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BIANCHI TYPE VIII BULK VISCOUS COSMOLOGICAL MODEL WITH BAROTROPIC EQUATION OF STATE AND DARK ENERGY

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ABSTRACT- Bianchi type VIII bulk viscous cosmological model with barotropic equation of state and dark energy is investigated. To achieve a deterministic solution, we assume that the coefficient of shear viscosity(σ) is proportional to expansion (θ), which leads to $A = B^m$, where A and B are metric potentials. Also coefficient of bulk viscosity (ξ) is assumed proportional to expansion, i.e. $\xi\theta = K$, where K is constant. Barotropic fluid equation $p = \gamma\rho$, where (γ) is a constant known as the barotropic index, (p) is pressure and (ρ) is energy density, is taken into consideration to determine cosmological constant (Λ). The behavior of the model from physical and geometrical aspects is discussed in detail.

KEYWORDS: Bianchi Type VIII, Bulk Viscosity, Barotropic Equation, Dark Energy.

1. INTRODUCTION

Bianchi space-times are key sources in understanding the initial stages of the developing universe. Since anisotropy is crucial to these early stages so models representing both homogeneity and anisotropy such as Bianchi type VIII have more importance to be explored in general relativity. Many researchers developed model of universe within the context of these space-times [1, 4, 7, 9, 16, and 20]. Chhajed et al. [6] examined behavior of such anisotropic models during the phase of inflation, focusing on a stiff perfect fluid distribution and a flat potential. Kushinova et al. [10] developed non-stationary models including various sources of gravity and studied the effect of rotating dark energy on them in general relativity.

Many academics have proposed viscous fluid-based cosmological models to examine the evolution of the cosmos. Such theories help to explain the significant increase in entropy per baryon observed in the current universe. Padmanabhan and Chitre [15] studied the models that include bulk viscosity within the cosmic fluids. It is observed that inflation-like solutions can result from bulk viscosity. Banerjee and Santos [5] investigated both viscous and non-viscous fluids in different Bianchi space-times (II, VIII and IX). Singh et al. [21] explored Bianchi type-VI_0 cosmological model including matter with bulk viscosity and dust using C-field theory. Sharma and Poonia [19] developed Bianchi type-IX inflationary universe model using the framework of the effects of flat potential and bulk viscous fluid. Recently bulk viscosity in the context of string Bianchi type-III space-time is examined by Patra et al. [17].

In the study of universe's modeling, barotropic equation of state (Eos) is defined by a linear relationship between pressure p and energy density ρ of cosmic fluid which can be written as $p = \gamma\rho$, where γ is a constant known as the barotropic index. Furthermore, the concept of dark energy being the fundamental component of cosmos, explains its rapid expansion and acceleration more accurately. The simplest theoretical explanation for dark energy is the cosmological constant. Since approximately 70% of the total energy density of the universe is in this form of dark energy so the models containing cosmological constant become more important to explore. Several authors have worked on models including these components. Bali and Singh [3] considered an inflationary case for a barotropic fluid distribution with decreasing vacuum energy density and fluctuating bulk viscosity in Bianchi Type V space-time

Nojiri et al. [13] studied the relationship between deceleration-acceleration and matter dominance-dark energy transitions. Spatially homogenous Bianchi type-II, VIII, and IX cosmological models containing dark energy and barotropic fluid are produced by Rao and Sireesha [18]. Mishra et al. [12] examined the anisotropic behavior of the cosmological model built in two fluid scenarios: the dark energy fluid and the typical bulk viscous fluid. Tyagi et al. [22] investigated Bianchi type-VI_0 cosmological model with and without a magnetic field for barotropic fluid distribution with dark energy. Odintsov et al. [14] studied cosmological models in which bulk viscosity is considered to represent dark energy, resulting in an effective pressure that can account for the late-time accelerated expansion. Yadav et al. [23] investigated a bulk viscous universe dominated by dark energy (DE) within the framework of Bianchi type I space-time. Gusu [8] constructed a cosmological model of Bianchi type-I that contains dark energy and bulk viscous fluid. Bali and Goyal [2] studied an inflationary scenario in Bianchi type V space-time with varying bulk viscosity and dark energy in the radiation-dominated phase.

Motivated by the research work previously provided, this paper includes the investigation of the barotropic bulk viscous Bianchi type VIII cosmological model with dark energy. In order to obtain the deterministic model we assume that σ (shear) is proportional to θ (expansion) which leads to $A = B^m$, where m is a constant, A and B are metric potentials and $\xi\theta = K$, where ξ is coefficient of bulk viscosity.

2. METRIC AND FIELD EQUATION

The Bianchi type VIII space - time can be given by following line element:

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - \left(A^2 \sinh^2 y + B^2 \cosh^2 y\right)dz^2$$
$$- 2B^2 \cosh y dx dz \qquad (1)$$

Here A and B are functions of single variable t.

For the fluid with bulk viscosity, energy momentum tensor T_i^j can be expressed as follows:

$$T_i^j = (\bar{p} + \rho)v_i v^j - \bar{p} g_i^j \qquad (2)$$

The density of matter is represented here by ρ, pressure is indicated by $\bar{p}$, a space-like unit vector is denoted by $x^i = \left(\frac{1}{B},0,0,0\right)$ and the four-velocity vector is described by $v^i = (0,0,0,1)$. This vector also adheres to the following relation

$$g_{ij}v^i v^j = 1 \qquad (3)$$

The effective pressure $\bar{p}$ is related to equilibrium pressure p by relation

$$\bar{p} = p - \xi\theta \qquad (4)$$

where ξ is the co-efficient of bulk viscosity and θ is the expansion scalar.

In the context of geometrized unit $(8\pi G = c = 1)$, the Einstein field equation can be expressed as:

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \qquad (5)$$

Einstein's field equations (5) for the metric (1) are obtained as follows:

$$\frac{2A_{44}}{A} + \frac{A_4^{\ 2}}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} - \Lambda = -p + \xi\theta \qquad (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} - \Lambda = -p + \xi\theta \qquad (7)$$

$$\frac{2A_4 B_4}{AB} + \frac{A_4^{\ 2}}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} - \Lambda = \rho \qquad (8)$$

The subscript 4 associated with the symbols A and B indicates differentiation with respect to variable 't'.

The scalar expansion θ is described by

$$\theta = v^i_{;i} = \frac{2A_4}{A} + \frac{B_4}{B} \qquad (9)$$

The shear scalar σ is given by

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \tag{10}$$

The Hubble parameter's generalized mean is expressed as follows $H = \frac{\theta}{3} = \frac{1}{3}\left(\frac{2A_4}{A} + \frac{B_4}{B}\right)$

(11)

Hence the deceleration parameter is characterized by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{12}$$

3. SOLUTION OF FIELD EQUATIONS

Here we have system of three equations (6) to (8) in eight unknowns A, B, p, ρ, ξ, θ, and Λ. For deterministic solution we assume, expansion θ is directly related to shear σ resulting in

$$A = B^m \tag{13}$$

Where m is arbitrary constant.

Also we assume coefficient of bulk viscosity ξ is inversely propotional to expansion, that is

$$\xi\theta = K \tag{14}$$

Where K is a constant

Using conditions (13) and (14) in field equations (6) and (7) we have equations

$$\frac{2mB_{44}}{B} + \frac{(3m^2 - 2m)B_4^{\,2}}{B^2} - \frac{1}{B^{2m}} - \frac{3}{4B^{4m-2}} - \Lambda = -p + K \tag{15}$$

$$\frac{(m+1)B_{44}}{B} + \frac{m^2B_4^{\,2}}{B^2} + \frac{1}{4B^{4m-2}} - \Lambda = -p + K \tag{16}$$

Equations (15) and (16) together lead to

$$\frac{(m-1)B_{44}}{B} + \frac{2m(m-1)B_4^{\,2}}{B^2} - \frac{1}{B^{4m-2}} - \frac{1}{B^{2m}} = 0 \tag{17}$$

Equation (17) can be written as

$$B_{44} + \frac{2mB_4^{\,2}}{B} = \frac{1}{(m-1)B^{4m-3}} + \frac{1}{(m-1)B^{2m-1}} \tag{18}$$

On putting $B_4 = l(B)$ and $B_{44} = ll'$ in equation (18), the following equation is obtained

$$\frac{dl^2}{dB}+\frac{4m^2l^2}{B}=\frac{2}{(m-1)B^{4m-3}}+\frac{2}{(m-1)B^{2m-1}} \tag{19}$$

Solving equation (19) we get

$$l^2=\frac{1}{2(m-1)B^{4m-4}}+\frac{1}{(m^2-1)B^{2m-2}}+\frac{L}{B^{4m}} \tag{20}$$

In equation (20), L denotes the constant of integration. On integrating this equation we obtain

$$\int\frac{dB}{\sqrt{\frac{1}{2(m-1)B^{4m-4}}+\frac{1}{(m^2-1)B^{2m-2}}+\frac{L}{B^{4m}}}}=t+L' \tag{21}$$

where L' serves as the constant of integration. The value of B can be derived from equation (21). Consequently, by suitably transforming the co-ordinates, namely $B=\tau$, $x=X$, $y=Y$ and $z=Z$, metric (21) is modified to

$$ds^2=\frac{d\tau^2}{\frac{1}{2(m-1)\tau^{4m-4}}+\frac{1}{(m^2-1)\tau^{2m-2}}+\frac{L}{\tau^{4m}}}-\tau^2dX^2-\tau^{2m}dY^2$$
$$-\left(\tau^{2m}\sinh^2 Y+\tau^2\cosh^2 Y\right)dZ^2-2\tau^2\cosh Y dXdZ \tag{22}$$

4. PHYSICAL AND GEOMETRICAL CHARACTERISTICS

Model (22) provides the formulas for energy density (ρ), pressure (p), as follows (for $m\neq -1, m\neq 1$):

$$\rho=\frac{(2m+1)}{(m^2-1)\tau^{2m}}+\frac{(2m^2+3m+1)}{4(m-1)\tau^{4m-2}}+\frac{L(m^2+2m)}{\tau^{4m+2}}-\Lambda \tag{23}$$

$$p=-\frac{1}{(m^2-1)\tau^{2n}}+\frac{(2m^2-m-3)}{4(m-1)\tau^{4m-2}}+\frac{L(m^2+2m)}{\tau^{4m+2}}+\Lambda+K \tag{24}$$

Cosmological constant can be determined by taking barotropic fluid condition.

$$p=\gamma\rho; 0\leq\gamma\leq 1 \tag{25}$$

Hence by equations (23), (24) and (25) we obtain value of cosmological constant as

$$\Lambda=\frac{(2m^2\gamma-2m^2+3m\gamma+\gamma+m+3)}{4(1+\gamma)(m-1)\tau^{4m-2}}+\frac{1+2m\gamma+\gamma}{(1+\gamma)(m^2-1)\tau^{2m}}$$
$$+\frac{(\gamma-1)(m^2+2m)L}{(1+\gamma)\tau^{4m+2}}-\frac{K}{(1+\gamma)} \tag{26}$$

Using equation (26) in (23) and (24), we get density and pressure in terms of γ as follows:

$$\rho = \frac{(2m^2 + m - 1)}{2(1+\gamma)(m-1)\tau^{4m-2}} + \frac{2m}{(1+\gamma)(m^2-1)\tau^{2m}} + \frac{2L(m^2+2m)}{(1+\gamma)\tau^{4m+2}} + \frac{K}{(1+\gamma)} \tag{27}$$

$$p = \frac{(2m^2 + m - 1)\gamma}{2(1+\gamma)(m-1)\tau^{4m-2}} + \frac{2m\gamma}{(1+\gamma)(m^2-1)\tau^{2m}} + \frac{2L(m^2+2m)\gamma}{(1+\gamma)\tau^{4m+2}} + \frac{K\gamma}{(1+\gamma)} \tag{28}$$

Also the expansion scalar (θ), shear scalar (σ), Hubble parameter (H) and the deceleration parameter (q) are derived as:

$$\theta = (2m+1)\left(\frac{1}{2(m-1)\tau^{4m-2}} + \frac{1}{(m^2-1)\tau^{2m}} + \frac{L}{\tau^{4m+2}}\right)^{1/2} \tag{29}$$

$$\sigma = \frac{(m-1)}{\sqrt{3}}\left(\frac{1}{2(m-1)\tau^{4m-2}} + \frac{1}{(m^2-1)\tau^{2m}} + \frac{L}{\tau^{4m+2}}\right)^{1/2} \tag{30}$$

$$H = \frac{(2m+1)}{3}\left(\frac{1}{2(m-1)\tau^{4m-2}} + \frac{1}{(m^2-1)\tau^{2m}} + \frac{L}{\tau^{4m+2}}\right)^{1/2} \tag{31}$$

$$q = -1 + \frac{3}{2m+1}\left\{\frac{\frac{(2m-1)}{2(m-1)\tau^{4m-2}} + \frac{m}{(m^2-1)\tau^{2m}} + \frac{(2m+1)L}{\tau^{4m+2}}}{\frac{1}{2(m-1)\tau^{4m-2}} + \frac{1}{(m^2-1)\tau^{2m}} + \frac{L}{\tau^{4m+2}}}\right\} \tag{32}$$

For this model the dominant energy condition $p + \rho \geq 0$ leads to

$$\frac{(2m^2 + m - 1)}{2(m-1)\tau^{4m-2}} + \frac{2m}{(m^2-1)\tau^{2m}} + \frac{2L(m^2+2m)}{\tau^{4m+2}} + K \geq 0 \tag{33}$$

The strong energy condition $p + 3\rho \geq 0$ gives

$$\frac{(1+3\gamma)}{(1+\gamma)}\left[\frac{(2m^2 + m - 1)}{2(m-1)\tau^{4m-2}} + \frac{2m}{(m^2-1)\tau^{2m}} + \frac{L(m^2+2m)}{\tau^{4m+2}} + K\right] \geq 0 \tag{34}$$

For this model we get

$$\omega = 0 \tag{35}$$

Here are some graphs showing variation of cosmological parameters with cosmic time:

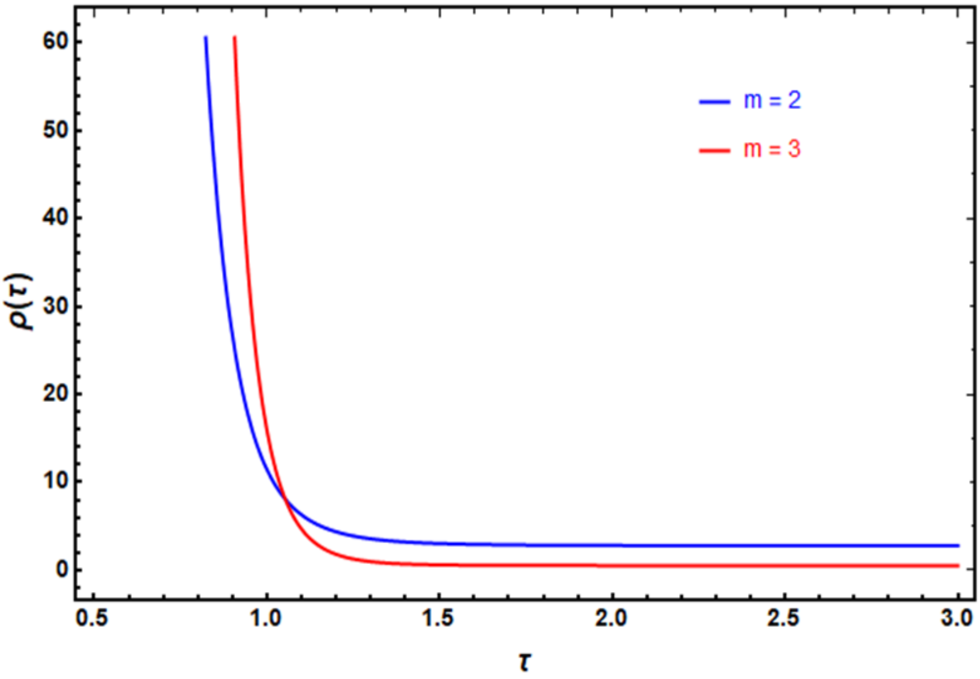

Figure 1: Variation of energy density ρ with cosmic time τ for $m = 2,3$.

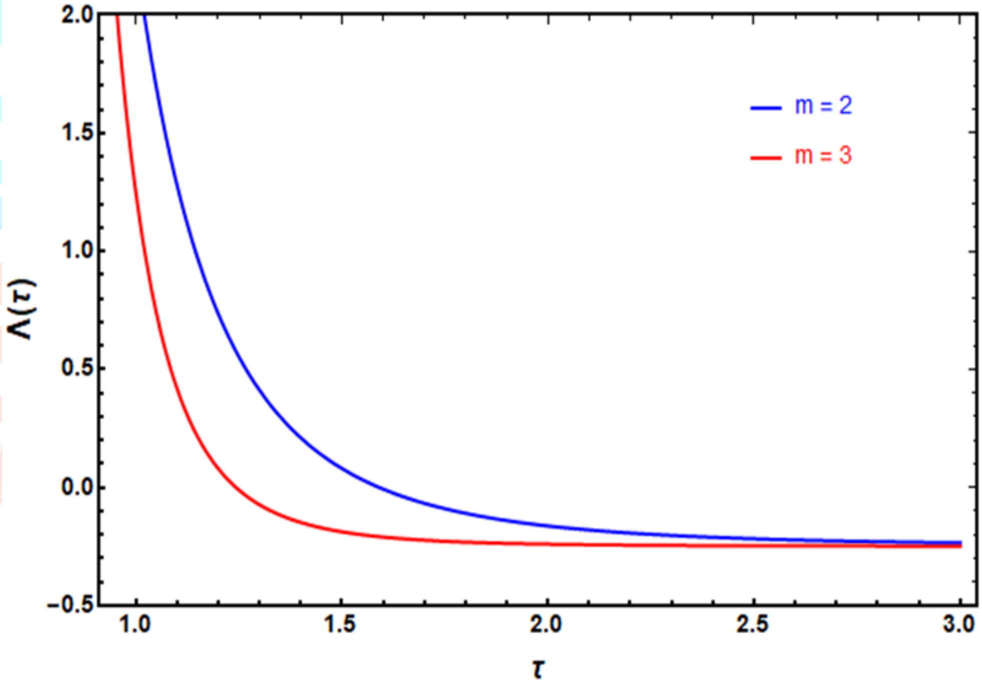

Figure 2: Variation of cosmological constant Λ with cosmic time τ for $m = 2,3$.

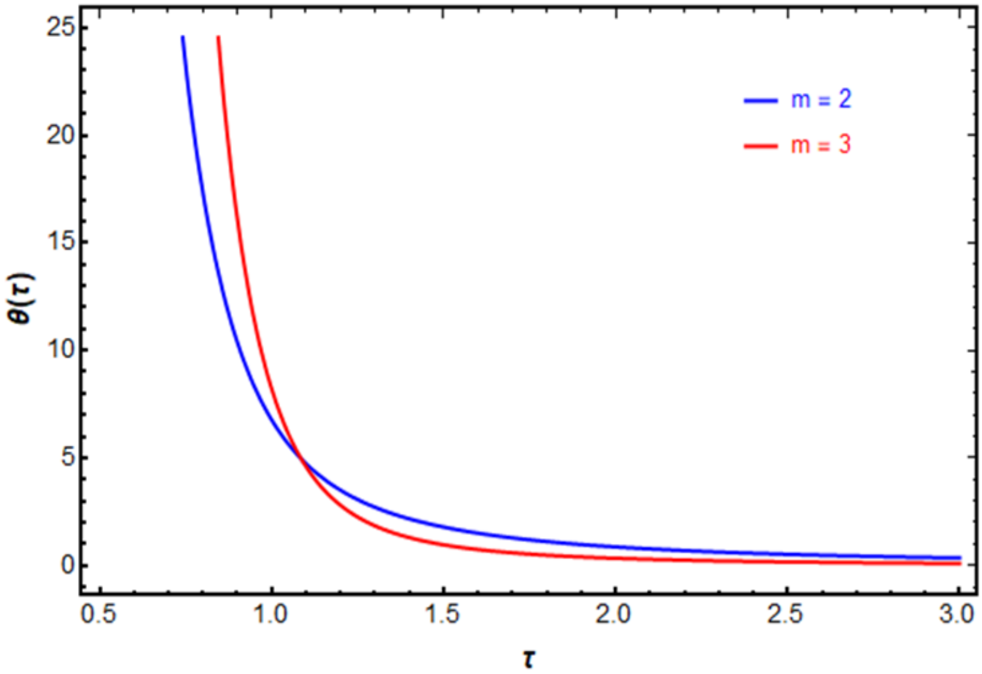

Figure 3: Variation of expansion scalar θ with cosmic time τ for $m = 2,3$.

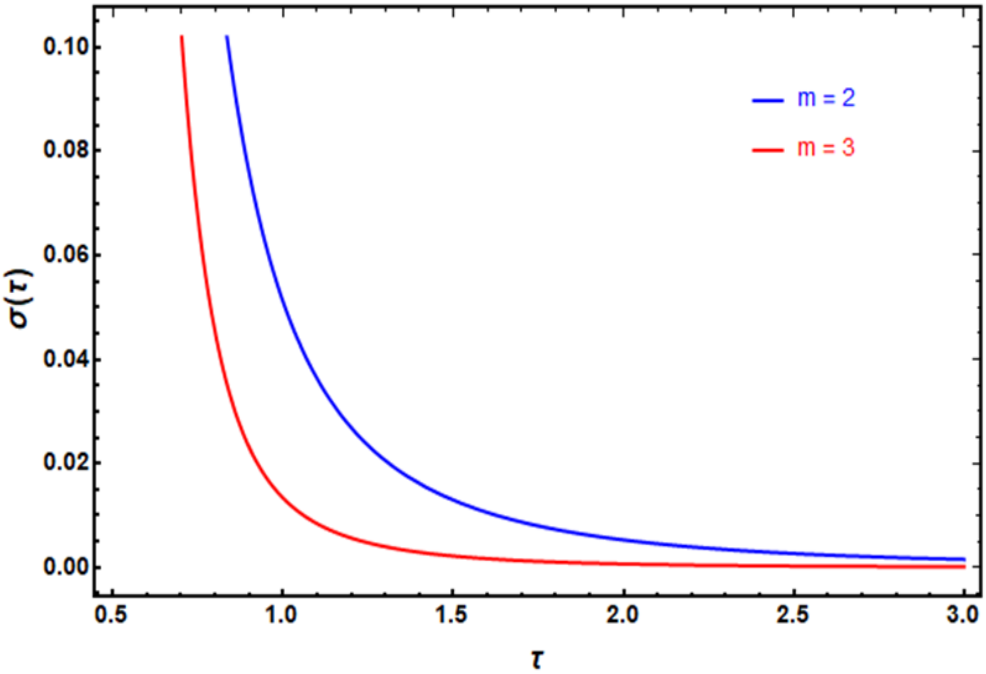

Figure 4: Variation of shear scalar σ with cosmic time τ for $m = 2,3$.

5. CONCLUSION

In this paper, Bianchi type VIII cosmological model with bulk viscosity is investigated. To determine dark energy candidate (Λ) a barotropic fluid condition is taken into consideration. Model (22) initiates its expansion at the big bang and the rate of expansion (θ) decelerates with time for $m > 1$. For anisotropy of model, we observe that $\frac{\sigma}{\theta} = \frac{(m-1)}{\sqrt{3}(2m+1)} \neq 0$. Hence we conclude that the model exhibits anisotropic behavior for $m \neq 1$ and isotropization occurs for $m \to 1$.

The cosmological parameters, including energy density (ρ), pressure (p), shear (σ) and Hubble parameter (H), decrease over time for $m > 1$ and approach zero as $\tau \to \infty$. The energy condition holds true for all values of τ. Furthermore, the deceleration parameter $q \to -1$ as $\tau \to \infty$ for $m > 1$. This indicates that the model is in accelerating phase of the universe. For $m > 1$, cosmological constant also diminishes with cosmic time. We observed a point-type singularity [11] for $m > 0$. Therefore, the present model represents an expanding, shearing, non-rotating and anisotropic universe in general.

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