IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Some properties of Left Bi-quasi Ideals in Ternary Semirings

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ABSTRACT

Algebraic structures play a very outstanding role in mathematics. Iseki introduced the concept of quasi ideal for a semirings. The quasi ideals in ternary semirings are a generalization of left and right ideals. Bi-ideals are generalization of quasi ideals. In 2017, Marapureddy Murali Krishna Rao introduced bi-quasi ideals in semirings. With this connection, here we are introducing some properties of bi-quasi ideals in ternary semirings.We proved that every left(right) ideal of ternary semiring is bi-quasi ideal, every quasi ideal of ternary semiring is bi-quasi ideal and every bi-ideal of ternary semiring is a bi-quasi ideal.

Key words: Ternary semiring, ideal,bi-ideal,bi-quasi ideal

1.Introduction

Algebraic structures play a very outstanding role in mathematics. Ternary generalizations of algebraic structures are the very natural ways for further development and in depth comprehension of their basic traits. The formal study of semigroups began in the early 20th century. But the concept of semiring was first introduced by Vandiver in 1934. However the developments of the theory in semirings and ordered semirings have been taking place since 1950s. The quasi ideals are generalization of left and right ideals, where as the bi-ideals are generalization of quasi ideals. The concept of bi ideals for associative rings were introduced by Lajos and Szasz**.** Quasi ideals are a generalization of left and right ideals. Bi ideals are a generalization of quasi ideals. Munir and M.Habib[1] characterized some classes of the semirings i.e regular and weakly regular semirings using their quasi and bi ideals. Iseki [2] introduced the concept of quasi ideal for a semiring. M. Henriksen [3] studied ideals in semirings**.** W. J. Liu[4] defined and studied fuzzy subrings as well as fuzzy ideals in rings. D. Mandal[5] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring. In 2017, Marapureddy Murali Krishna Rao[6] introduced left bi-quasi-ideals in semirings, This paper consists of some properties of Bi-quasi ideals of ternary semirings.

2.Preliminaries

Definition 2.1 A non-empty subset T of S is a sub ternary semiring of S if (T, +) is a sub ternary semigroup of (S, +) and abc ∈ T for all a, b,c ∈ T.

Definition 2.2 A non-empty subset T of S is a left (respectively lateral, right) ideal of S if T is a sub ternary semigroup of (S, +) and xya ∈ T (respectively xay ∈ T, axy ∈ T) for all a ∈ T, x,y ∈ S.

Definition 2.3 A non-empty subset L of a ternary semiring S is a quasi-ideal if Q is a sub ternary semigroup of (S, +) and (SSL) ∩ (SLS) ∩ (LSS) ⊆ Q.

obviously, every quasi ideal of S is a sub ternary semiring of S.

Example 2.4 Let N be the set of natural numbers. Then N is a ternary semiring with respect to usual addition and multiplication. Then L = 3N is a quasi-ideal of a ternary semiring N.

Definition 2.5 A non-empty subset B of ternary semiring S is said to be a bi-ideal of S if B is a sub ternary semiring of S and BSBSB ⊆ B. Clearly every bi-ideal is a sub ternary semiring but converse is not true. observe the example.

Definition 2.6 Let S be a ternary semiring. A non-empty subset L of **S** is said to be left bi-quasi (right bi-quasi, lateral bi-quasi) ideal of S if L is a sub ternary semigroup of (**S**, +) and SSL ⋂ LSLSLSL ⊆ L (LSS ⋂ LSLSLSL ⊆ L,SLS ⋂ LSLSLSL ⊆ L).

Definition 2.7. Let S be a ternary semiring. A non-empty subset L of S is said to be bi-quasi ideal of S if L is a subsemigroup of (S, +), SSL⋂LSLSLSL⊆L

and LSS⋂LSLSLSL⊆L.

Definition 2.8. A ternary semiring S is called a left bi-quasi simple ternary semiring if S has no left bi-quasi ideals other than itself

Example2.9: Let Q be the set of all rational numbers.

If S = $\left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} / a,b,c \in Q, c \neq 0 \right\}$ then S is a ternary semiring with respect to binary operations, usual addition and multiplication of matrices.

If $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix} / a,c \in Q, c \neq 0 \right\}$ then A is a left bi-quasi ideal of ternary semiring S and A is not a bi-ideal of semiring S**.**

Let $\begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} . \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} . \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} \in A$ and $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \cdot \begin{pmatrix} r & s \\ 0 & t \end{pmatrix} \in S$.

Then $\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} = \begin{pmatrix} axp & bxq \\ 0 & cyq \end{pmatrix}$. Therefore A is not a bi-ideal of semiring S.

Suppose

$$\begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}\begin{pmatrix} r & s \\ 0 & t \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} = \begin{pmatrix} l & 0 \\ 0 & m \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} ur & us \\ 0 & vt \end{pmatrix} = \begin{pmatrix} axp & bxq \\ 0 & cyq \end{pmatrix} = \begin{pmatrix} l & 0 \\ 0 & m \end{pmatrix}$$

$$\Longrightarrow ur = axp = l$$

$$us = bxq = 0$$

$$vt = cyq = m \neq 0.$$

These equations have solutions.

Therefore $ASS\cap ASASASA\neq \phi$ and and $SSA\cap ASASASA\neq \phi$.

Hence A is a bi-quasi ideal of S.

3.Main Results

Theorem 3.1. If L is a left bi-quasi ideal of ternary semiring S then L is a subsemiring of S.

Proof. Suppose L is a left bi-quasi ideal of ternary semiring S.

Then. $SSL\cap LSLSLSL\subseteq L$

Let $x \in LLL$ then $x=pqr$ where $p,q,r \in L$

Now $x = pqr = plqlr \in LSLSL$ and $x\in LLL \subseteq SSL$

$$\Longrightarrow x\in SSL$$

$$\Longrightarrow x\in SSL\cap LSLSLSL\subseteq L$$

$$\Longrightarrow x\in L$$

Hence $LLL\subseteq L$

Theorem 3.2. Every left ideal of ternary semiring S is a bi-quasi ideal of S.

Proof: Let L be the left ideal of ternary semiring S

Then $SSL\subseteq L$.

Therefore

$SSL\cap LSLSLSL\subseteq LLL\subseteq L$

and $LSS\cap LSLSLSL\subseteq L$

Hence L is a bi-quasi ideal of S.

Theorem 3.3. Every right ideal of ternary semiring S is a bi-quasi ideal of S.

Proof. Let L be the right ideal of ternary semiring S.

Therefore SSL⊆ LLL⊆ L,SSL⋂LSLSLSL⊆LSLSLSL⊆L and

LSS⋂LSLSLSL⊆LSLSLSL⊆L.

Hence L is a bi-quasi ideal of S.

Theorem 3.4. Every quasi ideal of ternary semiring S is a bi- quasi ideal of S**.**

Proof. Let L be a quasi ideal of ternary semiring S. Then

LSS ∩ SLS ∩ SSL ⊆ L and LSS ∩ SSLSS ∩ SSL ⊆ L.

We have LSLSLSL ⊆ LSS, since SSL ⊆ L

Therefore SSL ∩ LSLSLSL⊆ LSLSLSL ⊆ L.

There fore L is a left bi-quasi ideal of S.

We have SLS ∩ LSLSLSL⊆ LSLSLSL ⊆ L.

So L is a lateral bi -quasi ideal of S.

And similarly LSS ∩ LSLSLSL⊆ LSLSLSL ⊆ L.

Similarly we can prove, L is a right bi-quasi ideal of S.

Hence L is a right bi - quasi ideal of S.

Theorem 3.5. Every bi-ideal of ternary semiring S is a bi-quasi ideal of S.

Proof. Let L be a bi-ideal of ternary ternary semiring S. Then LSLSL ⊆ L.

Therefore

SSL∩LSLSL⊆LSLSL⊆L, SLS ∩ LSLSL⊆ LSLSL ⊆ L and LSS ∩ LSLSL⊆ LSLSL ⊆ L.

Hence every bi-ideal of semiring S is a bi-quasi ideal of S**.**

Theorem 3.6: Arbitrary intersection of left bi-quasi ideals of ternary semiring S is either empty or a left bi-quasi ideal of S**.**

Proof. Let $\phi \neq L = \cap_{i\in I} L_i$; where L_i is a left bi-quasi ideal of ternary semiring S . We have $SSL_i \cap L_iSL_iSL_i \subseteq L_i$, for all i ∈ I. Then

$SSL \cap LSLSL \subseteq SSL_i \cap L_iSL_iSL_i \subseteq L_i$ for all $i \in I$.

Hence L is a left bi-quasi ideal of ternary semiring S**.**

Corollary 3.7 : Let S is a ternary semiring, L is a left ideal of S, I is a lateral ideal of S and R is a right ideal of S. Then $L \cap I \cap R$ is a bi-quasi ideal of S.

Theorem 3.8 : Let S be a ternary semiring. If $S=SSa$, for all $a \in S$ then every left bi-quasi of ternary semiring is a quasi ideal of ternary semiring S.

Proof. Suppose $S = SSa$, for all $a \in S$

and L be a left bi-quasi ideal of ternary semiring S.

Then $SSL \cap LSLSL \subseteq L$

$$\Rightarrow SSL \subseteq L$$

And $LSLSL \subseteq L \Rightarrow SLS \subseteq L,$

$$LSLSL \subseteq L$$

$$\Rightarrow LSS \subseteq L .$$

$$SSL \cap SLS \cap LSS \subseteq L ,$$

$$SSL \cap SSLSS \cap LSS \subseteq L.$$

since $S=SSa$, for all $a \in S$.

Hence L is a quasi ideal of ternary semiring S.

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