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Solving Fuzzy Multiobjective Linear Fractional Programming Problem by Weighted Method

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ABSTRACT

The multiobjective linear fractional programming problem (MOLFPP) has been studied in this research utilizing a fuzzy set theoretic approach. This method transforms MOLFPP into a multiobjective linear programming problem (MOLPP) through an appropriate transformation. In algorithm , fuzzy set theory is used to convert MOLFPP into MOLPP, and Zimmermann's min-operator model and simplex approach are used to determine the Pareto optimum solution of the modified MOLPP. Additionally, we modified the aforesaid methodology using the weighted method. The weighted technique is used in Algorithm to determine the Pareto optimum solution of MOLFPP. One numerical problem is solved to determine the Pareto optimum solution by using this method in order to demonstrate the applicability of the defined methodology.

Keywords

Weighted method, fuzzy linear programming, fuzzy numbers, min operator model, and multiobjective linear fractional programming problem.

1. INTRODUCTION

Real-world issues including industrial planning, production planning, financial and corporate planning, healthcare, and hospital planning are modeled using fractional programming[1,3].
Linear fractional programming (LFP) problems is an important planning tool for the past decades which is applied to different disciplines like engineering, business, finance, economics, etc[17]. Fractional programming is the ratio criteria which are often used for modelling of real life problems with one or more objectives such as profit-cost, inventory-sales, actual cost-standard cost, output-employees, debt-equity, etc. In many practical applications like stock problems, ore blending problems, shipping schedules problems, optimal policy for a Markovian chains, sensitivity of linear programming problem, optimization of ratios of criteria gives more insight into the situations than the optimization of each criterion [3]. Multiobjective linear fractional programming is an area of multiple criteria decision making is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously where multiple objectives are written by fractional formulas[7,10,11,16]. MOLFP is an interesting topic which has been used in production planning, financial sector, inventory management, banking sector, etc

Lotfi et al. proposed an LP approach to test the strongly and weakly efficient solutions in the MOLFP problems by applying a simple geometrical interpretation[10,11]. Nahar and Alim [2015] suggested a statistical average approach where a single-objective function is developed from multi-objective functions to optimize the objective function, compared the proposed technique with some other techniques, such as arithmetic averaging and geometric averaging, and showed the effectiveness of the approach. Bhati et al. (2017) presented a review of the MOFP problems excluding various technical parts of fractional programming problems[2,3] . Numerical example to illustrate the theorem.

2.DEFINITION AND PRELIMINARIES

Definition I.

Linear Fractional Programming- The general format of linear fractional programming (LFP) may be written as:

$$\text{Max } \frac{cx+\alpha}{dx+\beta},$$

subject to the constraints:

$$x \in S = \{x \mid Ax=b,\ x\geq 0\} \qquad (1)$$

where $A \in R^{m\times n}$, $b \in R^m$, $x \in R^n$,$c,d \in R^n$, $\alpha,\beta \in R$ and S is a non-empty and bounded set. For some values of x, $dx + \beta$ may be equal to zero. For that we need to make an additional assumption that If

$$Ax = b, x \geq 0 \text{ then} \qquad dx+\beta > 0 \quad \text{or} \quad dx+\beta < 0$$

For convenience, assume that LFP (1) satisfies the condition:

$$x \geq 0 \qquad \text{then}$$

$$dx + \beta > 0 \qquad (2)$$

Definition II

The two mathematical programming problems

(i) Max A(x), subject to, $x \in \Delta$,

(ii) Max B(x), subject to, $x \in \Gamma$,

will be said to be equivalent iff there is a one-one mapping q(.) of the feasible set of (i), onto the feasible set of (ii), such that A(x)=B(q(x)) for all $x \in \Delta$.

Definition III.

Multiobjective Linear fractional Programming Problem:

The general format of a multiobjective linear fractional programming which is stated as follows :

$$\text{Max}Z(x)=\{Z_1(x),Z_2(x),\ldots, Z_p(x)\}$$

Subject to the constraints:

$$x\in\Delta=\{ \{x\in R^n : Ax \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x\ge 0\}$$

with $b\in R^n$, $A\in R^{mxn}$ and

$$Z_p(x)= \frac{c_p x+\alpha_p}{d_p x+\beta_p} = \frac{N_p(x)}{D_p(x)} \qquad (3)$$

Where, $c_p, d_p \in R^n$ and $\alpha_p, \beta_p \in R$

3. ALGORITHM

First of all compute the solution of MOLFPP. Algorithm for finding optimal solution of MOLFPP can be summarized in a series of step by step which are as follows:

Step 1: First convert each objective function of MOLFPP to MOLPP by using transformation y=tx(t>0).

Step 2: Convert each constraint by substituting y=tx.

Step 3: The said problem can be reduced by using step 2.

Step 4: Finding the membership function .

Step 5: X_{n+1} is an optimal solution to the problem (P) and Max. Z (X) = Z(Xn+1)

Step 6: Apply Zimmermann's min operator [21] to transform the fuzzy model(10) can be transformed to the crisp one as follows: Maxλ

Step 7: By using standard LPP to find the values of $y_1, y_2, \ldots, y_p, t, \lambda$.

Step 8: Changing the above values $y_1, y_2, \ldots, y_p$, t, λ by using the transformation y=tx and then substitute the values in the given equation.

Step 9: Applying the values of $x_1, x_2, \ldots, x_p$ compute the values of above constraints.

Step 10: Process repeat till the result come.

4. METHODOLOGY

Weighted Method to Solve Multiobjective Fuzzy Linear Fractional Programming Problem Chenet.al[12,13] have used weighted average method in" Fuzzy goal programming with different importance and properties".Tiwari, Dharmar and Rao[19] have mentioned an additive model in fuzzy goal programming which incorporates each goal's weight W_k into the corresponding objective function

i.e. $\sum_{k=1}^{n} W_k Z_k$

where Z_k denotes the kth fuzzy goal and

$$\sum_{k=1}^{n} W_k = 1$$

In the additive model weights show the relative importance of the goals. Now for simplicity, the importance of these objectives (goals)are taken to be different. Hence all objective function can be reformulated as a single objective function without adding any more constraints.

$$\text{Max } Z=W_1Z_1+W_2 Z_2..........+W_pZ_p \qquad (4)$$

Such that under given constraints

where ,

$$\sum_{k=1}^{p} W_k = 1$$

Membership function using product operator

$$\mu_{f_p}[f_p(\bar{x})] = \prod_{p=1}^{P} \mu_p(\bar{x})$$

Where $\mu_p(\bar{x})$ is the membership function of the p^{th} objective function of a VMP and is given as:

$$\mu_p(\bar{x}) = \begin{cases} 0 & , \quad f_p(\bar{x}) < f_p^L \\ \dfrac{f_p(\bar{x}) - f_p^L}{f_p^U - f_p^L} & , \quad f_p^L \le f_p(\bar{x}) \le f_p^U \\ 1 & , \quad f_p(\bar{x}) > f_p^U \end{cases} \qquad (5)$$

Where f_p^U and f_p^L are the upper and lower bounds on $f_p(\bar{x})$.

5.NUMERICAL EXAMPLE

Let us consider the numerical example studied by Luhandjula[14] is apply to illustrate the above approach:

Consider a MOLFPP with two objective functions as follows:

$$\text{Max } Z(x)=(Z_1(x) = \frac{x_1+4}{x_2-3}, Z_2(x) = \frac{x_1-3}{x_2+2}$$

subject to the constraints:

$x_1 - 4x_2 \ge 0$,

$X_1 \le 7$,

where $x_1, x_2 \ge 0$.

Solution:

Here, $Z_1(x) > 0$ for some x in the feasible region and $Z_2(x) \leq 0$,for each x in the feasible region.

The above MOLFPP is equivalent to the following MOLPP .

Max $\{f_1(y,t) = -y_1 + 4t,\ f_2(y,t) = -y_1 - 4t\}$

subject to the constraints:

$y_1 + t \leq 1,$

$-y_2 + 4t \leq 1,$

$-y_1 + 4y_2 \leq 0$

$-y_1 + 7t \leq 0,$

$y_2 - 3t = 1,$

$y_1, y_2, t \geq 0.$

On solving the problem we get, $f_1(y,t) = 0.88$ and $f_2(y,t) = 0.89$. Applying the membership functions defined in (05) , the above multiobjective linear programming problem reduces to the crisp model as follows:

Max λ

subject to the constraints:

$-y_1 + 3t - \lambda \geq 0,$

$-1.01y_1 + 4.04t - \lambda \geq 0,$

$-y_2 + t \leq 1,\ -y_1 + 4t \leq 1,$

$-y_1 + 3y_2 \leq 0,$

$y_1 - 6t \leq 0,$

$-y_2 + 3t = 1,$

$y_1, y_2, t, \lambda \geq 0.$

After solving the above problem by LP package, we get, $y_1 = 0.43$, $y_2 = 0$, $t = 0.42$ and $\lambda = 0.58$.

the Pareto optimal solution of the given problem is : $x_1 = .9, x_2 = 1$,$Z_1(x) = -2.3$, $Z_2(x) = 4.2$

6. CONCLUSIONS

In this paper, the Pareto optimum solution of MOLFPP has been studied. To illustrate our method, a algorithms have been suggested. Algorithm describes how to convert a MOLFPP into a MOLPP using Zimmermann's min-operator model and an appropriate transformation. Later on, a standard LP packaging was used to solve it. In the algorithm steps are followed one by one. After transforming MOLPP into a single objective linear programming problem using the weighted approach and membership function, the standard LP package was used to solve the problem. The suggested method for addressing MOLFPP produces an effective solution that is simple to compute and lowers the complexity of issue solving.

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