), which consider both membership and non-membership degrees, thereby allowing for a more flexible representation of uncertainty. Seeking even greater flexibility, Yager introduced Pythagorean Fuzzy Sets (PFSs), where the squared sum of the membership and non-membership degrees is constrained to be less than or equal to one. Yager later demonstrated the effectiveness of Pythagorean fuzzy membership grades in solving multi-criteria decision-making (MCDM) problems, highlighting their practical significance. Consequently, Zhang and Xu successfully extended the classical TOPSIS method to the Pythagorean fuzzy environment to enable more reliable decision-making. Despite these advancements, standard fuzzy models often fail to adequately parameterize the specific attributes of an object. This led to the adoption of Soft Set theory, which provides a parameterized mathematical tool for uncertainty. However, standard Soft Sets treat attributes as a single set. In real-world scenarios, attributes often possess their own sub-attributes. This limitation necessitated the transition to Hypersoft Sets, which replace the single attribute function with a multi-dimensional Cartesian product of disjoint attribute sub-domains.

To manage indeterminate and inconsistent data within these complex parameterizations, the neutrosophic vague set theory was integrated. This evolution has culminated in the concept of Neutrosophic Vague Hypersoft Sets (NVHSS).

Broaden the discussion on the evolution from Soft Sets to Hypersoft Sets, and subsequently to Neutrosophic Vague sets. Discuss the limitations of standard matrices in these domains to better frame why NVHSMs are necessary.

Methodology/Proofs: If you haven't already, provide formal mathematical proofs (e.g., proving Commutative, Associative, or De Morgan's laws) for the Union, Intersection, AND, and OR matrices.

Application Section: Expand the "illustrative examples" into a full, real-world case study for personnel selection or a management problem. Show the step-by-step calculation of the matrices to reach a final decision score.

Preliminaries:

In the following section, we recalled fundamental concepts that help us to develop the structure of the current article such as NV, NVS, NSS, VHSS, and NVHSS.

Definition 1.1:

Let U be the universal set and E be the set of attributes with respect to U. Let (U) be the power set of U and A ⊆ E. A pair (F, A) is called a soft set over U, and its mapping is given as

$$F: A \longrightarrow P(U). \qquad (1)$$

It is also defined as

$$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset, \text{ if } e \notin A\}. \qquad (2)$$

Definition 1.2:

Let U be a universe and A be an NVS on U defined as

$A = \{v, (T_A(vx), I_A(vx), C_A(vx)): v \in U\}$, where $T, I, C: U \longrightarrow [0^-, 1^+]$, and

$0^- \leq T_A(vx) + I_A(vx) + C_A(vx) \leq 3^+$.

Definition 1.3:

Let U be the universal set and E be the set of attributes with respect to U. Let (U) be the set of neutrosophic values of U and A ⊆ E. A pair (F, A) is called a neutrosophic soft set over U, and its mapping is given as

$$F: A \longrightarrow P(U). \qquad (3)$$

Definition 1.4:

Let U be the universal set and (U) be the power set of U. Consider $\ell^{1a}, \ell^{2a}, \ell^{3a}, \ldots, \ell^{na}$ for $n \geq 1$ be n well-defined attributes, whose corresponding attributive values are, respectively, the set $L^{1a}, L^{2a}, L^{3a}, \ldots, L^{na}$ with $L^{ia} \cap L^{ja} = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \ldots, n\}$, then the pair $(F, L^{1a} \times L^{2a} \times L^{3a}, \ldots, L^{na})$ is said to be hypersoft set over U, where

$$F: L^{1a} \times L^{2a} \times L^{3a}, \ldots, L^{na} \longrightarrow P(U). \qquad (4)$$

Table 1: Tabular representation of the characteristic function.

	$\mathbf{L_1^a}$	$\mathbf{L_2^b}$	. . .	$\boldsymbol{L_ß^z}$
$\boldsymbol{U^1}$	$\boldsymbol{X^{a1}(U^{-a1}, U^{+a1}), (L_1^{+a1}, L_1^{-a1}}$	$\boldsymbol{X^{a1}(U^{+ab}, U^{-ab}), (L_1^{+ab}, L_1^{-ab}}$	. . .	$\boldsymbol{X^{a1}(U^{+a1}, U^{-a1}), (L_ß^{+az}, L_ß^{-az}}$
$\boldsymbol{U^2}$	$\boldsymbol{X^{a1}(U^{+a2}, U^{-a2}), (L_1^{+aa}, L_1^{-aa}}$	$\boldsymbol{X^{a1}(U^{+a2}, U^{-a2}), (L_1^{+ab}, L_1^{-ab}}$	. . .	$\boldsymbol{X^{a1}(U^{+a2}, U^{-a2}), (L_ß^{+az}, L_ß^{-az}}$
. . .	. . .	. . .	. . .	. . .
$\boldsymbol{U^n}$	$\boldsymbol{X^{a1}(U^{+a\alpha}, U^{-a\alpha}), (L_1^{+aa}, L_1^{-aa}}$	$\boldsymbol{X^{a1}(U^{+a\alpha}, U^{-a\alpha}), (L_1^{+ab}, L_1^{-ab}}$	. . .	$\boldsymbol{X^{a1}(U^{+a\alpha}, U^{-a\alpha}), (L_ß^{+az}, L_ß^{-az}}$

Definition 1.5:

Let U be the universal set and (U) be the power set of E. Consider ℓ^{1a}, ℓ^{2a}, ℓ^{3a}, . . . , ℓ^{na} for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are, respectively, the set L^{1a}, L^{2a}, L^{3a}, . . . , L^{na} with $L^{ia} \cap L^{ja} = \emptyset$, for $i \neq j$ and i, j ϵ {1, 2, 3, . . . , n} and their relation L^{1a}, L^{2a}, L^{3a}, . . . , L^{na} = S, then the pair (F, S) is said to be neutrosophic vague hypersoft set (NVHSS) over U, where

F: $L^{1a} \times L^{2a}$, L^{3a}, . . . , $L^{na} \longrightarrow$ P(U).

$$F(L^{1a} \times L^{2a}, L^{3a}, \dots, L^{na}) = \{x, T(F(S)), I(F(S)), F(F(S)), x \in U\} \quad (5)$$

where T is the membership value of truthiness, I is the membership value of indeterminacy, and F is the membership value of falsity such that T, I, F: U $\longrightarrow$ [0, 1] also $0 \leq T(F(S)) + I(F(S)) + F(F(S)) \leq 3$.

2) Logical Operations for NVHSMs with Their Properties

Definition 2.1:

Let O = $[O_{aij}]$ and M = $[M_{aij}]$ be two NVHSMs, where O_{aij} = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aijk}^{-0}, F_{aijk}^{+0}))$, $M_{ij} = ((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aijk}^{-M}, F_{aijk}^{+M}))$. Then, OR – operation between them is defined as follows:

F (O v M) = max $\{(T_{aijk}^{-O}, T_{aijk}^{+0}), (T_{aijk}^{-M}, T_{aijk}^{+M})\}$

F (O v M) = min $\{(I_{aijk}^{-0}, I_{aijk}^{+0}), (I_{aijk}^{-M}, I_{aijk}^{+M})\}$

F (O v M) = min $\{(F_{aijk}^{-0}, F_{aijk}^{+0}),\ (F_{aijk}^{-M}, F_{aijk}^{+M})\}$

Definition 2.2:

Let O = $[O_{aij}]$ and M = $[M_{aij}]$ be two NVHSMs, where O_{aij} = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aijk}^{-0}, F_{aijk}^{+0}))$, $M_{aij} = ((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aijk}^{-M}, F_{aijk}^{+M}))$. Then, AND – operation between them is defined as follows:

F(O∧M) = min $\{(T_{aijk}^{-0}, T_{aijk}^{+0}),\ (T_{aijk}^{-M}, T_{aijk}^{+M})\}$

F(O∧M) = max $\{(I_{aijk}^{-0}, I_{aijk}^{+0}),\ (I_{aijk}^{-M}, I_{aijk}^{+M})\}$

F(O∧M) = max $\{(F_{aijk}^{-0}, F_{aijk}^{+0}),\ (F_{aijk}^{-M}, F_{aijk}^{+M})\}$.

Propostion 2.3:

Let O = $[O_{aij}]$, M = $[M_{aij}]$ and N = $[N_{aij}]$ be three NVHSMs, where O_{aij} = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aijk}^{-0}, F_{aijk}^{+0}))$, $M_{aij} = ((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aijk}^{-M}, F_{aijk}^{+M}))$ and N_{aij} = $((T_{aijk}^{-N}, T_{aijk}^{+N}), (I_{aijk}^{-N}, I_{aijk}^{+N}), (F_{aijk}^{-N}, F_{aijk}^{+N}))$ Then,

1) M∨O = O∨M.
2) M∧O = O∧M.
3) O∨(M∨N) = (O∨M) ∨N.
4) O∧ (M∧N) = (O∧M) ∧N.

Proof: The proof of the above proposition is straightforward.

Definition 2.4:

Let O = $[O_{aij}]$ and an NVHSMs, where $O_{aij} = ((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aijk}^{-0}, F_{aijk}^{+0}))$. Then, the necessity operator for NVHSM is represented as ⊕ and defined as follows:

⊕ O = $((T_{aijk}^{-0}, T_{aijk}^{+0}),\ (I_{aijk}^{-0}, I_{aijk}^{+0}),\ 1 - (T_{aik}^{-0}, T_{aik}^{+0})$

Definition 2.5:

Let O = $[O_{aij}]$ and an NVHSMs, where $O_{aij} = ((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aijk}^{-0}, F_{aijk}^{+0}))$. Then, the possibility operator for NVHSM is represented as ⊕ and defined as follows:

⊕ O = (1- $(F_{aijk}^{-0}, F_{aijk}^{+0}),\ (I_{aijk}^{-0}, I_{aijk}^{+0}),\ (T_{aik}^{-0}, T_{aik}^{+0})$).

Proposition 2.6:

Let O = $[O_{aij}]$, M = $[M_{aij}]$ and N = $[N_{aij}]$ be two NVHSMs, where O_{aij} = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aik}^{-0}, F_{aik}^{+0}))$, $M_{aij} = ((T_{aijk}^{-M}, T_{aijk}^{+M}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aik}^{-M}, F_{aik}^{+M}))$ and Then,

1) ⊕ (O∪M) = ⊕O∪⊕M.
2) ⊕ (O∩M) = ⊕O∩⊕M.
3) □ (O∪M) = □O∪⊕M.
4) □ (O∩M) = □O∩⊕M.

Proof : We know that O = $[O_{aij}]$ = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aik}^{-0}, F_{aik}^{+0}))$ and $M = [M_{aij}]$ = $((T_{aijk}^{-M}, T_{aijk}^{+M}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aik}^{-M}, F_{aik}^{+M}))$ are two NVHSMs. Then, Utilizing Definition 13, we get

O∪M = $((T_{aijk}^{-0}, T_{aijk}^{+0}), (I_{aijk}^{-0}, I_{aijk}^{+0}), (F_{aik}^{-0}, F_{aik}^{+0})) \cup ((T_{aijk}^{-M}, T_{aijk}^{+M}), (I_{aijk}^{-M}, I_{aijk}^{+M}), (F_{aik}^{-M}, F_{aik}^{+M}))$,

$$O \cup M = \left(max\left((T_{aijk}^{-0}, T_{aijk}^{+0}), (T_{aijk}^{-M}, T_{aijk}^{+M})\right), \frac{\left((I_{aijk}^{-0}, I_{aijk}^{+0}) + (I_{aijk}^{-M}, I_{aijk}^{+M})\right)}{2}, min\ ((F_{aik}^{-0}, F_{aik}^{+0}), (F_{aik}^{-M}, F_{aik}^{+M}) \right)$$

Utilizing Definition 2.4, we get

$$\oplus\,(\mathrm{OUM}) = \left(max\left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right)\right), \frac{\left(\left(I^{-0}_{aijk}, I^{+0}_{aijk}\right) + \left(I^{-M}_{aijk}, I^{+M}_{aijk}\right)\right)}{2}, 1 - max\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right) \right)$$

Again, using Definition 2.4, we have

$\oplus\mathrm{O} = \left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(I^{-0}_{aijk}, I^{+0}_{aijk}\right), 1 - \left(T^{-0}_{aik}, T^{+0}_{aik}\right)\right)$

$\oplus\mathrm{M} = \left(\left(T^{-M}_{aijk}, T^{+M}_{aijk}\right), \left(I^{-M}_{aijk}, I^{+M}_{aijk}\right), 1 - \left(T^{-M}_{aik}, T^{+M}_{aik}\right)\right) \left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right)$

Then, utilizing Definition 13, we have

$$\oplus\mathrm{OU}\oplus\mathrm{M} = \left(max\left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right)\right), \frac{\left(\left(I^{-0}_{aijk}, I^{+0}_{aijk}\right) + \left(I^{-M}_{aijk}, I^{+M}_{aijk}\right)\right)}{2}, min\,(1 - \left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), 1 - \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right) \right)$$

$$\oplus\mathrm{OU}\oplus\mathrm{M} = \left(max\left(T^{0}_{aijk}, T^{M}_{aijk}\right), \frac{\left(I^{O}_{aijk} + I^{M}_{aijk}\right)}{2}, 1 - max\left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right)\right) \right)$$

Hence,

$\oplus\,(\mathrm{OUM}) = \oplus\mathrm{OU}\oplus\mathrm{M}$.

Proof: It is similar to assertion 1.

Proof: We know that O = $[O_{ij}]$ = $\left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(I^{-0}_{aijk}, I^{+0}_{aijk}\right), \left(F^{-0}_{aik}, F^{+0}_{aik}\right)\right)$ and $M = [M_{aij}] = \left(\left(T^{-M}_{aijk}, T^{+M}_{aijk}\right), \left(I^{-M}_{aijk}, I^{+M}_{aijk}\right), \left(F^{-M}_{aik}, F^{+M}_{aik}\right)\right)$ are two NVHSMs. Then,

Utilizing Definition 2.1, we get

$\mathrm{OUM} = \left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(I^{-0}_{aijk}, I^{+0}_{aijk}\right), \left(F^{-0}_{aik}, F^{+0}_{aik}\right)\right) \cup \left(\left(T^{-M}_{aijk}, T^{+M}_{aijk}\right), \left(I^{-M}_{aijk}, I^{+M}_{aijk}\right), \left(F^{-M}_{aik}, F^{+M}_{aik}\right)\right)$,

$$\oplus\,\mathrm{O} \cup \oplus\,\mathrm{M} = \left(max\left(\left(T^{-0}_{aijk}, T^{+0}_{aijk}\right), \left(T^{-M}_{aijk}, T^{+M}_{aijk}\right)\right), \frac{\left(I^{O}_{aijk} + I^{M}_{aijk}\right)}{2}, min\left(\left(F^{-0}_{aijk}, F^{+0}_{aijk}\right), \left(F^{-M}_{aijk}, F^{+M}_{aijk}\right)\right) \right).$$

Utilizing Definition 2.5, we get

$$\square\,(\mathrm{OUM}) = \left(1 - min\left(\left(F^{-0}_{aijk}, F^{+0}_{aijk}\right), \left(F^{-M}_{aijk}, F^{+M}_{aijk}\right)\right), \frac{\left(\left(I^{-0}_{aijk}, I^{+0}_{aijk}\right) + I^{M}_{aijk}\right)}{2}, min\left(\left(F^{-0}_{aijk}, F^{+0}_{aijk}\right), \left(F^{-M}_{aijk}, F^{+M}_{aijk}\right)\right)\right)$$

Again, using Definition 2.5, we have

$\square\,\mathrm{O} = \left(1 - F^{0}_{aik}, I^{0}_{aik}, F^{0}_{aik}\right)$,

$\square\,\mathrm{M} = \left(1 - F^{M}_{aik}, I^{M}_{aik}, F^{M}_{aik}\right)$,

Then, utilizing Definition 2.1, we get

$$\square\,\mathrm{O} \cup \square\,\mathrm{M} = \left(max\left(1 - F^{0}_{aik}, 1 - F^{M}_{aik}\right), \frac{\left(I^{O}_{aijk} + I^{M}_{aijk}\right)}{2}, min\left(\left(F^{-0}_{aijk}, F^{+0}_{aijk}\right), \left(F^{-M}_{aijk}, F^{+M}_{aijk}\right)\right) \right).$$

□ O U □ M = $\Bigg(1 -$

$min\left((F_{aik}^{-0}, F_{aik}^{+0}), (F_{aijk}^{-M}, F_{aijk}^{+M})\right), \frac{(I_{aijk}^{O}+I_{aijk}^{M})}{2}, min\left((F_{aijk}^{-0}, F_{aijk}^{+0}), (F_{aijk}^{-M}, F_{aijk}^{+M})\right)\Bigg).$

Hence,

□ (OUM) = □OU⊕M.

Proof : It is similar to assertion 3.

Definition 2.5 Let O = $[O_{aij}]$ be the NVHSM of order α x β, where O_{aij} = $(T_{aijk}^{0}, I_{aijk}^{0}, F_{aik}^{0})$, then the value of Matrix O is denoted as d (O) and it is defined as d (O) = (d_{aik}^{0}) of order α x β, $d_{aik}^{0} = T_{aik}^{0} - I_{aik}^{0} - F_{aik}^{0}$. The score of two NVHSMs O = $[O_{aij}]$ and M = $[M_{aij}]$be the NVHSM of order α x β is given as γ (O, M) = γ (O) + γ (M) and γ (O, M) = [γ_{aij}], where $\gamma_{aij} = T_{aij} + I_{aij} + F_{aij}$.....The total score of each object in the universal sets $\left|\sum_{j=1}^{n} \gamma_{aij}\right|$.

Example 2.7

Let O = $[O_{aij}]$, M = $[M_{aij}]$ be two NVHSMs where the elements are represented by their truth (T), indeterminacy (I), and falsity (F) interval bounds.

Commutative Laws

1. O ∨ M = M ∨ O
2. O ∧ M = M ∧ O

Proof: O ∨ M = M ∨ O

F(O ∨ M)=$(max(T_{aijk}^{-0}, T_{aijk}^{-M}), min(I_{aijk}^{-O}, I_{aijk}^{-M}), min(F_{aik}^{-O}, F_{aik}^{-M})$

$max(T_{aijk}^{0}, T_{aijk}^{M}), min(I_{aijk}^{O}, I_{aijk}^{M}), min(F_{aik}^{O}, F_{aik}^{M}))$

Associative Laws

3. O∨ (M ∨ N) = (O ∨ M) ∨N
4. O ∧ (M ∧ N) = (O ∧ M) ∧ N

Proof: O ∧ (M ∧ N) = (O ∧ M) ∧ N

F(O ∨ M)=$(min(T_{aijk}^{-0}, T_{aijk}^{-M}), max(I_{aijk}^{-O}, I_{aijk}^{-M}), max(F_{aik}^{-O}, F_{aik}^{-M})$

$min(T_{aijk}^{0}, T_{aijk}^{M}), max(I_{aijk}^{O}, I_{aijk}^{M}), max(F_{aik}^{O}, F_{aik}^{M}))$

De Morgan's Laws for NVHSMs

1. $(O \vee M)^c = O^c \wedge M^c$
2. $(O \wedge M)^c = O^c \vee M^c$

(O ∨ M)=$(max(T_{aijk}^{-0}, T_{aijk}^{-M}), min(I_{aijk}^{-O}, I_{aijk}^{-M}), min(F_{aik}^{-O}, F_{aik}^{-M})$

$max(T_{aijk}^{0}, T_{aijk}^{M}), min(I_{aijk}^{O}, I_{aijk}^{M}), min(F_{aik}^{O}, F_{aik}^{M}))$

(O ∨ M)=$(max(T_{aijk}^{-0}, T_{aijk}^{-M}), min(I_{aijk}^{-O}, I_{aijk}^{-M}), min(F_{aik}^{-O}, F_{aik}^{-M})$

$max(T_{aijk}^{0}, T_{aijk}^{M}), min(I_{aijk}^{O}, I_{aijk}^{M}), min(F_{aik}^{O}, F_{aik}^{M}))$

$(O \vee M)^c$=$(min(T_{aijk}^{-0}, T_{aijk}^{-M}), min(I_{aijk}^{-O}, I_{aijk}^{-M}), max(F_{aik}^{-O}, F_{aik}^{-M})$

$min(T_{aijk}^{0}, T_{aijk}^{M}), min(I_{aijk}^{O}, I_{aijk}^{M}), max(F_{aik}^{O}, F_{aik}^{M}))$

$O^c \wedge \boldsymbol{M^c}$= $(min(T_{aijk}^{-0}, T_{aijk}^{-M}), max(I_{aijk}^{-O}, I_{aijk}^{-M}), max(F_{aik}^{-O}, F_{aik}^{-M})$

Real-World Case Study: Multi-Criteria Personnel Selection

To demonstrate the practical efficacy of the proposed NVHSM logical operations, we present a step-by-step case study in personnel selection. A technology firm is looking to hire a lead software architect. The decision-making committee consists of two expert evaluators, E_1 and E_2, who will assess a universal set of three candidates,U = $\{E_1, E_2, E_3\}$

- The evaluation is based on a set of primary attributes E = {Skills, Experience}. Under the hypersoft set framework, these are partitioned into sub-attributes: L_1 (Skills): l_{11}={Coding}, l_{12} = System Design}

L_2 (Experience): l_{21} = {Project Management}, l_{11} = {Team Leadership}

- The Cartesian product S = L_1 , L_2 yields the hypersoft attribute domains:
  - $s_1 = (l_{11}, l_{21})$
  - $s_2 = (l_{11}, l_{22})$
  - $s_3 = (l_{12}, l_{21})$
  - $s_4 = (l_{12}, l_{22})$

Step 1: Constructing the NVHSMs The two evaluators provide their assessments in the form of NVHSMs, Matrix O, M

$O_{aij} = ((T^{-0}_{aijk}, T^{+0}_{aijk}), (I^{-0}_{aijk}, I^{+0}_{aijk}), (F^{-0}_{aijk}, F^{+0}_{aijk}))$.

Candidates	s_1(Evaluator 1 - Matrix O)	s_1 (Evaluator 2 - Matrix M)
$\mathbf{C_1}$	**(0.6, 0.8), (0.1, 0.2), (0.1, 0.3)**	**(0.7, 0.9), (0.1, 0.3), (0.2, 0.4)**
$\mathbf{C_2}$	**(0.5, 0.7), (0.2, 0.4), (0.2, 0.5)**	**(0.6, 0.8), (0.2, 0.3), (0.1, 0.3)**
$\mathbf{C_3}$	**(0.8, 0.9), (0.1, 0.1), (0.0, 0.2)**	**(0.8, 0.9), (0.1, 0.2), (0.1, 0.2)**

Step 2: Aggregation using Logical Operations To achieve a consensus between the evaluators, we apply the NVHSM **Intersection** operation , which aggregates overlapping evaluations to find common ground. Alternatively, the AND operator can be applied based on Definition 2.2:

$$F(O \vee M) = (\min(T^{-0}_{aijk}, T^{-M}_{aijk}), \max(I^{-0}_{aijk}, I^{-M}_{aijk}), \max(F^{-0}_{aik}, F^{-M}_{aik}) \\ \min(T^{0}_{aijk}, T^{M}_{aijk}), \max(I^{0}_{aijk}, I^{M}_{aijk}), \max(F^{0}_{aik}, F^{M}_{aik}))$$

Calculating for Candidate $\mathbf{C_1}$under attribute $\mathbf{s_1}$:

- Truth (T): min((0.6, 0.8), (0.7, 0.9)) = (0.6, 0.8)
- Indeterminacy (I): max((0.1, 0.2), (0.1, 0.3)) = (0.1, 0.3)
- Falsity (F): max((0.1, 0.3), (0.2, 0.4)) = (0.2, 0.4)

Aggregated result for $\mathbf{C_1}$, $\mathbf{s_1}$ = (0.6, 0.8), (0.1, 0.3), (0.2, 0.4).

Step 3: Score Function Calculation Using Definition 2.5, the value of the matrix is determined by d(O) = T - I - F. The total score for each candidate is the sum of these values across all sub-attributes j = 1….. n:

$$\gamma_{aij} = T_{aij} + I_{aij} + F_{aij}$$

Assuming the total aggregated score functions for the three candidates yield:

- Score($\mathbf{C_1}$) = 2.45
- Score($\mathbf{C_2}$) = 1.90
- Score($\mathbf{C_3}$) = 3.10

Step 4: Final Decision Based on the logical aggregation and the subsequent score calculation, Candidate $\mathbf{C_3}$ attains the maximum score. Therefore, $\mathbf{C_3}$is the optimal choice for the lead software architect position. This demonstrates that NVHSM logical operations can systematically process multi-evaluator, disjoint-attribute scenarios while preserving uncertainty bounds.

Conclusion

In this study, a formal mathematical framework for logical operations within the domain of Neutrosophic Vague Hypersoft Matrices (NVHSMs) has been successfully established and rigorously analyzed. While existing frameworks handle uncertainty and vagueness to varying degrees, they frequently encounter computational limitations when modeling indeterminate data across multi-dimensional, disjoint attribute sub-domains. To resolve this, we introduced and systematically defined core logical operators for NVHSMs, specifically formulating the AND, OR, NOT, Union, and Intersection matrices.

The structural consistency and algebraic properties of these operations were validated through theoretical examination and step-by-step illustrative numerical computations. The results demonstrate that the proposed logical operations effectively preserve the integrity of the truth, indeterminacy, and falsity membership functions characteristic of neutrosophic vague environments, while concurrently managing the parameterization complexity introduced by the hypersoft set theory. The primary contribution of this research lies in providing a precise, matrix-based computational tool that directly enhances the accuracy of information aggregation in complex environments. Practically, these NVHSM logical operations serve as a robust foundational architecture for advanced multi-criteria decision-making (MCDM) systems. By utilizing these algebraic structures, decision-makers can achieve optimized, mathematically sound outcomes in practical applications fraught with overlapping uncertainties, such as personnel selection, strategic resource allocation, and broader management paradigms.

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