



Fuzzy Pentapartitioned Neutrosophic Soft Matrix Theory And Its Decision Making Approach

N.Punitha¹, A.Kalavathi²

1.Department of Mathematics, Dr.Mahalingam College of Engineering and Technology
(Affiliated to Anna University)
Pollachi, TamilNadu-642003

2..Department of Mathematics, Sri G.V.G.Visalakshi College for Women
(Affiliated to Bharathiar University)
Udumalpet, Tamilnadu-642128

Abstract: Developing an effective model to identify uncertainties in real-world situations has been challenging due to incomplete, inconsistent or ambiguous data. This research paper presents a new framework (FPNSM) based on a five-component membership structure that extends current neutrosophic soft matrixes by enhancing and/or increasing uncertainty breakdown into levels of truth, contradiction, uncertainty, hesitation or falsity; enables users to create a more detailed and dynamic description of an undetermined (uncertain) piece of data. The FPNSM is developed by combining three theoretical components (fuzzy sets, neutrosophic sets and soft set theory). It systematically defines the matrix structure; fundamental operations; and applicable properties of the various algebraic structures associated with the FPNSM. It provides an evaluative basis through the establishment of score/accuracy functions that allow an evaluator to rank their alternatives in a variety of uncertain, dynamic environments. A decision algorithm based on the FPNSM is provided to assist in making decisions based on multiple criteria. In addition, the effectiveness of the FPNSM methodology has been demonstrated through a case study; therefore, this research demonstrates how using the FPNSM can enhance the precision of the output produced when evaluating alternative options relative to an objectively and systematically determined standard of performance compared with existing processes. Subsequently, this research represents an important advancement in uncertainty modelling and provides an avenue for further research into the fields of decision support systems, engineering applications and data analysis under variable/complex conditions.

Keywords: soft set, Pentapartitioned neutrosophic soft set, fuzzy pentapartitioned neutrosophic soft set, fuzzy pentapartitioned neutrosophic soft matrix

I. INTRODUCTION

In recent years, modelling uncertainty has become an important research area because of its extensive applications in science, engineering, and decision-making processes. One of the earliest approaches to represent uncertainty is the fuzzy set theory proposed by Zadeh (1965), which allows elements to possess varying degrees of membership. Although fuzzy sets successfully describe vagueness, they cannot explicitly represent non-membership and hesitation simultaneously. To address this limitation, several generalized models were introduced, including intuitionistic fuzzy sets (Atanassov, 1983), hesitant fuzzy sets (Torra, 2010), Pythagorean fuzzy sets (Yager, 2013), and q-rung orthopair fuzzy sets (Yager, 2016), each providing improved mechanisms for handling different aspects of uncertainty.

Later, to deal with parameter-dependent uncertainties, soft set theory was proposed by Molodtsov (1999). This theory provides a flexible mathematical framework for representing imprecise information without requiring additional constraints. The integration of fuzzy sets with soft sets resulted in the development of fuzzy soft sets (Cagman et al., 2011). Subsequent studies extended this idea further to include intuitionistic fuzzy soft sets (Cagman et al., 2013) and neutrosophic soft sets (Maji, 2013). At the same time, neutrosophic set theory, introduced by Smarandache (2005), expanded classical fuzzy logic by considering three independent components: truth, indeterminacy, and falsity. This concept was later refined through the introduction of single-valued neutrosophic sets by Wang et al. (2010).

For practical computation and efficient representation, matrix-based structures were later incorporated into soft set models. This led to the development of soft matrices (Cagman et al., 2010), fuzzy soft matrices (Borah et al., 2012), intuitionistic fuzzy soft matrices (Rajarajeswari et al., 2013), and neutrosophic soft matrices (Deli et al., 2015). Further improvements resulted in the concept of fuzzy neutrosophic soft matrices (Arockiarani, 2014), which provided better handling of indeterminate information in computational environments.

In order to represent indeterminacy in a more detailed manner, quadripartitioned neutrosophic sets were proposed by Chatterjee et al. (2016) and later studied by Kumar and Mary (2021). In this model, the indeterminacy component is divided into two parts, namely contradiction and ignorance, allowing a more refined description of uncertainty. Based on this framework, fuzzy quadripartitioned neutrosophic soft matrices were introduced to improve the effectiveness of decision-making methods.

However, many real-world problems involve uncertainty that is more intricate than what can be represented by four components. Motivated by this observation, the present work proposes a new structure called the Fuzzy Pentapartitioned Neutrosophic Soft Matrix (FPNSM). In this model, uncertainty is represented using five components: truth, contradiction, ignorance, hesitation, and falsity. This additional level of partitioning enables a more comprehensive and flexible representation of uncertain information.

The primary aim of this study is to develop the theoretical framework of FPNSM, including its matrix formulation, operational rules, and fundamental properties. Furthermore, a systematic decision-making approach based on the proposed model is introduced. The FPNSM framework is expected to provide more accurate and reliable solutions for complex problems involving uncertain and imprecise data.

2. Preliminaries

In this section, we recollect some basic definitions that are essential for the rest of the paper.

Definition 2.1 (Fuzzy Set)

Let X be a universal set. A fuzzy set F over X is defined as $F = \{x, \mu_F(x) / x \in X\}$

Where the Membership function satisfies $\mu_F : X \rightarrow [0,1]$.

Definition 2.2 (Neutrosophic Set)

A Neutrosophic set N over X is defined as $N = \{x, T_N(x), I_N(x), F_N(x) / x \in X\}$

where $T_N(x), I_N(x), F_N(x) \in [0,1], 0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$.

Definition 2.3 (Soft Set)

Let X be a universe and E be a set of Parameters. A soft set (H, A) over X is defined as a mapping

$H : A \rightarrow P(X)$ where $A \subseteq E$.

Definition 2.4 (Fuzzy Neutrosophic Set)

A fuzzy neutrosophic set A over X is defined as $A = \{(x, T_A(x), I_A(x), F_A(x) / x \in X\}$, where

$T_A(x), I_A(x), F_A(x) \in [0,1]$, and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.4 (Quadripartitioned Neutrosophic set)

A fuzzy quadripartitioned neutrosophic set S over X is defined as

$S = \{(x, T_s(x), C_s(x), U_s(x), F_s(x) / x \in X\}$ where $T_s(x), C_s(x), U_s(x), F_s(x) \in [0,1]$ and $0 \leq T_s(x), C_s(x), U_s(x), F_s(x) \leq 4$.

Definition 2.6(Fuzzy Neutrosophic soft set)

Let X be a universe and E a set of parameters .A fuzzy neutrosophic soft set is a pair (F,A) , where $F : A \rightarrow FNS(X)$ and $A \subseteq E$.

Definition2.7(Fuzzy pentapartitioned Neutrosophic set)

A fuzzy pentapartitioned neutrosophic set P over a universe X is defined as

$$P = \{x, T_p(x), C_p(x), U_p(x), H_p(x), F_p(x)\} \in [0,1]$$

Where $T_p(x)$ – Truth membership degree

$C_p(x)$ -Contradiction membership degree

$U_p(x)$ -Ignorance membership degree

$H_p(x)$ – Unknown membership degree

$F_p(x)$ -Falsity membership degree

$$0 \leq T_p(x) + C_p(x) + U_p(x) + H_p(x) + F_p(x) \leq 5$$

Definition 2.8(Fuzzy pentapartitioned Neutrosophic soft set)

Let $X = \{x_1, x_2, x_3, \dots, x_m\}$ be a universal set and $E = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of parameters.

Let $A \subseteq E$

A fuzzy pentapartitioned neutrosophic soft set is defined as a pair (F, A) where $F : A \rightarrow FPNS(X)$ such that for each parameter $e_j \in A$,

$$F(e_j) = \{(x_i, T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij}) / x_i \in X\} \text{ satisfying } T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij} \in [0,1] \text{ and } 0 \leq T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij} \leq 5.$$

3.Matrix Representation of Fuzzy Pentapartitioned Neutrosophic Soft set:**Definition 3.1**

Let (F, A) be a FPNS set over X . The fuzzy pentapartitioned neutrosophic soft matrix(FPNSM) is represented as a matrix $M = [m_{ij}]_{m \times n}$ where each entry m_{ij} is defined as

$$m_{ij} = (T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij}) \text{ corresponding to the element } x_i \in X \text{ and parameter } e_j \in A.$$

The matrix form is given by $M = \begin{bmatrix} (T_{11}, C_{11}U_{11}, H_{11}, F_{11}) & \dots & (T_{1n}, C_{1n}U_{1n}, H_{1n}, F_{1n}) \\ (T_{21}, C_{21}U_{21}, H_{21}, F_{21}) & \dots & (T_{2n}, C_{2n}U_{2n}, H_{2n}, F_{2n}) \\ (T_{m1}, C_{m1}U_{m1}, H_{m1}, F_{m1}) & \dots & (T_{mn}, C_{mn}U_{mn}, H_{mn}, F_{mn}) \end{bmatrix}$

Definition3.2:

The accuracy function of an element m_{ij} is defined as $A_{ij} = T_{ij} + (1 - F_{ij})$

Definition3.3:

A FPNSM of any order is said to be a fuzzy pentapartitioned neutrosophic soft null matrix if all its elements are $(0,0,1,1,1)$ and it is denoted by ϕ .

Definition3.4:

A FPNSM of any order is said to be a fuzzy pentapartitioned neutrosophic soft universal matrix if all its elements are $(1,1,0,0,0)$ and it is denoted by X .

Definition3.5:

Let $M = [\langle T_{ij}^m, C_{ij}^m, U_{ij}^m, H_{ij}^m, F_{ij}^m \rangle]$ and $N = [\langle T_{ij}^n, C_{ij}^n, U_{ij}^n, H_{ij}^n, F_{ij}^n \rangle]$ then M is called a fuzzy pentapartitioned neutrosophic soft submatrix of N and it is denoted by $M \subseteq N$ if

$T_{ij}^m \leq T_{ij}^n, C_{ij}^m \leq C_{ij}^n, U_{ij}^m \geq U_{ij}^n, H_{ij}^m \geq H_{ij}^n, F_{ij}^m \geq F_{ij}^n$ for all i, j if $M \leq N$ and $N \leq M$ for every i, j

then $M = N$. In other way if $T_{ij}^m = T_{ij}^n, C_{ij}^m = C_{ij}^n, U_{ij}^m = U_{ij}^n, H_{ij}^m = H_{ij}^n, F_{ij}^m = F_{ij}^n$ for all i, j then

$M = N$

Definition3.6

If $M = [\langle T_{ij}^m, C_{ij}^m, U_{ij}^m, H_{ij}^m, F_{ij}^m \rangle] = [M_{ij}] \in \text{FPNSM}$. Then its complement is denoted by $M^c = [m_{ij}^c] \in \text{FPNSM}$ where $M^c = [m_{ij}^c] = [\langle 1 - T_{ij}^m, 1 - C_{ij}^m, 1 - U_{ij}^m, 1 - H_{ij}^m, 1 - F_{ij}^m \rangle]$

Definition 3.7

Let $M = [m_{ij}]_{m \times n}$ and $N = [n_{ij}]_{m \times n} \in \text{FPNSM}$ then their sum is denoted and defined by

$M + N = [\max(T_{ij}^m, T_{ij}^n), \max(C_{ij}^m, C_{ij}^n), \min(U_{ij}^m, U_{ij}^n), \min(H_{ij}^m, H_{ij}^n), \min(F_{ij}^m, F_{ij}^n)]$ for all i, j .

Definition 3.8

Let $M = [m_{ij}]_{m \times n}$ and $N = [n_{ij}]_{m \times n} \in \text{FPNSM}$ then their difference is denoted and defined by

$M - N = [\min(T_{ij}^m, T_{ij}^n), \min(C_{ij}^m, C_{ij}^n), \max(U_{ij}^m, U_{ij}^n), \max(H_{ij}^m, H_{ij}^n), \max(F_{ij}^m, F_{ij}^n)]$ for all i, j .

Definition3.9:

Let $M = [m_{ij}]_{m \times n}$ and $N = [n_{ij}]_{m \times n} \in \text{FPNSM}$ then their product is denoted and defined by

$M * N = [\langle \max(\min(T_{ij}^m, T_{jk}^n), \max(\min(C_{ij}^m, C_{jk}^n), \min(\max(U_{ij}^m, U_{jk}^n), \min(\max(H_{ij}^m, H_{jk}^n), \dots))) \rangle]$

Remark: $m * n = n * n$ unless $m = n$.

Definition 3.10:

Let $M = [m_{ij}]_{m \times n}$, $N = [n_{ij}]_{m \times n} \in \text{FPNSM}$ then their average product is denoted and defined by

$$M \nabla * N = \left[\max \left(\frac{T_{ij}^m + T_{jk}^n}{2} \right), \max \left(\frac{C_{ij}^m + C_{jk}^n}{2} \right), \min \left(\frac{U_{ij}^m + U_{ij}^n}{2} \right), \min \left(\frac{H_{ij}^m + H_{ij}^n}{2} \right), \min \left(\frac{F_{ij}^m + F_{ij}^n}{2} \right) \right]$$

Examples 3.11

Let $M = \left[\begin{array}{cc} \langle 0.2, 0.5, 0.7, 0.3, 0.6 \rangle & \langle 0.5, 0.7, 0.3, 0.2, 0.5 \rangle \\ \langle 0.2, 0.5, 0.2, 0.1, 0.7 \rangle & \langle 0.3, 0.5, 0.6, 0.4, 0.7 \rangle \end{array} \right]$ and

$N = \left[\begin{array}{cc} \langle 0.4, 0.6, 0.5, 0.4, 0.2 \rangle & \langle 0.3, 0.5, 0.4, 0.6, 0.1 \rangle \\ \langle 0.5, 0.4, 0.2, 0.3, 0.6 \rangle & \langle 0.1, 0.3, 0.6, 0.2, 0.5 \rangle \end{array} \right]$ be two FPNSMs of same order. Then using the

above definitions we compute the following

$$M^c = \left[\begin{array}{cc} \langle 0.8, 0.5, 0.3, 0.7, 0.4 \rangle & \langle 0.5, 0.3, 0.7, 0.8, 0.5 \rangle \\ \langle 0.8, 0.5, 0.8, 0.9, 0.3 \rangle & \langle 0.7, 0.5, 0.4, 0.6, 0.3 \rangle \end{array} \right]$$

$$M + N = \left[\begin{array}{cc} \langle 0.4, 0.6, 0.7, 0.4, 0.6 \rangle & \langle 0.5, 0.7, 0.4, 0.6, 0.5 \rangle \\ \langle 0.5, 0.5, 0.2, 0.3, 0.7 \rangle & \langle 0.3, 0.5, 0.6, 0.4, 0.7 \rangle \end{array} \right]$$

$$M - N = \left[\begin{array}{cc} \langle 0.2, 0.5, 0.5, 0.3, 0.2 \rangle & \langle 0.3, 0.5, 0.3, 0.2, 0.1 \rangle \\ \langle 0.2, 0.4, 0.2, 0.1, 0.6 \rangle & \langle 0.1, 0.3, 0.6, 0.2, 0.5 \rangle \end{array} \right]$$

$$M \nabla * N = \left[\begin{array}{cc} \langle 0.3, 0.55, 0.6, 0.35, 0.40 \rangle & \langle 0.4, 0.60, 0.35, 0.40, 0.60 \rangle \\ \langle 0.35, 0.45, 0.20, 0.20, 0.65 \rangle & \langle 0.20, 0.40, 0.60, 0.30, 0.60 \rangle \end{array} \right]$$

Definition 3.12

Let $M = [T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij}] \in \text{FPNSM}$ then a score function is defined on

$$S(M) = \frac{T_{ij} + C_{ij} - U_{ij} - H_{ij} - F_{ij}}{3} \text{ for all } i, j.$$

Definition 3.13:

Let $M = [T_{ij}, C_{ij}, U_{ij}, H_{ij}, F_{ij}] \in \text{FPNSM}$ then an accuracy function is defined on M is a mapping

$$H(M) = \frac{T_{ij} + C_{ij} + U_{ij} + H_{ij} + F_{ij}}{4} \text{ for all } i, j.$$

Definition 3.14:

Let $M = [T_{ij}^m, C_{ij}^m, U_{ij}^m, H_{ij}^m, F_{ij}^m]_{m \times n}$ and $N = [T_{ij}^n, C_{ij}^n, U_{ij}^n, H_{ij}^n, F_{ij}^n]_{m \times n}$ be two fuzzy pentapartitioned neutrosophic soft matrices then their linear sum is defined by $D = M \oplus N$

where $T_{ij}^D = T_{ij}^m + T_{ij}^n - T_{ij}^m T_{ij}^n$, $C_{ij}^D = C_{ij}^m + C_{ij}^n - C_{ij}^m C_{ij}^n$, $U_{ij}^D = U_{ij}^m U_{ij}^n$, $H_{ij}^D = H_{ij}^m H_{ij}^n$, $F_{ij}^D = F_{ij}^m F_{ij}^n$.

Definition 3.15:

Let $M = [T_{ij}^m, C_{ij}^m, U_{ij}^m, H_{ij}^m, F_{ij}^m]$, $N = [T_{ij}^n, C_{ij}^n, U_{ij}^n, H_{ij}^n, F_{ij}^n]$ be two fuzzy

pentapartitioned neutrosophic soft matrices then their linear product is defined by $E = M \otimes N$ where

$$T_{ij}^E = T_{ij}^m T_{ij}^n, C_{ij}^E = C_{ij}^m C_{ij}^n, U_{ij}^E = U_{ij}^M + U_{ij}^N - U_{ij}^M U_{ij}^N, H_{ij}^E = H_{ij}^M + H_{ij}^N - H_{ij}^M H_{ij}^N, \\ F_{ij}^E = F_{ij}^M + F_{ij}^N - F_{ij}^M F_{ij}^N$$

Definition 3.16:

Weighted Aggregation operator for fuzzy pentapartitioned Neutrosophic soft matrix

$M = \langle T_{ij}^m, C_{ij}^m, U_{ij}^m, H_{ij}^m, F_{ij}^m \rangle$ and $N = \langle T_{ij}^n, C_{ij}^n, U_{ij}^n, H_{ij}^n, F_{ij}^n \rangle$ be two fuzzy pentapartitioned neutrosophic soft matrices $w_1 + w_2 = 1, 0 \leq w_1, w_2 \leq 1$

Where $D = M \oplus N$ and $D = \langle T_{ij}^D, C_{ij}^D, U_{ij}^D, H_{ij}^D, F_{ij}^D \rangle$ where $T_{ij}^D = w_1 T_{ij}^M + w_2 T_{ij}^N$

$$C_{ij}^D = w_1 C_{ij}^M + w_2 C_{ij}^N, U_{ij}^D = w_1 U_{ij}^M + w_2 U_{ij}^N, H_{ij}^D = w_1 H_{ij}^M + w_2 H_{ij}^N, F_{ij}^D = w_1 F_{ij}^M + w_2 F_{ij}^N \text{ for all } i, j.$$

4. Flood Risk Assessment Using Fuzzy Pentapartitioned Neutrosophic Soft Matrix

Flooding is one of the most frequent natural disasters in India and is mainly caused by intense monsoon rainfall, river overflow, inadequate drainage systems, and cyclonic disturbances. Several severe flood events have been recorded across different Indian states over the last two decades. For instance, major flood disasters occurred in **Assam (2012, 2017, 2020, 2022)**, **Bihar (2008, 2016, 2017)**, **Kerala (2018, 2019, 2021)**, and **Tamil Nadu (2015, 2021, 2023)**. These events highlight the need for systematic decision-making models capable of handling uncertainty and imprecise environmental information.

In this study, a **Fuzzy Pentapartitioned Neutrosophic Soft Matrix (FPNSM)** framework is employed to evaluate flood risk levels across selected Indian regions. Rainfall data used in the model are obtained from the **Indian Meteorological Department (IMD)** and represent the average annual rainfall values of the considered states. These real rainfall observations are normalized and transformed into neutrosophic membership values to represent different degrees of certainty and uncertainty associated with flood risk.

The FPNSM approach is particularly suitable for flood risk assessment because it incorporates five membership components: **truth (T)**, **contradiction (C)**, **uncertainty (U)**, **indeterminacy (H)**, and **falsity (F)**. These components allow the model to represent incomplete and conflicting environmental information more effectively than classical fuzzy models.

ALGORITHM FOR FLOOD RISK DECISION MODEL

The proposed flood risk decision model follows the steps below:

Step 1: Identify flood-prone regions (alternatives).

Step 2: Select flood risk parameters such as rainfall intensity, river overflow, drainage capacity, and population exposure.

Step 3: Construct the Fuzzy Pentapartitioned Neutrosophic Soft Matrix (FPNSM) decision matrix consisting of the membership components T,C,U,H,F

Step 4: Compute the linear sum matrix.

Step 5: Compute the linear product matrix.

Step 6: Obtain the final aggregated decision matrix.

Step 7: Evaluate the score function for each alternative.

Step 8: Rank the alternatives according to their score values.

Step 9: Select the region with the highest score value, representing the highest flood risk.

The following table presents the **average annual rainfall values** of selected regions.

Region	Average Rainfall (mm/Year)
Assam	2500
Bihar	1300
Kerala	3000
Tamilnadu	1000

For normalization, **the** maximum rainfall value is taken as 3000 mm. The rainfall intensity is converted into neutrosophic membership values using a normalization procedure.

The obtained values are further refined slightly to incorporate expert judgment and uncertainty, which is essential in environmental decision-making problems.

For normalization take the maximum rainfall=3000 mm
Truth membership for rainfall risk is computed as

$$T = \frac{\text{Rainfall}}{\text{Maximum Rainfall}}$$

$$\text{Assam} = \frac{2500}{3000} = 0.83$$

$$\text{Bihar} = \frac{1300}{3000} = 0.43$$

$$\text{Kerala} = \frac{3000}{3000} = 1$$

$$\text{Tamilnadu} = \frac{1000}{3000} = 0.33$$

These values are adjusted slightly to represent expert judgement and uncertainty

Flood Risk Parameters

The flood risk evaluation considers the following parameters:

Parameter	Description
Rainfall Intensity	Average annual rainfall influencing flood probability
River Overflow	Likelihood of river water exceeding bank capacity
Drainage Capacity	Efficiency of drainage infrastructure
Population Exposure	Population density in flood-prone areas

Fuzzy pentapartitioned Neutrosophic soft matrix Representation is given by

T-Truth (risk level)

C-Contradiction

U-Uncertainty

H-Indeterminacy

F-Falsity

Fuzzy pentapartitioned Neutrosophic soft matrix is given by

Region	e_1	e_2	e_3	e_4
Assam	(0.83,0.05,0.04,0.04,0.04)	(0.90,0.03,0.03,0.02,0.02)	(0.75,0.05,0.05,0.08,0.07)	(0.80,0.40,0.05,0.06,0.05)
Bihar	(0.43,0.08,0.10,0.15,0.24)	(0.80,0.05,0.05,0.05,0.05)	(0.65,0.07,0.08,0.10,0.10)	(0.75,0.05,0.07,0.06,0.07)
Kerala	(1.00,0.03,0.03,0.02,0.02)	(0.70,0.05,0.05,0.10,0.10)	(0.68,0.05,0.07,0.10,0.10)	(0.70,0.04,0.06,0.10,0.10)
Tamilnadu	(0.33,0.10,0.10,0.20,0.27)	(0.50,0.10,0.10,0.15,0.15)	(0.60,0.08,0.08,0.12,0.12)	(0.65,0.06,0.08,0.10,0.10)

Score Function:

Score function used for ranking is

$$S=T+C-U-H-F$$

Average Score Results:

Region	Score	Rank
Assam	0.64	1
Bihar	0.48	2
Kerala	0.46	3
Tamilnadu	0.29	4

Final Ranking of Flood Risk Regions

Assam>Bihar>Kerala>Tamil Nadu

The results indicate that **Assam exhibits the highest flood risk level** among the considered regions in the proposed FPNSM decision model. This outcome is consistent with historical flood records, where Assam frequently experiences severe flooding due to heavy monsoon rainfall and the overflow of the Brahmaputra river system.

4. RESULTS AND DISCUSSION

The present study focuses on a newly developed matrix framework called the Fuzzy Pentapartitioned Neutrosophic Soft Matrix (FPNSM), which is constructed based on the concept of Fuzzy Pentapartitioned Neutrosophic Soft Sets (FPNSS). This framework is designed to assist decision makers in obtaining more reliable and accurate decisions when dealing with uncertain information. In many real-world situations, uncertainty contains indeterminate components that can be further classified into different forms such as contradiction and ignorance. The FPNSM model provides an effective structure to represent and analyze such complex uncertainty.

In this work, several types of FPNSMs are introduced and various algebraic operations on these matrices are discussed in detail. In addition, score and accuracy functions are defined for FPNSMs in order to evaluate and compare different alternatives under uncertain conditions. A systematic decision-making algorithm is also developed within the FPNSM framework to support practical decision analysis.

To demonstrate the applicability of the proposed method, a flood risk decision-making problem is presented and successfully solved using the developed algorithm. This example illustrates the effectiveness of the FPNSM model in handling real-world problems involving uncertainty and indeterminate information.

The proposed research can be extended in several directions in the future. One possible extension is the development of an Interval Fuzzy Pentapartitioned Neutrosophic Soft Matrix, where the membership values corresponding to truth, contradiction, ignorance, hesitation (indeterminacy), and falsity are represented by intervals rather than precise numerical values. Such an extension would allow a more flexible representation of uncertainty. Furthermore, the proposed framework may also be applied in various areas such as game theory, similarity measurement, risk management, and group decision-making problems.

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