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New Maps In Fuzzy Topological Spaces

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ABSTRACT: The aim of this paper is to introduce a new type of maps called Fuzzy wgrα-irresolute maps,strongly Fuzzy wgrα-continuous maps and study some of these properties.

Keywords: Fuzzy wgrα-closed-sets,Fuzzy wgrα-open-sets, Fuzzy wgrα-continuous-maps,Fuzzy wgrα-irresolute maps,stronglyFuzzy wgrα-continuous maps.

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1.INTRODUCTION:

The concept of irresolute map was introduced byHildebrand[5] and strongly continuous functions was introduced by Levine[6]. Later Wali and Benchalli[2] introduced and studiedrw-irresolute maps and strongly rw-continuous maps.In this section we introduce the concept of Fuzzy wgrα –irresolute maps and strongly Fuzzy wgrα-continuous maps in topological space and investigate some of their properties.

2.PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c ,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i)Semi-open set [1] if A $\subseteq$ cl(int(A)) and semi-closed set if int(cl(A)) $\subseteq$ A.

(ii)Regular ω- closed **(**briefly rω -closed)set [2] if cl(A)⊆U whenever A⊆ U and U is regular semi- open in X.

(iii) Pre generalized pre regular ωeakly closed set[3](briefly pgprω-closed set) if pCl(A)⊆U whenever A⊆U and U is rgα open in (X,τ).

(iv)pre generalized pre-regular ωeakly open(briefly pgprω-open)[4]set in X if A^c is pgprω-closed in X.

Definition 2.2A map f: (X,τ)→(Y,σ) is said to

(i)Semi continuos map[1] if $f^{-1}(V)$ is a semi-closed of (X,τ)for every closed set of (Y,σ).

(ii)irresolute map [5] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.

(iii)Strongly–continuousmap[6]if $f^{-1}(V)$ is Clopen (both open and closed) in X for every subset V of Y.

(iv)A function f from a topological space X into a topological space Y is called Fuzzy wgrα-continuous map(Fuzzy wgrα–Continuous)[7] if $f^{-1}(V)$ is Fuzzy wgrα–Closed set in X for every closed set V in Y.

3. Fuzzy wgrα–irresolute and Strongly Fuzzy wgrα–Continuous functions

Definition 3.1: A function f from a topological space X into a topological space Y is called Fuzzy wgrα-irresolute (Fuzzy wgrα–irresolute) map if $f^{-1}(V)$ is Fuzzy wgrα–Closed set in X for every Fuzzy wgrα–closed set V in Y.

Definition 3.2: A function f from a topological space X into a topological space Y is called strongly Fuzzy wgrα continuous (strongly Fuzzy wgrα–continuous) map if $f^{-1}(V)$ is closed set in X for every Fuzzy wgrα–closed set V in Y.

Theorem 3.3: If A map f: (X,τ)→(Y,σ) is Fuzzy wgrα–irresolutemap,if and only if the inverse image $f^{-1}(V)$ is Fuzzy wgrα–open set in X for every Fuzzy wgrα–open set V in Y.

Proof: Assume that f: X→Y is Fuzzy wgrα–irresolute map. Let G be Fuzzy wgrα–open in Y. The $G^c$is Fuzzy wgrα–closed in Y. Since f is Fuzzy wgrα– irresolute, $f^{-1}(G^c)$ is Fuzzy wgrα–closed in X.
But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is Fuzzy wgrα–open in X. Converserly , Assume that the inverse image of each open set in Y is Fuzzy wgrα–open in X. Let F be any Fuzzy wgrα–closed set in Y. By assumption $f^{-1}(F^c)$ is Fuzzy wgrα–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus X– $f^{-1}(F)$ is Fuzzy wgrα–open in X and so $f^{-1}(F)$ is Fuzzy wgrα–closed in X. Therefore f is Fuzzy wgrα– irresolute map.

Theorem 3.4: If A map f: (X,τ)→(Y,σ) is Fuzzy wgrα–irresolute map, then it is Fuzzy wgrα–continuous map but not conversely.

Proof: Let f: X→Y be Fuzzy wgrα–irresolute map. Let F be any closed set in Y. Then F is Fuzzy wgrα–closed in Y. Since f is Fuzzy wgrα–irresolute map, the inverse image $f^{-1}(F)$ is Fuzzy wgrα–closed set in X. Therefore f is Fuzzy wgrα–continuous map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: X={a,b,c,d}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a}},

Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=a , f(d)=c then f is Fuzzy wgrα–continuous map but f is not Fuzzy wgrα–irresolute map, as Fuzzy wgrα-closed set F= {b} in Y, then f^{-1}(F)={a} in X , which is not Fuzzy wgrα–closed set in X.

Theorem 3.6: Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two maps. Then

(i) g o f : (X, τ)→(Z,η) is Fuzzy wgrα–irresolute map if g is Fuzzy wgrα–irresolute map and f is Fuzzy wgrα–irresolute map.

(ii) g o f : (X, τ)→(Z,η) is Fuzzy wgrα–continuous map if g is Fuzzy wgrα–continuous map and f is Fuzzy wgrα–irresolute map.

Proof:

(i) Let U be a Fuzzy wgrα–open set in (Z, η) . Since g is Fuzzy wgrα–irresolute map, g^{-1}(U) is Fuzzy wgrα–open set in (Y, σ). Since f is Fuzzy wgrα–irresolute map, $f^{-1}(g^{-1}(U))$ is aFuzzy wgrα–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an Fuzzy wgrα–open set in (X, τ) and hence gof is Fuzzy wgrα–irresolute map.

(iii) Let U be a open set in (Z, η) . Since g is continuous map, g^{-1}(U) is open set in (Y, σ). As every open set is Fuzzy wgrα–open, g^{-1}(U) is Fuzzy wgrα–open set in (Y, σ). Since f is Fuzzy wgrα– irresolute map,$f^{-1}(g^{-1}(U))$ is an Fuzzy wgrα–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is anFuzzy wgrα–open set in (X, τ) and hence gof is Fuzzy wgrα–continuous map.

Theorem 3.7: Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) and (Y,σ) are Topological-space .Where "every Fuzzy wgrα closed subset is closed" Then the following are equivalent:

(i) f is Fuzzy wgrα-irresolute map

(ii) f is Fuzzy wgrα-continuous map.

Proof :

(i) implies (ii) follows from theorem 3.4

(ii) implies (i) Let F be a Fuzzy wgrα closed set in (Y,σ) then F is a closed set in (Y,σ) by hypothesis. Since f is a Fuzzy wgrα-continuousmap,f^{-1}(F) is a Fuzzy wgrα closed set in (X,τ), Therefore f is Fuzzy wgrα-irresolute map.

Remark 3.8: The following examples show that the notation of irresolute maps and Fuzzy wgrα-irresolute maps are independent.

Example: X={a,b,c}, Y={a,b,c} τ = {X, ϕ,{a}} &σ ={Y, ϕ,{a},{b},{a,b}},
thenthe identity map f:(X, τ) ➔ (Y,σ) is Fuzzy wgrα–irresolute map but it is not–irresolute map as inverse image of the semi-open set {b} in (Y,σ) is {b} in X , which is not semi-open set in (X,τ)

Example: X={a,b,c}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b}} σ ={Y, ϕ,{a}},
thenthe identity mapf:(X, τ) ➔ (Y, σ) is–irresolutemap but it is notFuzzy wgrα-irresolute map, as inverse image of the Fuzzy wgrα-closed set {b} in (Y,σ) is {b} in X , which is not Fuzzy wgrα closedset in (X,τ).

Theorem 3.9: Let f: (X, τ)→(Y,σ) is strongly Fuzzy wgrα–continuous map if and only if $f^{-1}(G)$ is open set in X for every Fuzzy wgrα–open set G in Y.
Proof : Assume that f: X→Y is strongly Fuzzy wgrα–continuous map. Let G be Fuzzy wgrα–open in Y. The G^c is Fuzzy wgrα–closed in Y. Since f is strongly Fuzzy wgrα–continuous map, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$, Thus $f^{-1}(G)$ is open in X. Converserly, Assume that the inverse image of each open set in Y is Fuzzy wgrα–open in X. Let F be any Fuzzy wgrα–closed set in Y. By assumption F^c is Fuzzy wgrα–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus X– $f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X. Therefore f is strongly Fuzzy wgrα–continuous map.

Theorem 3.10: Let f: (X, τ)→(Y,σ) is strongly Fuzzy wgrα–continuous map then it is continuous map.
Proof: Assume that f: (X, τ)→(Y,σ) is strongly Fuzzy wgrα–continuous map, Let F be closed set in Y. As every closed is Fuzzy wgrα–closed, F is Fuzzy wgrα–closed in Y. Since f is strongly Fuzzy wgrα–continuous mapthen $f^{-1}(F)$ is closed set in X. Therefore f is continuous map.

Example 3.11: X=Y={a,b,c,d}, τ = {X, ϕ,{a},{b},{a,b},{a,b,c}}
σ ={Y, ϕ,{a},{b},{a,b},{a,b,c}}.Let map f: X→Y defined by , f(a)=a , f(b)=b , f(c)=b, f(d)=b. then f is continuous but f is not strongly Fuzzy wgrα–continuous, since forFuzzy wgrα-closed set F= {a,c,d} in Y, then $f^{-1}(F)$={a} in X , which is not closed set in X.

Theorem: 3.12: Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) and (Y,σ) are Topological-space .Where "every Fuzzy wgrα closed subset is closed" Then the following are equivalent:

(i) f is strongly Fuzzy wgrα-continuous map
(ii) f is continuous map.

Proof:

(i) =>(ii) Let U be any open set in (Y,σ). Since every open set is Fuzzy wgrα-open, U is Fuzzy wgrα-open in (Y,σ). Then f^{-1}(U) is open in (X,τ). Hence f is continuous map.

(ii) =>(i) Let U be any Fuzzy wgrα-open set in (Y,σ). Since (Y,σ) is a topoogical-space, U is open in (Y,σ). Since f is continuous map.Then f^{-1}(U) is open in (X,τ). Hence f is strongly Fuzzy wgrα-continuous map.

Theorem 3.13: Let f: (X, τ)→(Y,σ) is strongly continuous map then it is strongly Fuzzy wgrα–continuous map.

Proof: Assume that f: X→Y is strongly continuous map. Let G be Fuzzy wgrα–open in Y and also it is any subset of Y since f is strongly continuous map, f^{-1}(G) is open (and also closed) in X. f^{-1}(G) is open in X Therefore f is strongly Fuzzy wgrα–continuous map.

Theorem 3.14: Let f: (X, τ)→(Y,σ) is strongly Fuzzy wgrα–continuous map then it is Fuzzy wgrα–continuous map.

Proof: Let G be open in Y , every open is Fuzzy wgrα–open , G is Fuzzy wgrα–open in Y, since f is strongly Fuzzy wgrα–continuous map, f^{-1}(G) is open in X. and therefore f^{-1}(G) is pgprω–open in X . Hence f is Fuzzy wgrα–continuous map.

Theorem 3.15: In discrete space, a map f: (X, τ)→(Y,σ) is strongly Fuzzy wgrα–continuous map then it is strongly continuous map.

Proof:F be any subset of Y, in discrete space, Every subset F in Y is both open and closed , then subset F is both Fuzzy wgrα–open or Fuzzy wgrα–closed, i) let F is pgprω–closed in Y, since f is strongly pgprω–continuous map, then f^{-1}(F) is closed in X. ii) let F is Fuzzy wgrα–open in Y, since f is strongly Fuzzy wgrα–continuous map, then f^{-1}(F) is open in X. Therefore f^{-1}(F) is closed and open in X. Hence f is strongly continuous map.

Theorem 3.16 : Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two functions. Then

(i) g o f : (X, τ)→(Z,η) is strongly Fuzzy wgrα–continuous map if g is strongly Fuzzy wgrα–continuous map and f is strongly Fuzzy wgrα–continuous map.

(ii) g o f : (X, τ)→(Z,η) is strongly Fuzzy wgrα–continuous map if g is strongly Fuzzy wgrα–continuous map and f is continuous map.

(iii) g o f : (X, τ)→(Z,η) is Fuzzy wgrα–irresolute map if g is strongly Fuzzy wgrα–continuous map and f is Fuzzy wgrα–continuous map.

(iv) g o f : (X, τ)→(Z,η) is continuous map if g is Fuzzy wgrα–continuous map and f is strongly Fuzzy wgrα–continuous map.

Proof:

(i)Let U be a Fuzzy wgrα–open set in (Z, η) . Since g is strongly Fuzzy wgrα–continuous map, $g^{-1}(U)$ is open set in (Y, σ). As every open set is Fuzzy wgrα–open, $g^{-1}(U)$ is Fuzzy wgrα–open set in (Y, σ). Since f is strongly Fuzzy wgrα–continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly Fuzzy wgrα–continuous map.

(ii)Let U be a Fuzzy wgrα–open set in (Z, η) . Since g is strongly Fuzzy wgrα–continuous map, $g^{-1}(U)$ is open set in (Y, σ). Since f is continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly Fuzzy wgrα–continuous map.

(iii)Let U be a Fuzzy wgrα–open set in (Z, η) . Since g is strongly Fuzzy wgrα–continuous map, $g^{-1}(U)$ is open setin (Y, σ). Since f is Fuzzy wgrα–continuous map $f^{-1}(g^{-1}(U))$ is anFuzzy wgrα–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is anFuzzy wgrα–open set in (X, τ) and hence gof is Fuzzy wgrα–irresolute map.

(iv)Let U be open set in (Z, η) . Since g is Fuzzy wgrα–continuous map, $g^{-1}(U)$ is Fuzzy wgrα–open set in (Y, σ).Since f is strongly Fuzzy wgrα–continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous map.

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