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SOFT GENERALIZED CLOSED MAPPINGS AND HOMEOMORPHISM IN SOFT GRILL TOPOLOGICAL SPACES

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Abstract: In this article we introduce, in terms of with soft grill $\boldsymbol{\zeta_{\mathcal{S}}}$, the concepts soft $\boldsymbol{\zeta_{\mathcal{S}} - \mathcal{G}}$ closed map, $\boldsymbol{\zeta_{\mathcal{S}} - \mathcal{G}}$ open map and $\boldsymbol{\zeta_{\mathcal{S}} - \mathcal{G}}$ homeomorphism in soft grill topological spaces (X, $\boldsymbol{\tau_{\mathcal{S}}}, \boldsymbol{\zeta_{\mathcal{S}}}$, A). Several related results and properties of these concepts are investigated.

Keywords: Soft sets, soft homeomorphism, $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set, $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map, $\zeta_{\mathcal{S}} - \mathcal{G}$ open map, $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set homeomorphism

I. INTRODUCTION

The concept of soft grill topological spaces was first introduced by Rodyna A, et-al[8]. The concept of soft grill topological spaces was first introduced by Rodyna A, et-al[9]. We introduced [1] soft generalized closed set in soft grill topological spaces. In this paper we are going to introduce a $\zeta_{\mathcal{S}} - \mathcal{G}$ Homeomorphism. Further we are going to study their properties in detail.

II. PRELIMINARIES

2.1 Definition[10]

Let $\mathcal{X}$ be an initial universe set and $\mathcal{A}$ be a set of parameters. Let P($\mathcal{X}$) denote the power set of $\mathcal{X}$ and ß be a non empty subset of $\mathcal{A}$. A pair $(\mathcal{F}, ß)$ is denoted by $\mathcal{F}_{ß}$ is said to be soft set over $\mathcal{X}$, where $\mathcal{F}_{ß}$ is mapping given by $\mathcal{F}: ß \to P(\mathcal{X})$. In other words, a soft set over $\mathcal{X}$ is a parameterized family of subsets of the universe $\mathcal{X}$.

i.e., $\mathcal{F}_{ß}$={ $\mathcal{F}$(a): a $\in$ ß $\subseteq$ $\mathcal{A}$, $\mathcal{F}(a) = \emptyset$ if a $\notin$ ß}. If SS $(\mathcal{X}, \mathcal{A})$ denote is the family of all soft subsets over $\mathcal{X}$.

2.2 Definition[11]

Let $\tau_{\mathcal{S}}$ be the collection of soft sets over $\mathcal{X}$, then $\tau_{\mathcal{S}}$ is said to be a soft topology on $\mathcal{X}$. If the following axioms:

(i) $\widetilde{\emptyset_{\mathcal{A}}}, \widetilde{\mathcal{X}_{\mathcal{A}}}$ belong to $\tau_{\mathcal{S}}$
(ii) The union of any number of soft sets in $\tau_{\mathcal{S}}$ belongs to $\tau_{\mathcal{S}}$.
(iii) The intersection of any two number of soft sets in $\tau_{\mathcal{S}}$ belongs to $\tau_{\mathcal{S}}$.

The triplet $(\mathcal{X}, \tau_{\mathcal{S}}, \mathcal{A})$) is said to be a soft topological space or soft space.

2.3 Definition[13]

A non empty collection $\mathcal{G} \subseteq SS(X, A)$ of soft sets over X is called a soft grill, if the following conditions hold:

(i) If $\mathcal{F}_A \in \mathcal{G}$ and $\mathcal{F}_A \subseteq \mathcal{H}_A$, which implies $\mathcal{H}_ß \in \mathcal{G}$.

(ii) If $\mathcal{F}_A \subseteq \mathcal{H}_A \in \mathcal{G}$, which implies $\mathcal{F}_A \in \mathcal{G}$ or $\mathcal{H}_A \in \mathcal{G}$.

The quadruplet (X, τ, A, $\mathcal{G}$) is said to be soft grill topological space.

2.4 Definition[3]

Let $\zeta_{\mathcal{S}}$ be a soft grill over a soft topological space ($\mathcal{X}$, $\tau_{\mathcal{S}}$, $\mathcal{A}$). A soft set $\mathcal{F}_ß$ is called $\zeta_{\mathcal{S}}$ generalized closed set (briefly $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set), if $\chi_\zeta(\mathcal{F}_ß) \subseteq \mathcal{U}_{\mathcal{A}}$, whenever $\mathcal{F}_ß \subseteq \mathcal{U}_{\mathcal{A}}$ and $\mathcal{U}_{\mathcal{A}}$ is soft open in ($\mathcal{X}$, $\tau_{\mathcal{S}}$, $\mathcal{A}$). The complement of such set will be called $\zeta_{\mathcal{S}} - \mathcal{G}$ open set (resp. $\zeta_{\mathcal{S}}$- $\mathcal{G}$ open set).

2.5 Definition[4]

A function δ_{pu}: (X, $\tau_{\mathcal{S}}$, $\zeta_{\mathcal{S}}$, A)→ (Y, $\sigma_{\mathcal{S}}$, B) is called soft generalized continuous functions in soft grill topological spaces (shortly $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous) if for soft closed set L_B of (Y, $\sigma_{\mathcal{S}}$, B), $\delta_{pu}^{-1}((L_B) \in \zeta_{\mathcal{S}} - \mathcal{G}C(X, \tau_{\mathcal{S}}, \zeta_{\mathcal{S}}, A)$.

2.6 Definition[6]

A bijection f : (X,τ,E) → (Y,σ,K) is called Soft homeomorphism if f is both Soft continuous and Soft open map.

2.7 Definition[7]

A bijection f : (X,τ,E) → (Y,σ,K) is called Soft g homeomorphism if f is both Soft g continuous and Soft g open map.

III. SOFT GENERALIZED CLOSED AND OPEN MAPPINGS IN SOFT GRILL TOPOLOGY

In this section, we define a novel category of $\zeta_{\mathcal{S}} - \mathcal{G}$ closed and open mapping within the framework of soft grills as follows:

3.1 Definition

A mapping δ_{pu}: (X, $\tau_{\mathcal{S}}$, A)→ (Y, $\sigma_{\mathcal{S}}$, $\zeta_{\mathcal{S}}$, B) is said to be $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map (res. $\zeta_{\mathcal{S}} - \mathcal{G}$ open map) if the image of every soft closed set (res. soft open) in X is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set (res. $\zeta_{\mathcal{S}} - \mathcal{G}$ open set) in Y.

3.2 Example

Let X $=\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$=Y and $\mathcal{A}$=B $=\{\alpha_1, \alpha_2\}$, $\tau_{\mathcal{S}} = \{\widetilde{\emptyset}, \widetilde{X}, K_1, K_2, K_3\}$, $\zeta_{\mathcal{S}}{}^1$=$\{K_4, K_5, K_6, \widetilde{X}\}$,$\sigma_{\mathcal{S}} = \{\widetilde{\emptyset}, \widetilde{X}, S_1, S_2, S_3\}$ and $\zeta_{\mathcal{S}}{}^2 = \{S_5, S_6, S_7, \widetilde{X}\}$ where K_1, K_2, K_3, K_4, K_5, K_6, $S_1, S_2, S_3, S_4, S_5, S_6, S_7$ are soft subsets over $X_{\mathcal{A}}$, we get the following
$K_1 = \{\{\upsilon_1, \upsilon_2\}, \{\upsilon_3, \upsilon_4\}\}$, K_2=$\{\{\upsilon_3\}, \{\upsilon_1\}\}$, $K_3 = \{\{\upsilon_1, \upsilon_2, \upsilon_3\}, \{\upsilon_1, \upsilon_3, \upsilon_4\}\}$,
$K_4 = \{\{\upsilon_1\}, \{\upsilon_4\}\}$, $K_5 = \{\{\upsilon_1, \upsilon_2, \upsilon_3\}, \{\upsilon_1, \upsilon_2, \upsilon_4\}\}$, $K_6 = \{\{\upsilon_1, \upsilon_3\}, \{\upsilon_2, \upsilon_4\}\}$,
$S_1 = \{\{\upsilon_1, \upsilon_2\}, \{\upsilon_2, \upsilon_3\}$, $S_2 = \{\{\upsilon_2, \upsilon_3\}, \{\upsilon_3, \upsilon_4\}\}$, $S_3 = \{\{\upsilon_1, \upsilon_2, \upsilon_3\}, \{\upsilon_2, \upsilon_3, \upsilon_4\}\}$,
$S_4 = \{\{\upsilon_1\}, \{\upsilon_1, \upsilon_2, \upsilon_3\}\}$, $S_5 = \{\{\upsilon_2\}, X\}$, $S_6 = \{\{\upsilon_1, \upsilon_2\}, X\}$,$\delta_{pu}(K_2) = S_2$, $\delta_{pu}(K_1) = S_1$, $\delta_{pu}(K_3) = S_3$. Hence δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set in Y.

3.3 Theorem

Every soft closed map is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

Proof

Let δ_{pu}: (X, $\tau_{\mathcal{S}}$, A)→ (Y, $\sigma_{\mathcal{S}}$, $\zeta_{\mathcal{S}}$, B) is a closed map and ${F_A}^C$ be a soft closed set in X. Then $\delta_{pu}({F_A}^C)$ is soft closed in Y. Since every soft closed set is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set. So $\delta_{pu}({F_A}^C)$ is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set in Y. Hence δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

3.4 Theorem

Every soft open map is $\zeta_{\mathcal{S}} - \mathcal{G}$ open map.

Proof

Let δ_{pu}: (X, $\tau_{\mathcal{S}}$, A)→ (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}$, B) is a open map and F_A be an soft open set in X. Then $\delta_{pu}(F_A)$ is soft open in Y. So $\delta_{pu}(F_A)$ is $\zeta_{\mathcal{S}} - \mathcal{G}$ open set in Y. Hence δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ open map.

3.5 Theorem

A map δ_{pu}: (X $\tau_{\mathcal{S}}$, A)→ (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}$, B) in $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map if and only if soft subset E_B of Y and for each soft open set in U_A containing $\delta_{pu}^{-1}(E_B)$ there is a $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set F_B of Y such that $E_B \subseteq F_B$ and $\delta_{pu}^{-1}(F_B) \subseteq U_A$.

Proof

Suppose δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map. Let soft subset E_B of Y and U_A be an soft open set of X such that $\delta_{pu}^{-1}(E_B) \subseteq U_A$ then $F_B = Y - \delta_{pu}(X - U_A)$ is a $\zeta_{\mathcal{S}} - \mathcal{G}$ open set containing E_B such that $\delta_{pu}^{-1}(F_B) \subseteq U_A$.

Conversely, suppose that E_A is soft closed of X. Then $\delta_{pu}^{-1}\left(Y - \delta_{pu}(E_A)\right) \subseteq X - E_A$ and $X - E_A$ is soft open. By hypothesis, there is a $\zeta_{\mathcal{S}} - \mathcal{G}$ open set F_B of Y such that Y$-\delta_{pu}(E_A) \subseteq F_B$ and $\delta_{pu}^{-1}(F_B) \subseteq X - E_A$.Therefore, $E_A \subseteq X - \delta_{pu}^{-1}(F_B)$. Hence Y - $F_B \subseteq \delta_{pu}(E_A) \subseteq \delta_{pu}\left(X - \delta_{pu}^{-1}(F_B)\right) \subseteq Y - F_B$ which implies $\delta_{pu}(E_A) = Y - F_B$. Since $Y - F_B$ is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set, $\delta_{pu}(E_A)$ is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed and thus δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

3.6 Theorem

If a map δ_{pu} : X → Y is soft continuous and $\zeta_{\mathcal{S}} - \mathcal{G}$ closed and E_A is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set of X, δ_{pu} (E_A) is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed in Y.

Proof

Let δ_{pu} (E_A) $\subseteq U_A$, when U_A is a soft open set of Y. Since δ_{pu} is continuous $\delta_{pu}^{-1}(U_A)$ is a soft open set containing E_A. Hence $\chi_\zeta(E_A) \subseteq \delta_{pu}^{-1}(U_A)$ as E_A is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set. Since δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed. δ_{pu} $(\chi_\zeta(E_A))$ is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed set contained in the soft open set U_A, which implies χ_ζ $(\delta_{pu}$ $(cl$ $(E_A)) \subseteq U_A$ and hence χ_ζ $(\delta_{pu}$ $(E_A)) \subseteq U_A$. So, δ_{pu} (E_A) is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed in Y.

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IV. SOFT GENERALIZED HOMEOMORPHISM IN SOFT GRILL TOPOLOGY

In this section we introduce and study a new homeomorphism known as called soft generalized homeomorphism in soft grill topological spaces and we workout some basic theorem.

4.1 Definition

A bijective mapping δ_{pu}: (X, $\tau_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^1$, A)→ (Y, $\sigma_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^2$ B) is called soft generalized homeomorphism in soft grill topological spaces (shortly $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism) if δ_{pu} is both $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous and $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

4.2 Example

Let $X = \{a_1, a_2, a_3\}$, $Y = \{b_1, b_2, b_3\}$ and $\mathcal{A} = B = \{\alpha_1, \alpha_2\}$,
$\tau_{\mathcal{S}} = \{\tilde{\emptyset}, \tilde{X}, K_1, K_2, K_3, K_4, K_5\}$, ${\zeta_{\mathcal{S}}}^1 = \{K_3, K_4, K_5, \tilde{X}\}$, $\sigma_{\mathcal{S}} = \{\tilde{\emptyset}, \tilde{Y}, S_1, S_2, S_3, S_4, S_5\}$ and
${\zeta_{\mathcal{S}}}^2 = \{S_3, S_4, S_5, \tilde{X}\}$ where $K_1, K_2, K_3, K_4, K_5, K_6, S_1, S_2, S_3, S_4, S_5$, are soft subsets over $X_{\mathcal{A}}$, we get the following : $K_1 = \{\{a_1\}, \{a_2\}\}$, $K_2 = \{\{a_2\}, \{a_3\}\}$, $K_3 = \{\{a_1, a_2\}, X\}$,
$K_4 = \{\{a_1, a_2\}, \{a_2, a_3\}\}$, $K_5 = \{X, \{a_1, a_2\}\}$, $S_1 = \{\{b_2, b_3\}, Y\}$, $S_2 = \{\{b_2\}, \{b_3\}\}$,
$S_3 = \{\{b_3\}, \{b_1\}\}$, $S_4 = \{\{b_2, b_3\}, \{b_1, b_3\}\}$, $S_5 = \{Y, \{b_1, b_3\}\}$.
let δ_{pu}: (X, $\tau_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^1$, A)→ (Y, $\sigma_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^2$ B) is a $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism.

4.3 Theorem

If a bijective mapping δ_{pu}: (X, $\tau_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^1$, A)→ (Y, $\sigma_{\mathcal{S}}, {\zeta_{\mathcal{S}}}^2$ B). Then δ_{pu} is a $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism if and only if δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map and $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous.

Proof

Let δ_{pu}: (X, $\tau_S, \zeta_S{}^1$, A)→ (Y, $\sigma_S, \zeta_S{}^2$ B) is a soft $\zeta_S - \mathcal{G}$ homeomorphism, then both δ_{pu} and $\delta_{pu}{}^{-1}$ soft $\zeta_S - \mathcal{G}$ continuous. For any soft $\zeta_S - \mathcal{G}$ closed set $F_A \subseteq X_A$, $\delta_{pu}(F_A) = (\delta_{pu}{}^{-1})^{-1}(F_A)$ soft $\zeta_S - \mathcal{G}$ closed in (Y, $\sigma_S, \zeta_S{}^2$ B). Hence δ_{pu} is a soft $\zeta_S - \mathcal{G}$ closed map.

Conversely, if δ_{pu} is $\zeta_S - \mathcal{G}$ closed map and $\zeta_S - \mathcal{G}$ continuous, then any soft $\zeta_S - \mathcal{G}$ closed $F_A \subseteq X_A$, $(\delta_{pu}{}^{-1})^{-1}(F_A) = \delta_{pu}(F_A)$ is a soft $\zeta_S - \mathcal{G}$ closed in (Y, $\sigma_S, \zeta_S{}^2$ B), so $\delta_{pu}{}^{-1}$ is a soft $\zeta_S - \mathcal{G}$ continuous. Thus δ_{pu} is a soft $\zeta_S - \mathcal{G}$ homeomorphism.

4.4 Theorem

If a bijective mapping δ_{pu}: (X, $\tau_S, \zeta_S{}^1$, A)→ (Y, $\sigma_S, \zeta_S{}^2$ B). Then the following are equivalent.

1) The inverse mapping $\delta_{pu}{}^{-1}$: (X, $\tau_S, \zeta_S{}^1$, A)→ (Y, $\sigma_S, \zeta_S{}^2$ B) is $\zeta_S - \mathcal{G}$ continuous map.
2) δ_{pu} is $\zeta_S - \mathcal{G}$ open map.
3) δ_{pu} is $\zeta_S - \mathcal{G}$ closed map.

Proof

(1)→ (2): Let F_A be a soft open in (X, $\tau_S, \zeta_S{}^1$, A). So $\left(\delta_{pu}{}^{-1}\right)^{-1}(F_A) = \delta_{pu}(F_A)$ is soft open in (Y, $\sigma_S, \zeta_S{}^2$ B). Therefore $\delta_{pu}(F_A)$ is $\zeta_S - \mathcal{G}$ open map (Y, $\sigma_S, \zeta_S{}^2$ B).

(2)→(3): Let E_A be a soft $\zeta_S - \mathcal{G}$ closed set in (X, $\tau_S, \zeta_S{}^1$, A).Then $(E_A)^c$is soft $\zeta_S - \mathcal{G}$ open set in (X, $\tau_S, \zeta_S{}^1$, A). By assumption $\delta_{pu}((E_A)^c)$ is $\zeta_S - \mathcal{G}$ open map in (Y, $\sigma_S, \zeta_S{}^2$ B). $\delta_{pu}((E_A)^c) = (\delta_{pu}(E_A))^c$ is $\zeta_S - \mathcal{G}$ open map in (Y, $\sigma_S, \zeta_S{}^2$ B). $\delta_{pu}(E_A)$ is $\zeta_S - \mathcal{G}$ closed map(Y, $\sigma_S, \zeta_S{}^2$ B).

(3)→(1): Let E_A be a soft $\zeta_S - \mathcal{G}$ closed set in (X, $\tau_S, \zeta_S{}^1$, A). $\delta_{pu}(E_A)$ is $\zeta_S - \mathcal{G}$ closed map in (Y, $\sigma_S, \zeta_S{}^2$,B). $\delta_{pu}(E_A) = \left(\delta_{pu}{}^{-1}\right)^{-1}(E_A)$ is $\zeta_S - \mathcal{G}$ is closed in Y. Hence $\delta_{pu}{}^{-1}$ is $\zeta_S - \mathcal{G}$ continuous map.

4.5 Theorem

If a bijective mapping δ_{pu}: (X, $\tau_S, \zeta_S{}^1$, A)→ (Y, $\sigma_S, \zeta_S{}^2$ B) be a soft $\zeta_S - \mathcal{G}$ homeomorphism, then $\delta_{pu}{}^{-1}$: (Y, $\sigma_S, \zeta_S{}^2$ B) → (X, $\tau_S, \zeta_S{}^1$, A) is also a soft $\zeta_S - \mathcal{G}$ homeomorphism.

Proof

By definition of a soft $\zeta_S - \mathcal{G}$ homeomorphism is bijective δ_{pu} with both δ_{pu} and $\delta_{pu}{}^{-1}$ are soft $\zeta_S - \mathcal{G}$ continuous. Let $\delta_{pu}{}^{-1}$ is soft $\zeta_S - \mathcal{G}$ continuous, and the inverse of δ_{pu} and $\delta_{pu}{}^{-1}$, which soft $\zeta_S - \mathcal{G}$ continuous by hypothesis. Hence $\delta_{pu}{}^{-1}$ satisfies the Definition 4.1 and it is a soft $\zeta_S - \mathcal{G}$ homeomorphism.

By above example 4.2 is satisfied to Theorem 4.5.

4.6 Theorem

If a bijective mapping δ_{pu}: (X, $\tau_S, \zeta_S{}^1$, A)→ (Y, $\sigma_S, \zeta_S{}^2$ B) be a soft $\zeta_S - \mathcal{G}$ homeomorphism if and only if δ_{pu} and $\delta_{pu}{}^{-1}$ each map soft $\zeta_S - \mathcal{G}$ open sets to soft $\zeta_S - \mathcal{G}$ open sets.

Proof

Let δ_{pu} is a soft $\zeta_S - \mathcal{G}$ homeomorphism, then δ_{pu} and $\delta_{pu}{}^{-1}$ are soft $\zeta_S - \mathcal{G}$ continuous. For any soft $\zeta_S - \mathcal{G}$ open $U_A \subseteq X_A$, $U_A = X_A - E_A$ for some soft $\zeta_S - \mathcal{G}$ closed set in (X, $\tau_S, \zeta_S{}^1$, A), then $\delta_{pu}(U_A) = Y_B - \delta_{pu}(E_A)$. Because δ_{pu} (or $\delta_{pu}{}^{-1}$) is soft $\zeta_S - \mathcal{G}$ continuous, so $\delta_{pu}(U_A)$ is a soft $\zeta_S - \mathcal{G}$ open set.

Conversely, if δ_{pu} and $\delta_{pu}{}^{-1}$ each map soft $\zeta_S - \mathcal{G}$ open sets to soft $\zeta_S - \mathcal{G}$ open sets. Then for any soft $\zeta_S - \mathcal{G}$ closed set $D_B \subseteq Y_B$, $\delta_{pu}{}^{-1}(D_B) = X_A - \delta_{pu}{}^{-1}(Y_B - D_B)$ is a complement of the image under $\delta_{pu}{}^{-1}$ of a soft $\zeta_S - \mathcal{G}$ open set, by hypothesis that is soft $\zeta_S - \mathcal{G}$ open set, so $\delta_{pu}{}^{-1}(D_B)$ is a soft $\zeta_S - \mathcal{G}$ closed. Hence $\delta_{pu}{}^{-1}$ is a soft $\zeta_S - \mathcal{G}$ continuous. Similarly δ_{pu} is soft $\zeta_S - \mathcal{G}$ continuous. Therefore δ_{pu} is soft $\zeta_S - \mathcal{G}$ is homeomorphism.

4.7 Theorem

Every soft homeomorphism is soft $\zeta_S - \mathcal{G}$ homeomorphism.

Proof

Let δ_{pu} be a soft homeomorphism. Then δ_{pu} and $\delta_{pu}{}^{-1}$ are soft continuous and δ_{pu} is bijective. Since every soft continuous function is $\zeta_S - \mathcal{G}$ continuous, we have δ_{pu} and $\delta_{pu}{}^{-1}$ are $\zeta_S - \mathcal{G}$ continuous. Therefore δ_{pu} is $\zeta_S - \mathcal{G}$ homeomorphism.

4.8 Remark

The converse of the above theorem is not true as seen from the following example.

4.9 Example

Let X={1,2,3,4}=Y, A=B={m,n}. Let H_1, H_2, H_3 ,H_4, H_5, H_6 functions defined from A to P(X) as follows, $H_1 = \{\{3\},\{1\}\}$, $H_2 = \{\{4\},\{2\}\}$, $H_3 = \{\{3,4\},\{1,2\}\}$, $H_4 = \{\{1,4\},\{2,4\}\}$, $H_5 = \{\{2,3,4\},\{1,2,3\}\}$, $H_6 = \{\{1,3,4\},\{1,2,4\}\}$.

Then $\tau_{\mathcal{S}} = \{\emptyset_{\mathcal{S}}, X_{\mathcal{S}}, H_1, H_2, H_3, H_4\}$ and $\zeta_{\mathcal{S}}{}^1 = \{X_{\mathcal{S}}, H_3, H_4, H_5, H_6\}$ is soft grill topology on X. Let I_1, I_2, I_3 , I_4 functions defined from B to P(Y) as follows, $I_1 = \{\{a\},\{d\}\}$, $I_2 = \{\{a\},\{c\}\}$, I_3={{a,b},{c,d}},$I_4 = \{\{b,c,d\},\{a,b,c\}\}$.

Then $\sigma_{\mathcal{S}} = \{\emptyset_{\mathcal{S}}, X_{\mathcal{S}}, I_1, I_2, I_3, I_4\}$ and $\zeta_{\mathcal{S}}{}^2 = \zeta_{\mathcal{S}}{}^1$is soft grill topology on Y. Let δ_{pu} is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism but not soft homeomorphism.

4.10 Theorem

Every soft generalized homeomorphism is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism.

Proof

Let δ_{pu} be a soft $\mathcal{G}$ homeomorphism. Then δ_{pu} and $\delta_{pu}{}^{-1}$ are soft $\zeta_{\mathcal{S}}$ continuous and δ_{pu} is bijective. Since every soft generalized continuous mapping is $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous, we have δ_{pu} and $\delta_{pu}{}^{-1}$ are $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous. Therefore δ_{pu} is $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism.

4.11 Remark

The converse of the above theorem is not true as seen from the following example.

4.12 Example

Let X= Y ={$\alpha_1, \alpha_2, \alpha_3$}, A=B={$a_1, a_2$}, $\tau_{\mathcal{S}} = \{\widetilde{\emptyset}, \widetilde{\mathcal{X}}, K_1, K_2, K_3, K_4, K_5, K_6, K_7\}$,
$\zeta_{\mathcal{S}}{}^1 = \zeta_{\mathcal{S}}{}^2 = \tau_{\mathcal{S}} - \widetilde{\emptyset}$, $\sigma_{\mathcal{S}} = \{\widetilde{\emptyset}, \widetilde{Y}, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$ and
where $K_1, K_2, K_3, K_4, K_5, K_6, K_7, S_1, S_2, S_3, S_4, S_5, S_6, S_7$are soft subsets over $\mathcal{X}_{\mathcal{A}}$, we get the following: $K_1 = \{\{\alpha_1\},\{\alpha_2\}\}$, K_2=$\{\{\alpha_1\},\{\alpha_2,\alpha_3\}\}$, $K_3 = \{\{\alpha_1\},\{\alpha_1,\alpha_2\}\}$,
$K_4 = \{\{\alpha_1,\alpha_2\},\{\alpha_3\}\}$, $K_5 = \{\{\alpha_1,\alpha_2\},\{\alpha_1,\alpha_2\}\}$, $K_6 = \{\{\alpha_2\},\{\alpha_2,\alpha_3\}\}$,
$K_7 = \{\{\alpha_2,\alpha_3\},\{\alpha_2,\alpha_3\}\}$, $S_1 = \{\{\alpha_2\},\{\alpha_3\}\}$, $S_2 = \{\{\alpha_2\},\{\alpha_1,\alpha_3\}\}$, $S_3 = \{\{\alpha_3\},\{\alpha_1,\alpha_2\}\}$,
$S_4 = \{\{\alpha_2,\alpha_3\},\{\alpha_3\}\}$, $S_5 = \{\{\alpha_2\},\{\alpha_2,\alpha_3\}\}$, $S_6 = \{\{\alpha_2\},\{\alpha_1,\}\}$, $S_7 = \{\{\alpha_2,\alpha_3\},\{\alpha_2,\alpha_3\}\}$.
So δ_{pu} is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism, but notsoft generalized homeomorphism.

4.13 Theorem

Let δ_{pu}: (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A)→ (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B) be a soft bijective and soft $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous, the following statement are equivalent.

1) δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open map.
2) δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism.
3) δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

Proof

(1)→ (2): Let δ_{pu} is soft bijective and soft $\zeta_{\mathcal{S}} - \mathcal{G}$ continuous and $\zeta_{\mathcal{S}} - \mathcal{G}$ open map, By definition 4.1, δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism.

(2) →(3): Let δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ homeomorphism and soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open map. Let F_A be a soft $\zeta_{\mathcal{S}}$ closed in (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A).Then $F_A{}^C$ is a soft $\zeta_{\mathcal{S}}$ open set in (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A).By assumptions $\delta_{pu}(F_A{}^C)$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open in (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B). That is $\delta_{pu}(F_A{}^C) = (\delta_{pu}(F_A))^C$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open set in (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B) and $\delta_{pu}(F_A)$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed in (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B). Hence δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed map.

(3) → (1):): Let E_A be a soft $\zeta_{\mathcal{S}}$ open in (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A). Then $E_A{}^C$ be a soft $\zeta_{\mathcal{S}}$ closed in (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A). By the given hypothesis, $\delta_{pu}(E_A{}^C)$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ closed in (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B). Now $\delta_{pu}(E_A{}^C) = (\delta_{pu}(E_A))^C$ is soft $\zeta_{\mathcal{S}} - \mathcal{G}$, (i.e) $\delta_{pu}(E_A)$ is a soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open in (Y, $\sigma_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^2$ B) for every soft $\zeta_{\mathcal{S}}$ open set E_A in (X, $\tau_{\mathcal{S}}, \zeta_{\mathcal{S}}{}^1$, A). Hence δ_{pu} is soft $\zeta_{\mathcal{S}} - \mathcal{G}$ open map.

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