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NEUTROSOPHIC PYTHAGOREAN CUBIC SOFT SET ALGEBRA

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Abstract: This study examines, the structural aspects of Neutrosophic Pythagorean Cubic Soft Sets in the context of BCK/BCI-algebra and subalgebras along with their related properties. Moreover, we present several significant results associated with them.

1.INTRODUCTION

Molodtsov introduced soft set theory in 1999 [6] as a versatile tool for addressing uncertain situations without relying on additional requirements such as membership functions or probability measures. Later, the concept of cubic sets formed by merging fuzzy sets with interval-valued fuzzy sets [1,4] was developed to provide a more enriched framework for expressing two-dimensional uncertainty. Pythagorean fuzzy sets further advanced intuitionistic fuzzy sets by allowing a broader range of representation, where the sum of the squares of membership and non-membership values does not exceed one.

Neutrosophic sets, proposed by Smarandache [2], extend this scope even further by introducing an independent degree of indeterminacy alongside truth and falsity measures. Combining these advanced models results in a unified system capable of representing multiple layers of uncertainty simultaneously. In this study, we investigate Neutrosophic Pythagorean Cubic Soft Sets within the context of algebraic frameworks, with a particular emphasis on BCK and BCI algebras and their subalgebras.

2.PRELIMINARY

Definition 2.1 [5]

Let U be an initial universe set. Let NPC(K) denote the set of all Neutrosophic Pythagorean cubic sets and E be the set of parameters. Let $D \subset E$, then $(Q, D) = \{Q(ei)= \{\langle u, A_{ei}(u), \lambda_{ei}(u)\rangle : u \in U\} e_i \in D\}$ Where $A_{ei}(u)=\{\langle u, A^T{}_{ei}(u), A^I{}_{ei}(u) A^F{}_{ei}(u)\rangle / u\in U\}$ is an Interval Neutrosophic Pythagorean Set, $\lambda_{ei}(u) =\{\langle u,\lambda^T{}_{ei}(u), \lambda^I{}_{ei}(u)\, \lambda^F{}_{ei}(u)\rangle / u\in U\}$ is a Neutrosophic Pythagorean set. The pair (Q, D) is termed to be the Neutrosophic Pythagorean Cubic Soft Set over U where Q is a mapping given by $Q: D \rightarrow NPC(K)$. The sets of all Neutrosophic Pythagorean cubic soft sets over U will be denoted by C_N^U.

Definition 2.2 [3,9]
A d−algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following axioms.

(i) $x * x = 0$

(ii) $0 * x = 0$

(iii) $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y \in X$.

A BCK-algebra is a d−algebra satisfying additional axioms:

(iv) $((x * y) * (x * z)) * (z * y) = 0$

(v) $(x * (x * y)) * y = 0$ for all $x, y, z \in X$.

In X we can define a binary relation "≤" by $x \leq y$ if and only if $x * y = 0$.

A mapping $f: X \to Y$ of d−algebra is called a d−homomorphism if

$$f(x * y) = f(x) * f(y) \text{ for all } x, y \in X.$$

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3. NEUTROSOPHIC PYTHAGOREAN CUBIC SOFT SET ON BCK/BCI ALGEBRA

Definition 3.1 A Neutrosophic Pyhtagorean Cubic Soft Set (S, R) over V is said to be Neutrosophic Pythagorean Cubic Soft Set BCK/BCI algebra over V based on a parameter e_{i_α} if there $e_{i_\alpha} \in R$ such that

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq \min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}, \quad (3.1)$$

$$\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \leq \max\left\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}, \quad (3.2)$$

Where

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) = \mathcal{A}_{e_{i_\alpha}}(x_1) = \{\langle x_1, \mathcal{A}_{e_{i_\alpha}}{}^{T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{F}(x_1)\rangle / x_1 \in V\}$$

$$=\{\langle x_1, \mathcal{A}_{e_{i_\alpha}}{}^{-T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(x_1)], [\mathcal{A}_{e_{i_\alpha}}{}^{-I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(x_1), [\mathcal{A}_{e_{i_\alpha}}{}^{-F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(x_1)]\rangle / x_1 \in V\}$$

$$\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) = \delta_{e_{i_\alpha}}(x_1) = \{\langle x_1, \delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{F}(x_1)\rangle / x_1 \in V\}$$

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq \min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} = \left\{\begin{matrix} \mathcal{A}_{e_{i_\alpha}}{}^{T}(x_1) \geq \min\{\mathcal{A}_{e_{i_\alpha}}{}^{T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T}(y_1)\} \\ \mathcal{A}_{e_{i_\alpha}}{}^{I}(x_1) \geq \min\{\mathcal{A}_{e_{i_\alpha}}{}^{I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{I}(y_1)\} \\ \mathcal{A}_{e_{i_\alpha}}{}^{F}(x_1) \geq \min\{\mathcal{A}_{e_{i_\alpha}}{}^{F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{F}(y_1)\} \end{matrix}\right\}$$

$$\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \leq max\left\{\delta_{e_{i_\alpha}}(x_1), \delta_{e_{i_\alpha}}(y_1)\right\} = \left\{\begin{matrix} \delta_{e_{i_\alpha}}{}^{T}(x_1) \leq max\{\delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{T}(y_1)\} \\ \delta_{e_{i_\alpha}}{}^{I}(x_1) \leq max\{\delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{I}(y_1)\} \\ \delta_{e_{i_\alpha}}{}^{F}(x_1) \leq max\{\delta_{e_{i_\alpha}}{}^{F}(x_1), \delta_{e_{i_\alpha}}{}^{F}(y_1)\} \end{matrix}\right\}$$

If (S, R) is a NPCSS BCK/BCI algebra over V based on all parameter, we say that (S, R) is a NPCSS BCK/BCI algebra over V.

Proposition 3.2 If (S,R) is a NPCSS BCK/BCI algebra over V based on a parameter e_{i_α} in R then ${\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(0) \geq {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1)$ and ${\delta_{e_{i_\alpha}}}^{T,I,F}(0) \geq {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1)$ for all $x_1 \in V$.

Proof. For any $x_1 \in V$ we have

$${\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(0) = {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1 * x_1) \geq min\left\{{\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1)\right\}$$

$$= \min\{[{\mathcal{A}_{e_{i_\alpha}}}^{-T}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+T}(x_1)], [{\mathcal{A}_{e_{i_\alpha}}}^{-I}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+I}(x_1), [{\mathcal{A}_{e_{i_\alpha}}}^{-F}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+F}(x_1)],$$

$$\min\{[{\mathcal{A}_{e_{i_\alpha}}}^{-T}(y_1), {\mathcal{A}_{e_{i_\alpha}}}^{+T}(y_1)], [{\mathcal{A}_{e_{i_\alpha}}}^{-I}(y_1), {\mathcal{A}_{e_{i_\alpha}}}^{+I}(y_1), [{\mathcal{A}_{e_{i_\alpha}}}^{-F}(y_1), {\mathcal{A}_{e_{i_\alpha}}}^{+F}(y_1)]$$

$$= \{[{\mathcal{A}_{e_{i_\alpha}}}^{-T}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+T}(x_1)], [{\mathcal{A}_{e_{i_\alpha}}}^{-I}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+I}(x_1), [{\mathcal{A}_{e_{i_\alpha}}}^{-F}(x_1), {\mathcal{A}_{e_{i_\alpha}}}^{+F}(x_1)]$$

$$= {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1)$$

$${\delta_{e_{i_\alpha}}}^{T,I,F}(0) = {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1 * x_1) \leq max\left\{{\delta_{e_{i_\alpha}}}^{T,I,F}(x_1), {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1)\right\}$$

$$= \max\{({\delta_{e_{i_\alpha}}}^{T}(x_1), {\delta_{e_{i_\alpha}}}^{I}(x_1), {\delta_{e_{i_\alpha}}}^{F}(x_1)), ({\delta_{e_{i_\alpha}}}^{T}(x_1), {\delta_{e_{i_\alpha}}}^{I}(x_1), {\delta_{e_{i_\alpha}}}^{F}(x_1))$$

$$= \left({\delta_{e_{i_\alpha}}}^{T}(x_1), {\delta_{e_{i_\alpha}}}^{I}(x_1), {\delta_{e_{i_\alpha}}}^{F}(x_1)\right)$$

$$= {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1) \text{ for all } x_1 \in V$$

Corollary 3.3. If (S,R) is a NPCSS BCK/BCI algebra over V then ${\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(0) \geq {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1)$ and ${\delta_{e_{i_\alpha}}}^{T,I,F}(0) \leq {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1)$ for all $e_{i_\alpha} \in R$ and $x_1 \in V$.

Proof. Let (S, R) is a NPCSS BCK/BCI algebra over V. For a parameter $e_{i_\alpha} \in R, w^{T,I,F}, [d,l]^{T,I,F} \in [w]^{T,I,F}$ we define a set

$$V_{e_{i_\alpha}}(S,R)_{[d,l]}{}^{w}: \{x_1 \in V / {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1) \geq [d,l]^{T,I,F},\ {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1) \leq w^{T,I,F}\}$$

If we put

$$\mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}: \{x_1 \in V / {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1) \geq [d,l]^{T,I,F} \text{ and}$$

$$\delta_{e_{i_\alpha}}(S,R)^{w}: \{x_1 \in V / {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1) \leq w^{T,I,F}\}$$

That is

$$\mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}: \{x_1 \in V / {\mathcal{A}_{e_{i_\alpha}}}^{T}(x_1) \geq [d,l]^{T}, {\mathcal{A}_{e_{i_\alpha}}}^{I}(x_1) \geq [d,l]^{I}, {\mathcal{A}_{e_{i_\alpha}}}^{F}(x_1) \geq [d,l]^{F}\} \text{ and}$$

$$\delta_{e_{i_\alpha}}(S,R)^{w} :: \{x_1 \in V / {\delta_{e_{i_\alpha}}}^{T}(x_1) \leq w^{T}, {\delta_{e_{i_\alpha}}}^{I}(x_1) \leq w^{I}, {\delta_{e_{i_\alpha}}}^{F}(x_1) \leq w^{F}\} \text{ then}$$

$$V_{e_{i_\alpha}}(S,R)_{[d,l]}{}^{w}: \{{\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(S,R)_{[d,l]} \cap {\delta_{e_{i_\alpha}}}^{T,I,F}(S,R)^{w}\}$$

Theorem 3.4 For a NPCSS (S, R) over V, the conditions listed below all are equivalent

1. (S, R) is a NPCSS BCK/BCI algebra over V based on the parameter $e_{i_\alpha} \in$ R.
2. The sets

$$\mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]} = \{x_1 \in V / {\mathcal{A}_{e_{i_\alpha}}}^{T,I,F}(x_1) \geq [d,l]^{T,I,F}\} \text{ and}$$

$$\delta_{e_{i_\alpha}}(S,R)^{w} = \{x_1 \in V / {\delta_{e_{i_\alpha}}}^{T,I,F}(x_1) \leq w^{T,I,F}\}$$

are subalgebras of V for $w^{T,I,F} \in [0,1]$ and $[d,l]^{T,I,F} \in [w]^{T,I,F}$ whenever they are nonempty.

Proof. Assume that (S, R) is a NPCSS BCK/BCI algebra over V based on the parameters $e_{i_\alpha} \in$ R. Let $x_1, y_1 \in V$ and a$[d,l]^{T,I,F} \in [I]^{T,I,F}$ be such $x_1, y_1 \in \mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}$. Then $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \geq [d,l]^{T,I,F}$ and $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1) \geq [d,l]^{T,I,F}$. It follows from (3.1) that

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \geq min[d,l]^{T,I,F}, [d,l]^{T,I,F} = [d,l]^{T,I,F}$$

Here $x_1 * y_1 \in \mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}$ and therefore $\mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}$ is a subalgebra of V. Now let $x_1, y_1 \in V$ and $[w]^{T,I,F} \in [0,1]$ be such that $x_1 * y_1 \in (S,R)^w$ then $\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) \leq w^{T,I,F}$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(y_1) \leq w^{T,I,F}$

Using (3.2) we have $\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \leq max\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\} \leq w$ and so $x_1, y_1 \in (S,R)^w$. Therefore $\delta_{e_{i_\alpha}}(S,R)^w$ is a subalgebra of V.

Conversely, suppose that the sets

$\mathcal{A}_{e_{i_\alpha}}(S,R)_{[d,l]}: \{x_1 \in V / \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \geq [d,l]^{T,I,F}\}$ and $\delta_{e_{i_\alpha}}(S,R)^w \coloneqq \{x_1 \in V / \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) \leq w^{T,I,F}\}$ are subalgebras of V for all $w \in [0,1]$ and $[d,l]^{T,I,F} \in [w]^{T,I,F}$ whenever they are nonempty.

Assume that there exists a, b$\in V$ such that

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \ngeq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(b)\right\}.$$

If we take

$$\mathcal{A}_{e_{i_\alpha}}{}^{T}(a) = [d_a^T, l_a^T], \mathcal{A}_{e_{i_\alpha}}{}^{I}(a) = [d_a^I, l_a^I], \mathcal{A}_{e_{i_\alpha}}{}^{F}(a) = [d_a^F, l_a^F],$$

$\mathcal{A}_{e_{i_\alpha}}{}^{T}(b) = [d_b^T, l_b^T], \mathcal{A}_{e_{i_\alpha}}{}^{I}(a) = [d_a^I, l_a^I], \mathcal{A}_{e_{i_\alpha}}{}^{F}(a) = [d_a^F, l_a^F]$, and

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a * b) = [d_0^T, l_0^T], \mathcal{A}_{e_{i_\alpha}}{}^{I}(a * b) = [d_0^I, l_0^I], \mathcal{A}_{e_{i_\alpha}}{}^{F}(a * b) = [d_0^F, l_0^F]$$

then

$$[d_0^T, l_0^T] \geq min\{[d_a^T, l_a^T], [d_a^T, l_a^T]\} = [min[\{d_a^T, d_a^T\}, min\{l_a^T, l_a^T\}],$$

$$[d_0^I, l_0^I] \geq min\{[d_a^I, l_a^I], [d_a^I, l_a^I]\} = [min[\{d_a^I, d_a^I\}, min\{l_a^I, l_a^I\}],$$

$$[d_0^F, l_0^F] \geq min\{[d_a^F, l_a^F], [d_a^F, l_a^F]\} = [min[\{d_a^F, d_a^F\}, min\{l_a^F, l_a^F\}]$$

Hence we have the following three cases:

1) (a) $d_0^T \geq min\{d_a^T, d_b^T\}$ and $l_0^T \leq min\{l_a^T, l_b^T\}$
(b) $d_0^I \geq min\{d_a^I, d_b^I\}$ and $l_0^I \leq min\{l_a^I, l_b^I\}$
(c) $d_0^F \geq min\{d_a^F, d_b^F\}$ and $l_0^F \leq min\{l_a^F, l_b^F\}$

2) (a) $d_0^T < min\{d_a^T, d_b^T\}$ and $l_0^T \leq min\{l_a^T, l_b^T\}$
(b) $d_0^I < min\{d_a^I, d_b^I\}$ and $l_0^I \leq min\{l_a^I, l_b^I\}$
(c) $d_0^F < min\{d_a^F, d_b^F\}$ and $l_0^F \leq min\{l_a^F, l_b^F\}$

3) (a) $d_0^T < min\{d_a^T, d_b^T\}$ and $l_0^T < min\{l_a^T, l_b^T\}$
(b) $d_0^I < min\{d_a^I, d_b^I\}$ and $l_0^I < min\{l_a^I, l_b^I\}$
(c) $d_0^F < min\{d_a^F, d_b^F\}$ and $l_0^F < min\{l_a^F, l_b^F\}$

For the first case we have

$\mathcal{A}_{e_{i_\alpha}}{}^{T}(a) = [d_a^T, l_a^T] \geq [min\{d_a^T, d_b^T\}, min\{l_a^T, l_b^T\}] \geq [min\{\{d_a^T, d_b^T\}, l_0^T],$
$\mathcal{A}_{e_{i_\alpha}}{}^{I}(a) = [d_a^I, l_a^I] \geq [min\{d_a^I, d_b^I\}, min\{l_a^I, l_b^I\}] \geq [min\{\{d_a^I, d_b^I\}, l_0^I],$
$\mathcal{A}_{e_{i_\alpha}}{}^{F}(a) = [d_a^F, l_a^F] \geq [min\{d_a^F, d_b^F\}, min\{l_a^F, l_b^F\}] \geq [min\{\{d_a^F, d_b^F\}, l_0^F],$

$\mathcal{A}_{e_{i_\alpha}}{}^{T}(b) = [d_a^T, l_a^T] \geq [min\{d_a^T, d_b^T\}, min\{l_a^T, l_b^T\}] \geq [min\{\{d_a^T, d_b^T\}, l_0^T],$

$\mathcal{A}_{e_{i_\alpha}}{}^{I}(b) = [d_a^I, l_a^I] \geq [min\{d_a^I, d_b^I\}, min\{l_a^I, l_b^I\}] \geq [min\{\{d_a^I, d_b^I\}, l_0^I],$
$\mathcal{A}_{e_{i_\alpha}}{}^{F}(b) = [d_a^F, l_a^F] \geq [min\{d_a^F, d_b^F\}, min\{l_a^F, l_b^F\}] \geq [min\{\{d_a^F, d_b^F\}, l_0^F],$

And so $a * b \in \mathcal{A}_{e_{i_\alpha}}(S, R)_{[min\{d_a^{T,I,F}, d_b^{T,I,F}\}, l_0^{T,I,F}]}$ but $a * b \notin \mathcal{A}_{e_{i_\alpha}}(S, R)_{[min\{d_a^{T,I,F}, d_b^{T,I,F}\}, l_0^{T,I,F}]}$, a contradiction.
By the similar way the (2) and (3) include a contradiction.

Thus $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}$ and for all $x_1, y_1 \in V$. Now suppose (3.2) is false. Then $\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) > w \geq min\left\{\delta_{e_{i_\alpha}}{}^{T,I,F}(a), \delta_{e_{i_\alpha}}{}^{T,I,F}(b)\right\}$ for some $x_1, y_1 \in V$ and $w \in [0,1]$ which implies $a, b \in \delta_{e_{i_\alpha}}(S, R)$, this is a contradiction and hence (3.2) is valid. There (S, R) is a NPCSS BCK/BCI algebra over V based on the parameter $e_{i_\alpha} \in R$.

Corollary 3.5 If a NPCSS (S, R) over V is a BCK/BCI algebra over V based on parameter $e_{i_\alpha} \in R$ then $V_{i_\alpha}(S, R) = \{x_1 \in V / \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \geq [d, l], \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) \leq w\}$ is a subalgebra of V for all w∈[0,1] and $[d, l]^{T,I,F} \in [w]^{T,I,F}$ whenever it is nonempty.

Theorem 3.6 The R-intersection of two NPCSS BCK/BCI algebra over V is also a NPCSS BCK/BCI algebra over V.

Proof. Let (S, R) and (Q, N) be NPCSS BCK/BI-algebra over V and let (H, C) = (S, R) $\cap_R$(Q, N) be the R-intersection of (S, R) and (Q, N). Then C = $R \cup N$. For any $e_{i_\alpha} \in C$, if $e_{i_\alpha} \in R/N$ then

$$\text{H}\,(e_{i_\alpha}) = S(e_{i_\alpha}) \wedge_R Q(e_{i_\alpha}) \text{ and } e_{i_\alpha} \in R \cap N$$
$$\text{H}\,(e_{i_\alpha}) = \{\langle x_1, C_{e_{i_\alpha}}(x_1), \zeta_{e_{i_\alpha}}(x_1)\rangle : x_1 \in V\} e_{i_\alpha} \in R \cap N$$
$$\begin{aligned} C_{e_{i_\alpha}}(x_1 * y_1) &= S_{e_{i_\alpha}}(x_1 * y_1) \\ &= S_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \\ &\geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \\ &= min\left\{C_{e_{i_\alpha}}{}^{T,I,F}(x_1), C_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \end{aligned}$$

and
$$\begin{aligned} \zeta_{e_{i_\alpha}}(x_1 * y_1) &= S_{e_{i_\alpha}}(x_1 * y_1) \\ &= S_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \\ &\leq max\left\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \\ &= max\left\{\zeta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \zeta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \end{aligned}$$

For all $x_1, y_1 \in V$. Similarly, if $e_{i_\alpha} \in R/N$ then $C_{e_{i_\alpha}}(x_1 * y_1) \geq min\left\{C_{e_{i_\alpha}}{}^{T,I,F}(x_1), C_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}$ and $\zeta_{e_{i_\alpha}}(x_1 * y_1) \leq max\left\{\zeta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \zeta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}$ for all $x_1, y_1 \in V$. Suppose that $e_{i_\alpha} \in R \cap N$. Then

$$\begin{aligned} C_{e_{i_\alpha}}(x_1 * y_1) &= S(e_{i_\alpha}) \wedge_R Q(e_{i_\alpha})(x_1 * y_1) \\ &\geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \\ &= min\left\{C_{e_{i_\alpha}}{}^{T,I,F}(x_1), C_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \\ &= min\left\{C_{e_{i_\alpha}}(x_1), C_{e_{i_\alpha}}(y_1)\right\} \end{aligned}$$

and

$$\begin{aligned} \zeta_{e_{i_\alpha}}(x_1 * y_1) &= S(e_{i_\alpha}) \wedge_R Q(e_{i_\alpha})(x_1 * y_1) \\ &\leq max\left\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \end{aligned}$$

$$= max\left\{\zeta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \zeta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}$$
$$= max\left\{\zeta_{e_{i_\alpha}}(x_1), \zeta_{e_{i_\alpha}}(y_1)\right\}$$

For all $x_1, y_1 \in V$. Therefore (H, C) = (S, R) $\cap_R$(Q, N) is a NPCSS BCK/BCI algebra over V.

SUBALGEBRAS OF BCK/BCI ALGEBRA BASED ON NEUTROSOPHIC PYTHAGOREAN CUBIC SOFT SET

Definition 3.7 A Neutrosophic Pythagorean Cubic Soft Set (S, R) over V is said to be a Neutrosophic Pythagorean Cubic Soft set BCK/BCI algebra over V based on a parameter e (briefly e – Neutrosophic Pythagorean Cubic Soft subalgebra over V) if there exists a parameter e $\in R$ such that

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}\ (x_1 * y_1) \geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \tag{3.3}$$

$$\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \leq max\left\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\} \tag{3.4}$$

For all $x_1, y_1 \in V$. If (S, R) is an e – Neutrosophic Pythagorean Cubic Soft subalgebra over V for all e $\in \mathcal{A}$, we say that (S, R) is a Neutrosophic Pythagorean Cubic Soft subalgebra over V.

Theorem 3.8 Let (S, R) and (Q, N) be Neutrosophic Pythagorean Cubic Soft Set subalgebra over V. If R and N are disjoint, then the R-union of (S, R) and (Q, N) is a NPCSS over V.

Theorem 3.9 Given a parameter $e_{i_\alpha} \in R$ a NPCSS (S, R) over V is an e- NPCSS subalgebra over V if and only if the sets

$$\mathcal{A}_{e_{i_\alpha}}[\eta_1, \eta_2] = \{x_1 \in V / \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \leq [\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}]\},$$
$$\delta_{e_{i_\alpha}}(t) = \{x_1 \in V / \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \leq t^{T,I,F}\}$$

are subalgebras of V for all $[\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}] \in w^{T,I,F}$ and $t^{T,I,F} \in [0,1]$

Proof. Assume that a NPCSS (S, R) over Vis an e-NPCSS subalgebra over V and let $x_1, y_1 \in V$. If $x_1, y_1 \in V$. If $x_1, y_1 \in V$ $\mathcal{A}_{e_{i_\alpha}}[\eta_1, \eta_2]$ for all $[\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}] \in$ $w^{T,I,F}$ then $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) \geq [\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}]$ and $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1) \geq [\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}]$ from (3.3) and (3.4)

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\right\}$$

$= min\{[\mathcal{A}_{e_{i_\alpha}}{}^{-T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(x_1)], [\mathcal{A}_{e_{i_\alpha}}{}^{-I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(x_1), [\mathcal{A}_{e_{i_\alpha}}{}^{-F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(x_1)],$
$\min\{[\mathcal{A}_{e_{i_\alpha}}{}^{-T}(y_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(y_1)], [\mathcal{A}_{e_{i_\alpha}}{}^{-I}(y_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(y_1), [\mathcal{A}_{e_{i_\alpha}}{}^{-F}(y_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(y_1)],$
$= min\{\eta_1{}^T(x_1), \eta_1{}^I(x_1), \eta_1{}^F(x_1)\}, min\{\eta_1{}^T(y_1), \eta_1{}^I(y_1), \eta_1{}^F(y_1)\}$
$= min\{[\eta_1{}^T(x_1), \eta_2{}^T(y_1)], [\eta_1{}^I(x_1), \eta_2{}^I(y_1)], [\eta_1{}^F(x_1), \eta_2{}^F(y)]\}$
$= [\eta_1{}^{T,I,F}(x_1), \eta_2{}^{T,I,F}(y_1)]$

Hence $x_1, y_1 \in \mathcal{A}_{e_{i_\alpha}}[\eta_1, \eta_2]$. Now if $x_1, y_1 \in \delta_{e_{i_\alpha}}(t)$for all t = $t^T, t^I, t^F \in [0,1]$ then $\delta_{e_{i_\alpha}}(x_1) \leq t$ and $\delta_{e_{i_\alpha}}(y_1) \leq t$. From (3.4) we have

$\delta_{e_{i_\alpha}}(x_1 * y_1) \leq max\{ \delta_{e_{i_\alpha}}(x_1), \delta_{e_{i_\alpha}}(y_1)\}$
$\leq max\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) , \delta_{e_{i_\alpha}}{}^{T,I,F}(y_1)\}$
$= \max \{(\delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{F}(x_1)),(\delta_{e_{i_\alpha}}{}^{T}(y_1), \delta_{e_{i_\alpha}}{}^{I}(y_1), \delta_{e_{i_\alpha}}{}^{F}(y_1))\}$
$= t^T, t^I, t^F$
$= t^T, t^I, t^F = t.$

And so $x_1, y_1 \in \delta_{e_{i_\alpha}}(x_1 * y_1)$. Therefore$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}[\eta_1, \eta_2]$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(t)$ subalgebra over V. Conversely suppose that $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}[\eta_1, \eta_2]$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(t)$ subalgebra over V for all $[\eta_1{}^{T,I,F}(x_1), \eta_2{}^{T,I,F}(y_1)] \in w^{T,I,F}$ and $t^{T,I,F} \in [0,1]$.

Assume that there exists a, b ∈ V such that

$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a * b) \geq min\left\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(b)\right\}$.

Let $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a) = [\zeta_1{}^{T,I,F}, \zeta_2{}^{T,I,F}], \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(b) = [\zeta_3{}^{T,I,F}, \zeta_4{}^{T,I,F}], \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a * b) = [\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}]$ then

$$[\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}] \leq min\{[\zeta_3{}^{T,I,F}, \zeta_2{}^{T,I,F}], [\zeta_4{}^{T,I,F}, \zeta_2{}^{T,I,F}]\}$$
$$= min\{[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}], [\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]\}$$

Hence $\eta_1{}^{T,I,F} \leq min\{[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}]$ and $\eta_2{}^{T,I,F} \leq [\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]$

taking$[\gamma_1, \gamma_2] = \frac{1}{2}\ (\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a * b) + min\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(b)\})$

$= \frac{1}{2}\ ([\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}] + min\{[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}], [\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]\})$

$= \frac{1}{2}(\eta_1{}^{T,I,F} + min[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}]) + \frac{1}{2}(\eta_2{}^{T,I,F} + min[\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}])$

It follows that

$$min[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}] > \gamma_1 = \frac{1}{2}(\eta_1{}^{T,I,F} + min[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}]) > \eta_1{}^{T,I,F}$$
$$min[\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}] > \gamma_2 = \frac{1}{2}(\eta_2{}^{T,I,F} + min[\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]) > \eta_2{}^{T,I,F}$$

And so that $min[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}], min[\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}] > [\gamma_1, \gamma_2] > [\eta_1{}^{T,I,F}, \eta_2{}^{T,I,F}] = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a * b)$. $(a * b) \notin \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}[\gamma_1, \gamma_2]$

On the other hand we know that

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(a) = [\zeta_1{}^{T,I,F}, \zeta_2{}^{T,I,F}] \geq min\{[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}], [\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]\} \geq [\gamma_1, \gamma_2]$$
$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(b) = [\zeta_3{}^{T,I,F}, \zeta_4{}^{T,I,F}] \geq min\{[\zeta_1{}^{T,I,F}, \zeta_3{}^{T,I,F}], [\zeta_2{}^{T,I,F}, \zeta_4{}^{T,I,F}]\} \geq [\gamma_3, \gamma_4]$$

Which implies that a, b ∈ $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}[\gamma_1, \gamma_2]$. This is a contradiction and so $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq min\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\}$ for all $x_1, y_1 \in V$.

Now assume that $\delta_{e_{i_\alpha}}{}^{T,I,F}(a * b) > max\{\delta_{e_{i_\alpha}}{}^{T,I,F}(a), \delta_{e_{i_\alpha}}{}^{T,I,F}(b)\}$ for some a, b $\in V$ then there exist $t_0{}^{T,I,F} \in [0,1]$ such that $\delta_{e_{i_\alpha}}{}^{T,I,F}(a * b) > max\{\delta_{e_{i_\alpha}}{}^{T,I,F}(a), \delta_{e_{i_\alpha}}{}^{T,I,F}(b)\}$. Hence a, b ∈ $t_0{}^{T,I,F}$ but a, b ∉ $t_0{}^{T,I,F}$. This is a contradiction and therefore $\delta_{e_{i_\alpha}}(x_1 * y_1) \leq max\{\delta_{e_{i_\alpha}}(x_1) , \delta_{e_{i_\alpha}}(y_1)\}$ for all $x_1, y_1 \in V$. Consequently, (S,R) is an e-NPCSS subalgebra over V.

Proposition 3.10 Given a parameter $e_{i_\alpha} \in R$ if a NPCSS (S,R) over V is an e-NPCSS subalgebra over V then $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) > \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1)$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(0) \leq \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1)$ for all $x_1 \in V$.

Proof. For any $x_1 \in V$ we have

$$\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * x_1) \geq min\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\}$$

$$=\min\left\{\begin{matrix}\left[\mathcal{A}_{e_{i_\alpha}}{}^{-T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(x_1)\right], [\mathcal{A}_{e_{i_\alpha}}{}^{-I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(x_1)], [\mathcal{A}_{e_{i_\alpha}}{}^{-F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(x_1)] \\ \left[\mathcal{A}_{e_{i_\alpha}}{}^{-T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(x_1)\right], [\mathcal{A}_{e_{i_\alpha}}{}^{-I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(x_1)], [\mathcal{A}_{e_{i_\alpha}}{}^{-F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(x_1)]\end{matrix}\right\}$$
$$=\left[\mathcal{A}_{e_{i_\alpha}}{}^{-T}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+T}(x_1)\right], \left[\mathcal{A}_{e_{i_\alpha}}{}^{-I}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+I}(x_1)\right]\left[\mathcal{A}_{e_{i_\alpha}}{}^{-F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{+F}(x_1)\right]$$
$$=\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1)$$

$$\delta_{e_{i_\alpha}}{}^{T,I,F}(0) = \delta_{e_{i_\alpha}}(x_1 * x_1) \leq max\{\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1)\}$$
$$= \max\{(\delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{F}(x_1)), (\delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{F}(x_1))\}$$
$$= (\delta_{e_{i_\alpha}}{}^{T}(x_1), \delta_{e_{i_\alpha}}{}^{I}(x_1), \delta_{e_{i_\alpha}}{}^{F}(x_1))$$
$$=\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) \text{ for all } x_1 \in V.$$

Theorem 3.11 Let (S, R) be a e-NPCSS subalgebra over V for a parameter $e_{i_\alpha} \in R$. If there is a sequence in V such that and $\lim_{n\to\infty} \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_n) = [1,1]$ and $\lim_{n\to\infty} \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_n) = 0$ then $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) = [1,1]$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(0) = 0$.

Proof. Since $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) \geq \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1)$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(0) \leq \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1)$ for all $x_1 \in V$. We have $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) \geq \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \delta_{e_{i_\alpha}}{}^{T,I,F}(0) \leq \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1)$, for every positive integer n. Note that [1,1] $\geq \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) \geq \lim_{n\to\infty} \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_n) = [1,1]$ and $0 \leq \delta_{e_{i_\alpha}}{}^{T,I,F}(0) \leq \delta_{e_{i_\alpha}}{}^{T,I,F}(x_n) = 0$. Hence $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) = [1.1]$ and $\delta_{e_{i_\alpha}}{}^{T,I,F}(0) = 0$.

Theorem 3.12 Given a parameter $e_{i_\alpha} \in R$ if a NPCSS (S, R) over V is an e-NPCSS subalgebra over V then the sets $V_{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}} := \{x_1 \in V / \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0)\}$ and $V_{\delta_{e_{i_\alpha}}{}^{T,I,F}} := \{x_1 \in V / \delta_{e_{i_\alpha}}{}^{T,I,F}(x_1) = \delta_{e_{i_\alpha}}{}^{T,I,F}(0)\}$ are subalgebra of V.

Proof. Let $x_1, y_1 \in V$. If $x_1, y_1 \in V_{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0)}$ then $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1) = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0) = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)$. Hence $\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) \geq min\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(x_1), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(y_1)\} = min\{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0), \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0)\} = \mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}(0)$ and
$\delta_{e_{i_\alpha}}{}^{T,I,F}(x_1 * y_1) = \delta_{e_{i_\alpha}}{}^{T,I,F}(0)$. This shows that $x_1 * y_1 \in V_{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}}$. Therefore $x_1 * y_1 \in V_{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}}$ and $x_1 * y_1 \in V_{\delta_{e_{i_\alpha}}{}^{T,I,F}}$ are subalgebras of V.

Corollary 3.13 Given a parameter $e_{i_\alpha} \in R$ if a NPCSS (S,R) over V is an e-NPCSS subalgebra over V then the set $V_{\mathcal{A}_{e_{i_\alpha}}{}^{T,I,F}} \cap_{\delta_{e_{i_\alpha}}{}^{T,I,F}} V$ is a subalgebra of V.

Proof. The proof is straight forward.

CONCLUSION

This paper investigates the properties of Neutrosophic Pythagorean Cubic Soft Sets (NPCSS) within the framework of BCK/BCI algebras. This research focuses on the representation of uncertainty and indeterminacy in algebraic structures. By applying NPCSS to BCK/BCI environments, we enhance the theoretical understanding of Neutrosophic Soft Set systems. The work contributes to building a more comprehensive foundation for algebra-based decision-making models. The study also highlights the role of NPCSS in soft computing. Future work may extend these results to broader algebraic structures. Overall, this research offers a solid basis for further exploration of NPCSS in theoretical and applied contexts.

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