



## A Study On PR- Closed Sets in Bitopological Space

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### ABSTRACT

This paper studies a new type of mathematical set called pr-closed sets in bitopological spaces. We examine how these sets interact with pre-regular open, pre-regular closed, and pre-regular semi-open sets. Our main results show that the union of two pr-closed sets is also pr-closed, and any set "sandwiched" between a pr-closed set and its closure is pr-closed as well. We also prove that sets sharing overlapping properties become "clopen" (both open and closed) or standard closed sets under specific conditions. Ultimately, these findings expand our understanding of bitopological structures and provide a basis for future research on continuous maps and separation axioms.

### I. INTRODUCTION

Kelly introduced the concept of bitopological space in 1961. Jelly introduced the concept of closed sets in 1963. The author aimed to explain about the further properties of closed sets. The paper have been studied about the concept the pr- closed sets in bitopological space and its properties

### II. PREMILINARIES

Throughout this paper  $(X, \tau_1, \tau_2)$  denote the bitopological spaces.  $X \setminus A$  is the complement of a subset  $A$  of  $X$ . The interior and closure operators are respectively denoted by  $\text{int}_1(A)$  and  $\text{cl}_2(A)$ . The following definitions and results will be useful in sequel. The following definitions and result will be useful in this paper.

#### 2.1 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre – open** if  $A \subseteq \text{int}_1 [\text{cl}_2 (A)]$

#### 2.2 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre - closed** if  $A \subseteq \text{cl}_2 [\text{int}_1 (A)]$

#### 2.3 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Semi -open** if  $A \subseteq \text{int}_1 [\text{cl}_2 (A)]$

#### 2.4 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Semi- closed** if  $A \subseteq \text{cl}_2 [\text{int}_1 (A)]$

### III PR - CLOSED SETS

#### 3.1 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre regular open** if  $A = \text{int}_1 [\text{cl}_2 (\text{int}_1 (A))]$

#### 3.2 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre - regular closed** if  $A = \text{cl}_2 [\text{int}_1 (\text{cl}_2 (A))]$

#### 3.3 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre – regular semi open** if  $A \subseteq \text{int}_1 [\text{cl}_2 (\text{int}_1(A))]$

#### 3 4 DEFINITION

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called **Pre – regular semi closed** if  $A \subseteq \text{cl}_2 [\text{int}_1(\text{cl}_2 (A))]$

#### 3.5 RESULT

For any subset  $A$  of  $(X, \tau_1, \tau_2)$  is  $A \subset \text{prsker}(A)$

#### 3.6 RESULT

The space  $(X, \tau_1, \tau_2)$  is general bitopology  $\mu = \tau_1$  then  $\text{pr}\mu$ - closed sets become equivalent to pr closed set

#### 3.7 RESULT

The intersection of two pr -closed sets in  $X$  is generally not an pr closed set in  $X$

#### 3.8 THEOREM

The union of two pr- closed subsets of  $X$  is also an pr- closed subset of  $X$ .

Proof.

Assume that  $A$  and  $B$  are pr- closed sets in  $X$ . Let  $U$  be pre regular semi open in  $X$  such that  $A \cup B \subset U$ . Then  $A \subseteq U$  and  $B \subseteq U$ . Since  $A$  and  $B$  are pr closed,  $\text{cl}(A) \subset U$  and  $\text{cl}(B) \subset U$ .

Hence  $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) \subset U$ . That is  $\text{cl}(A \cup B) \subset U$ . Therefore  $A \cup B$  is an pr-closed set in  $X$ .

#### 3.9 THEOREM

For an element  $x \in X$ , the set  $X \setminus \{x\}$  is pr-closed or pre-regular semi open.

Proof.

Suppose  $X \setminus \{x\}$  is not pre-regular semi open. Then  $X$  is the only pre regular semi open set containing  $X \setminus \{x\}$ . This implies  $\text{cl}(X \setminus \{x\}) \subset X$ . Hence  $X \setminus \{x\}$  is an pr -closed set in  $X$ .

**3.10 THEOREM**

If  $A$  is pre regular open and pr - closed , then  $A$  is pre regular closed and hence clopen .

Proof .

Suppose  $A$  is pre regular open and pr - closed . As every pre regular open set is pre regular semiopen and  $A \subset A$ , we have  $\text{cl}(A) \subset A$ . Also  $A \subset \text{cl}(A)$ . Therefore  $\text{cl}(A) = A$ . That means  $A$  is closed . Since  $A$  is pre regular open  $A$  is open . Now  $\text{cl}(\text{int}(A)) = \text{cl}(A) = A$ . Therefore  $A$  is pre regular closed and clopen .

**3.11 THEOREM**

If  $A$  is an pr - closed subset of  $X$  such that  $A \subset B \subset \text{cl}(A)$ , then  $B$  is an pr - closed set in  $X$ .

Proof .

Let  $A$  be an pr - closed set of  $X$  such that  $A \subset B \subset \text{cl}(A)$ . Let  $U$  be a pre regular semi open set of  $X$  such that  $B \subset U$ . Then  $A \subset U$ . Since  $A$  is pr - closed , we have  $\text{cl}(A) \subset U$ . Now  $\text{cl}(B) \subset \text{cl}(\text{cl}(A)) = \text{cl}(A) \subset U$ . Therefore  $B$  is an pr - closed set in  $X$ .

**3.12 THEOREM**

Let  $A$  be pr - closed in  $(X, \tau_1, \tau_2)$ . Then  $A$  is closed if and only if  $\text{cl}(A) \setminus A$  is pre regular semiopen .

Proof .

Suppose  $A$  is closed in  $X$ . Then  $\text{cl}(A) = A$  and so  $\text{cl}(A) \setminus A = \emptyset$ , which is pre regular semiopen in  $X$ .

Conversely , suppose  $\text{cl}(A) \setminus A$  is pre regular semiopen in  $X$ . Since  $A$  is pr - closed ,  $\text{cl}(A) \setminus A$  does not contain any nonempty pre regular semiopen set in  $X$ . Then  $\text{cl}(A) \setminus A = \emptyset$ , hence  $A$  is closed in  $X$ .

**3.13 THEOREM**

If  $A$  is pre regular open and pr - closed , then  $A$  is pr - closed in  $X$ .

Proof .

Let  $A$  be pre regular open and pr - closed in  $X$ . We prove that  $A$  is an pr - closed set in  $X$ . Let  $U$  be any pre regular semiopen set in  $X$  such that  $A \subset U$ . Since  $A$  is pre regular open and pr - closed , we have  $\text{cl}(A) \subset A$ . Then  $\text{cl}(A) \subset A \subset U$ . Hence  $A$  is pr closed in  $X$ .

**3.14 THEOREM**

If a subset  $A$  of a bitopological space  $X$  is both pre regular semi open and pr - closed , then it is closed .

Proof .

Suppose a subset  $A$  of a bitopological space  $X$  is both pre regular semiopen and pr - closed .

Now  $A \subset A$ . Then  $\text{cl}(A) \subset A$ . Hence  $A$  is closed

## CONCLUSION

In this paper, we introduced and investigated the properties of PR-closed sets within the framework of bitopological spaces  $(X, \tau_1, \tau_2)$ , building upon the foundational concepts of bitopology and generalizing the notion of closed sets. Our findings established that the union of two PR-closed sets preserves this property, and we proved that if a set  $B$  is sandwiched between a PR-closed set  $A$  and its closure such that  $A \subset B \subset \text{cl}(A)$ , then  $B$  is also a PR-closed set. Furthermore, we demonstrated that if a set is both pre-regular open and PR-closed, it becomes pre-regular closed and thus clopen, while also providing a necessary and sufficient condition for a PR-closed set to be a standard closed set based on the set difference  $\text{cl}(A) \setminus A$ . Ultimately, this study enriches the literature on bitopological structures and these newly defined sets can be utilized in future research to explore new classes of continuous map, irresolute functions, and separation axioms.

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