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Addition And Subtraction Of Generalized Fuzzy Numbers

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ABSTRACT

In this paper , we discuss the addition and subtraction of generalized trapezoidal fuzzy numbers with example , which shows that the existing operations make some approximation, thereby causing loss of information.

Keywords:- Fuzzy set theory , Generalized Fuzzy Numbers, function Principle

1.1 Introduction

The generalized fuzzy number (GFN) is realized to be more flexible than the normalized fuzzy number since it considers the degrees of confidence of experts' opinions. In 1985,Chen introduced the concepts of GFNs and also formulated different arithmetic operations on GFNs by proposing *function principle*. Based on Chen's (1985) arithmetic operations, a few researchers, of late, have provided important theoretical foundations on GFNs and their applications (e.g., Chen and Chen 2003, Chen and Hsieh 1999, Chen and Wang 2009, Chen *et al*. 2007, Hsieh and Chen 1999, Islam and Roy 2006, Mahapatra and Roy 2006, Wei and Chen 2009).

Although *function principle* (Chen 1985) was employed to develop arithmetic operations on GFNs, eventually some shortcomings of these operations have been observed. From mathematical point of view computation of different arithmetic operations using *function principle* is basically a point-wise operation. Due to this reason, it has been observed that Chen's arithmetic operations on generalized trapezoidal (and also triangular) fuzzy numbers causes loss of information, leading to inexact results. In view of this, the aim of this paper to discuss the addition and subtraction of generalized trapezoidal fuzzy numbers with example , which shows that the existing operations make some approximation, thereby causing loss of information.

Definition 1.1 **Positive Trapezoidal Fuzzy Number**

A generalized trapezoidal fuzzy number denoted as $\tilde{A} = (m_1, m_2\,; \beta\,, \gamma\,; w)$ is said to be positive if $m_1 - \beta \geq 0$ and $m_1 - \beta\,, m_1, m_2$ and $m_2 + \gamma$ are not identical.

1.2 The fuzzy addition and subtraction operations using function principle (Chen 1985)

Let us consider two positive generalized trapezoidal fuzzy numbers

$\tilde{A}_1 = (m_1, m_2\,; \beta_1\,, \gamma_1\,; w_1\,)$ and $\tilde{A}_2 = (m_3, m_4\,; \beta_2\,, \gamma_2\,; w_2\,)$.

Then the operational laws for these two fuzzy numbers are presented as

follows:

• The addition of $\tilde{A}_1$ *and* $\tilde{A}_2$:

$\tilde{A}_1(+)\tilde{A}_2 = (m_1 + m_3\,, m_2 + m_4; \beta_1 + \beta_2\,, \gamma_1 + \gamma_1; \min(w_1, w_2)\,)$.

• The subtraction of $\tilde{A}_1$ *and* $\tilde{A}_2$:

$$\tilde{A}_1(-)\tilde{A}_2 = (m_1 - m_4\,, m_2 - m_3; \beta_1 + \,, \gamma_2, \beta_2 + \gamma_1; \min(w_1, w_2)\,)$$

Setting $m_1 = m_2$ and $m_3 = m_4$ in the above operations, the corresponding results for generalized triangular fuzzy numbers (GTFNs) may be obtained. However, in course of the study some shortcomings of Chen's operations have been observed which are better reflected through GTFNs compared to generalized trapezoidal fuzzy numbers. Therefore, in the following section the shortcomings of Chen's method have been illustrated by utilizing example of GTFNs.

1.3 Example

Consider two GTFNs $\tilde{A}_1 = (0.8; 0.1{,}0.1; 0.5)$ *and* $\tilde{A}_2 = (0.9; 0.1{,}0.1; 1.0)$. After performing Chen's addition operation (defined in the section 1.2) between $\tilde{A}_1$ *and* $\tilde{A}_2$, a GTFN is obtained as $\tilde{A}_1(+)\tilde{A}_2 = \tilde{B} = (1.7; 0.2{,}0.2; 0.5)$. The result has been illustrated with the help of Figure 1.1.

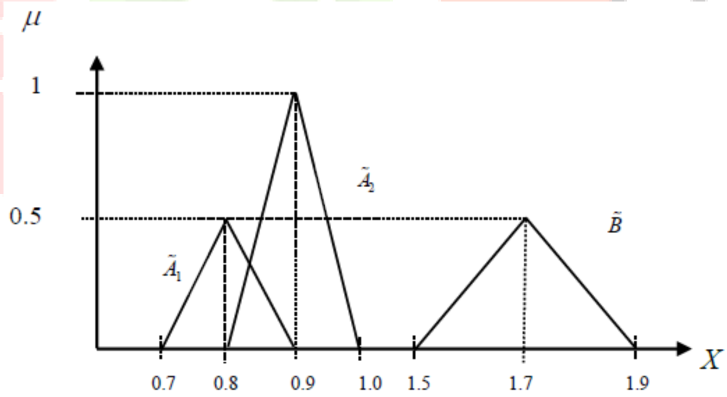

Figure 1.1: Sum of two generalized triangular fuzzy numbers (GTFNs)

It may be observed from Figure 1.1 that min (1,0.5) = 0.5 . If both the fuzzy numbers are taken to the same level by 'truncating the higher one', i.e., if we take 0.5 (since 0.5 <1.0) cut of $\tilde{A}_2$ then $\tilde{A}_2$ is transformed into a generalized trapezoidal (flat) fuzzy number. Therefore, there is a necessity to conserve this flatness for retaining the inherent information into the resultant GFN. In this respect, Chen's approach is incomplete and hence, loses its significance. Thus, the arithmetic operation on GFNs needs to be calculated more appropriately. In the following sections, we have presented methodologies to determine the improved arithmetic operations on generalized trapezoidal (and also triangular) fuzzy numbers.

1.4 Operations on GFNs

Let $\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1)$ and $\tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2)$ are two GFNs with the following membership functions:

$$\mu_{\tilde{A}_1}(x) = \begin{cases} 0 & \text{if} \quad -\infty < x \le m_1 - \beta_1 \\ w_1 f_a(x) & \text{if} \quad m_1 - \beta_1 \le x \le m_1 \\ w_1 & \text{if} \quad m_1 \le x \le m_2 \\ w_1 h_a(x) & \text{if} \quad m_2 \le x \le m_2 + \gamma_1 \\ 0 & \text{if} \quad m_2 + \gamma_1 \le x < \infty \end{cases}$$

$$\mu_{\tilde{A}_2}(y) = \begin{cases} 0 & \text{if} \quad -\infty < x \le m_3 - \beta_2 \\ w_2 f_b(y) & \text{if} \quad m_3 - \beta_2 \le y \le m_3 \\ w_2 & \text{if} \quad m_3 \le y \le m_4 \\ w_2 h_b(y) & \text{if} \quad m_4 \le y \le m_4 + \gamma_2 \\ 0 & \text{if} \quad m_4 + \gamma_2 \le y < \infty \end{cases}$$

Here $f_a(x)$ *and* $f_b(y)$ both are strictly increasing functions, where $f_a(x): [m_1 - \beta_1, m_1] \to [0,1]$ *and* $f_b(y): [m_3 - \beta_2, m_3] \to [0,1]$. Again $h_a(x)$ *and* $h_b(y)$ both are strictly decreasing functions, where $h_a(x) : [m_2, m_2 + \gamma_1] \to [0,1]$ *and* $h_b(y) : [m_4, m_4 + \gamma_2] \to [0,1]$.

Let $\tilde{C} = \tilde{A}_1(*)\tilde{A}_2$ (where $(*)$ is an extended fuzzy binary operation to combine $\tilde{A}_1$ and $\tilde{A}_2$), then by utilizing *extension principle* it can be said that $\tilde{C}$ will be a GFN.

1.5 Sum of two GFNs

Let us consider two GFNs denoted as $\tilde{A}_1$ and $\tilde{A}_2$. If it is assumed that $\tilde{C} = \tilde{A}_1(+)\tilde{A}_2$, then Theorem 1.1 may be established as follows:

Theorem 1.1

Let $\mu_{\tilde{C}}$ be the membership function of $\tilde{A}_1(+)\tilde{A}_2$; then

$$\mu_{\tilde{C}}(z) = \begin{cases} = 0 & \forall\, z \; (-\infty, (m_1 + m_3) - (\beta_1 + \beta_2)] \cup [(m_2 + m_4) + (\gamma_1 + \gamma_2), \infty) \\ \in [0, w_{\tilde{C}}] & \forall\, z \in \; [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}})] \\ & \cup\, [h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}}), (m_2 + m_4) + (\gamma_1 + \gamma_2)] \\ = w_{\tilde{C}} & \forall\, z \in \; [f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}}), h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}})] \end{cases}$$

where $w_{\tilde{C}} = \min(w_1, w_2)$

Proof: For $\tilde{C} = \tilde{A}_1(+)\tilde{A}_2 = \left\{\left(z, \mu_{\tilde{C}}(z)\right): z = x + y \;\; and\; \tilde{A}_1 == \left(x, \mu_{\tilde{A}_1}(x)\right), \tilde{A}_2 = \left(y, \mu_{\tilde{A}_2}(y)\right)\right\}$,

depending on the positions of *x* and *y* the following three cases may arise:

Case 1: Let us consider

$$x \in (-\infty, m_1 - \beta_1\,] and \;\; y \in (-\infty, m_3 - \beta_2].$$

Therefore, we obtain

$$x + y = z \in (-\infty, (m_1 + m_3) - (\beta_1 + \beta_2)].$$

As

$\mu_{\tilde{A}_1}(x) = 0\ \forall x \in (-\infty, m_1 - \beta_1]\ and\ \mu_{\tilde{A}_2}(y) = 0\ \forall y \in (-\infty, m_3 - \beta_2],$

we need to prove

$$\mu_{\tilde{C}}(z) = 0\ \forall z \in (-\infty, m_1 + m_3) - (\beta_1 + \beta_2)].$$

In order to evaluate $\mu_{\tilde{C}}(z)$ we must consider every pair (p, r) such that $z = p + r$. Now for every pair (p, r) (for $p + r = z$) the following two possibilities may be considered:

(i) For $p < x$ and $r > y$, $\min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0\ as\ \mu_{\tilde{A}_1}(p) = 0\ \forall\ p < x.$

(ii) For $p > x$ and $r < y$, $\min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0\ as\ \mu_{\tilde{A}_2}(r) = 0\ \forall\ r < y.$

From both (i) and (ii),

$$\sup_{z=p+r} min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r) = 0 \Rightarrow \mu_{\tilde{C}}(z) = 0.$$

Since this result holds for any arbitrary value of z, therefore, it may be said that

$$\mu_{\tilde{C}}(z) = 0\ \forall z \in (-\infty, (m_1 + m_3) - (\beta_1 + \beta_2)].$$

In a similar manner it may also be proved that

$$\mu_{\tilde{C}}(z) = 0\ \forall\ z \in [(m_2 + m_4) + (\gamma_1 + \gamma_2), \infty).$$

Thus,

$\mu_{\tilde{C}}(z) = 0\ \forall\ z \in (-\infty, (m_1 + m_3) - (\beta_1 + \beta_2)] \cup [(m_2 + m_4) + (\gamma_1 + \gamma_2), \infty).$

Case 2: Let us consider $min(w_1, w_2) = w_1$.

For $x \in [m_1 - \beta_1, m_1]$ and $y \in [m_3 - \beta_2, f_b^{-1}(w_1)], i.e.,\ x \in [m_1 - \beta_1, f_a^{-1}(w_1)]$ and $y \in [m_3 - \beta_2, f_b^{-1}(w_1)] \Rightarrow\ x + y = z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)]$

We need to prove

$$\mu_{\tilde{C}}(z) \in [0, w_1] \forall z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)].$$

Now when

$z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1))$, for every pair (p, r)

(where $p + r = z$) the following two possibilities may be considered:

(i) For $p < x$ and $r > y$,

a. For $p > m_1 - \beta_1$ and $r < f_b^{-1}(w_1)$, we have $min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} < w_1.$

b. For $p > m_1 - \beta_1$ and $r > f_b^{-1}(w_1)$, we have $min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} < w_1.$

c. For $p < m_1 - \beta_1$ and $r > f_b^{-1}(w_1)$, we obtain $min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0.$

d. For $p < m_1 - \beta_1$ and $r < f_b^{-1}(w_1)$, we have $min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0.$

(ii) For $p > x$ and $r < y$,

a. For $p < f_a^{-1}(w_1) = m_1\ and\ r > m_3 - \beta_2, we\ have\ \min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} < w_1.$

b. For $p < m_1\ and\ r < m_3 - \beta_2, we\ obtain\ \min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0.$

c. For $p > m_1\ and\ r > m_3 - \beta_2, we\ have\ \min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} < w_1.$

d. For $p > m_1\ and\ r < m_3 - \beta_2, we\ have\ \min\{\mu_{\tilde{A}_1}(p), \mu_{\tilde{A}_2}(r)\} = 0.$

Consequently for both (i) and (ii), we can obtain

$$0 \le \sup_{z=p+r} \min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} < w_1 \forall z \in \left[(m_1 + m_2) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)\right).$$

In particular for $z = f_a^{-1}(w_1) + f_b^{-1}(w_1)$, when $x = f_a^{-1}(w_1)$ and $y = f_b^{-1}(w_1)$, it is clear that $min\{\mu_{\bar{A}_1}(x), \mu_{\bar{A}_2}(y)\} = w_1$. Again as before, for this particular case the following possibilities may also be considered:

(iii) $For\ p < x\ and\ r > y, we\ get \min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} < w_1.$

(iv) $For\ p > x\ and\ r < y, we\ get \min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} < w_1.$

(v) $For\ p = x\ and\ r = y\ only, we\ get \min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} = w_1.$

Consequently for (iii), (iv) and (v) we have

$\sup_{z=p+r} min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} = w_1.$ Therefore, from the above five possibilities, we can say that for any arbitrary

$$z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)],$$

the following relation holds:

$$0 \le \sup_{z=p+r} min\{\mu_{\bar{A}_1}(p), \mu_{\bar{A}_2}(r)\} \le w_1.$$

Since this result holds for any arbitrary

$$z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)],$$

we can state that this is true for all

$$z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)],$$

Therefore,

$\mu_{\bar{c}}(z) \in [0, w_1] \forall z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_1) + f_b^{-1}(w_1)]$ In a similar manner

$$\mu_{\bar{c}}(z) \in [0, w_1]\ \forall z \in [h_a^{-1}(w_1) + h_b^{-1}(w_1), (m_2 + m_4) + (\gamma_1 + \gamma_2)].$$

Similar result can be obtained if we consider $min(w_1, w_2\ ) = w_2$.

Hence in general, if it can be written that $min(w_1, w_2\ ) = w_{\bar{c}}$., then the following holds:

$$\mu_{\bar{c}}(z) \in [0, w_{\bar{c}}]\ \forall z \in [(m_1 + m_3) - (\beta_1 + \beta_2), f_a^{-1}(w_{\bar{c}}) + f_b^{-1}(w_{\bar{c}})]$$
$$\cup\ [h_a^{-1}(w_{\bar{c}}) + h_b^{-1}(w_{\bar{c}}), (m_2 + m_4) + (\gamma_1 + \gamma_2)].$$

Case 3: Let us consider

$$x \in \left[f_a^{-1}(w_{\tilde{C}}), h_a^{-1}\left((w_{\tilde{C}})\right)\right] \text{ and } y \in [f_b^{-1}(w_{\tilde{C}}), h_b^{-1}(w_{\tilde{C}})]$$

$\Rightarrow x + y = z \in [f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}}), h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}})]$. Now

$\mu_{\tilde{A}_1}(x) = w_{\tilde{C}} \forall x \in [f_a^{-1}(w_{\tilde{C}}), h_a^{-1}(w_{\tilde{C}})]$ and $\mu_{\tilde{A}_2}(y) = w_{\tilde{C}} \forall y \in [f_b^{-1}(w_{\tilde{C}}), h_b^{-1}(w_{\tilde{C}})]$.

Therefore, it is obvious that

$\mu_{\tilde{C}}(z) = w_{\tilde{C}}, \forall z \in [f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}}), h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}})]$.Therefore,for $\tilde{C} = \tilde{A}_1 (+) \tilde{A}_2$, the membership function $\mu_{\tilde{C}}(z)$ may be written as follows:

$$\mu_{\tilde{C}}(z) = \begin{cases} 0 & if \quad -\infty < z \le (m_1 + m_3) - (\beta_1 + \beta_2) \\ w_{\tilde{C}} f_c z & if \quad (m_1 + m_3) - (\beta_1 + \beta_2) \le z \le f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}}) \\ w_{\tilde{C}} & if \quad f_a^{-1}(w_{\tilde{C}}) + f_b^{-1}(w_{\tilde{C}}) \le z \le h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}}) \\ w_{\tilde{C}} h_c z & if \quad h_a^{-1}(w_{\tilde{C}}) + h_b^{-1}(w_{\tilde{C}}) \le z \le (m_2 + m_4) + (\gamma_1 + \gamma_2) \\ 0 & if \quad (m_2 + m_4) + (\gamma_1 + \gamma_2) \le z < \infty \end{cases}$$

where
$\boldsymbol{f_c(z) = \sup_{z=x+y} \min\{f_a(x), f_b(y)\} \text{ and } h_c(z) = \sup_{z=x+y} \min\{h_a(x), h_b(y)\}}$ by extension principle .

1.6 Particular case: for GTFNs

Let us consider two GTFNs $\tilde{A}_1 = (m_1; \beta_1, \gamma_1; w_1)$ and $\tilde{A}_2 = (m_2; \beta_2, \gamma_2; w_2)$ where $w_1 \ne w_2$.

Theorem 1.2

Addition of two GTFNs $\tilde{A}_1$ *and* $\tilde{A}_2$ generates a generalized trapezoidal fuzzy number as follows:

$$\tilde{C} = \tilde{A}_1 (+) \tilde{A}_2 = \left(\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}; \beta_{\tilde{C}}, \gamma_{\tilde{C}}; w_{\tilde{C}}\right) \text{ where,}$$

$$\underline{m}_{\tilde{C}} = (m_1 - \beta_1) + (m_2 - \beta_2) + (\beta_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2)$$

$$\overline{m}_{\tilde{C}} = (m_1 + \gamma_1) + (m_2 + \gamma_2) - (\gamma_1 w_{\tilde{C}}/w_1 + \gamma_2 w_{\tilde{C}}/w_2)$$

$$\beta_{\tilde{C}} = {}^{\beta_1 w_{\tilde{C}}}/_{w_1} + {}^{\beta_2 w_{\tilde{C}}}/_{w_2}$$

$$\gamma_{\tilde{C}} = {}^{\gamma_1 w_{\tilde{C}}}/_{w_1} + {}^{\gamma_2 w_{\tilde{C}}}/_{w_2}$$

and $w_{\tilde{C}} = \min(w_1, w_2)$; $w_1 \ne w_2$ (1.1)

Proof: GTFNs $\tilde{A}_1$ and $\tilde{A}_2$ have the membership functions of the following form:

$$\mu_{\tilde{A}_1}(x) = \begin{cases} 0 & if \quad -\infty \le x \le m_1 - \beta_1 \\ w_1(x - m_1 + \beta_1)/\beta_1 & if \quad m_1 - \beta_1 \le x \le m_1 \\ w_1(m_1 + \gamma_1 - x)/\gamma_1 & if \quad m_1 \le x \le m_1 + \gamma_1 \\ 0 & if \quad m_1 + \gamma_1 \le x \le \infty \end{cases} \qquad (1.2)$$

$$\mu_{\tilde{A}_2}(y) = \begin{cases} 0 & if \quad -\infty \le y \le m_2 - \beta_2 \\ w_1(y - m_2 + \beta_2)/\beta_2 & if \quad m_2 - \beta_2 \le y \le m_2 \\ w_1(m_2 + \gamma_2 - y)/\gamma_2 & if \quad m_2 \le y \le m_2 + \gamma_2 \\ 0 & if \quad m_2 + \gamma_2 \le y \le \infty \end{cases} \qquad (1.3)$$

For $\tilde{C} = \tilde{A}_1(+)\tilde{A}_2$, we need to have $\mu_{\tilde{C}}(z)$ where $z = x + y$. Now it can be said that for a fixed value of $w \in [0, \min(w_1, w_2)], \exists (x, y) \in \mathbb{R}^2$ such that $\mu_{\tilde{A}_1}(x) = \mu_{\tilde{A}_2}(y) = w = \mu_{\tilde{C}}(z)$ holds. For obtaining $\tilde{C}$ the following two cases may be considered:

Case 1: For

$x \in [m_1 - \beta_1, m_1]$ *and* $y \in [m_2 - \beta_2, m_2]$, the following may be written:

From (2.2), $\mu_{\tilde{A}_1}(x) = w \Longrightarrow x = m_1 - \beta_1(1 - w/w_1)$.

Similarly from (2.3), $\mu_{\tilde{A}_2}(y) = w \Longrightarrow y = m_2 - \beta_2(1 - w/w_2)$.

Thus, $\quad x + y = z = (m_1 + m_2) - \beta_1(1 - w/w_1) - \beta_2(1 - w/w_2)$

or $\quad w = \frac{z-(m_1+m_2-\beta_1-\beta_2)}{\beta_1/w_1+\beta_2/w_2} \qquad$ *(1.4)*

Again , $\mu_{\tilde{C}}(z) = w \Longrightarrow z = \underline{m}_{\tilde{C}} - \beta_{\tilde{C}}(1 - w/w_{\tilde{C}})$.

or $\quad w = \frac{z-(\underline{m}_{\tilde{C}}-\beta_{\tilde{C}})}{\beta_{\tilde{C}}/w_{\tilde{C}}} \qquad$ *(1.5)*

Comparing (1.4) and (1.5) the following can be written:

$$\underline{m}_{\tilde{C}} - \beta_{\tilde{C}} = (m_1 - \beta_1) + (m_2 - \beta_2) \ and \ \beta_{\tilde{C}} = \beta_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2$$

Therefore,

$$\underline{m}_{\tilde{C}} = (m_1 - \beta_1) + (m_2 - \beta_2) + \beta_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2$$

Case 2: For $x \in [m_1, m_1 + \gamma_1]$ *and* $y \in [m_2, m_2 + \gamma_2]$, in a way similar to case 1, we will obtain the vales of $\overline{m}_{\tilde{C}}$ *and* $\gamma_{\tilde{C}}$

1.7 More on the example of section 1.3

As shown in the section 1.3, after performing Chen's (1985) addition operation we have obtained $\tilde{A}_1(+)\tilde{A}_2 = \bar{B} = (1.7; 0.2, 0.2; 0.5)$. However, from Theorem 1.2 it may be said that addition of $\tilde{A}_1$ *and* $\tilde{A}_2$ generates a generalized trapezoidal fuzzy number $\tilde{C}$ = (1.65,1.75; 0.15, 0.15; 0.5) , thereby providing a better result. The difference between the two operations has been illustrated in Figure 1.2, which clearly revealed that Chen's addition causes loss of information.

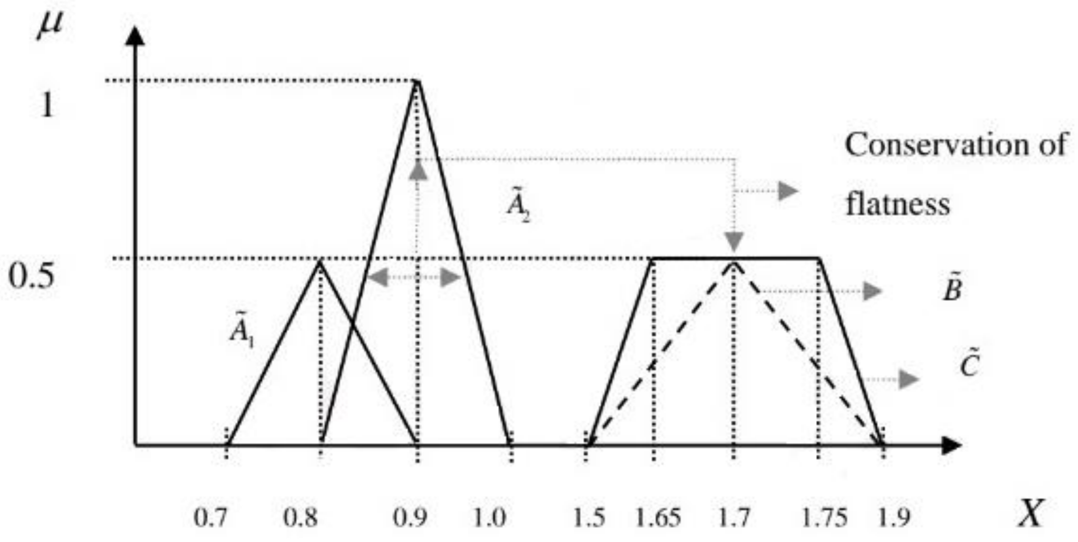

Figure 1.2: Comparison between the two additions

Theorem 1.3

Addition of two generalized trapezoidal fuzzy numbers $\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1)$ *and* $\tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2)$ generates a generalized trapezoidal fuzzy number as follows :

$$\tilde{C} = \tilde{A}_1(+)\tilde{A}_2 = (\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}; \beta_{\tilde{C}}, \gamma_{\tilde{C}}; w_{\tilde{C}})$$

where

$$\underline{m}_{\tilde{C}} = (m_1 - \beta_1) + (m_3 - \beta_2) + (\beta_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2)$$

$$\overline{m}_{\tilde{C}} = (m_2 + \gamma_1) + (m_4 + \gamma_2) - (\gamma_1 w_{\tilde{C}}/w_1 + \gamma_2 w_{\tilde{C}}/w_2)$$

$$\beta_{\tilde{C}} = \beta_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2$$

$$\gamma_{\tilde{C}} = \gamma_1 w_{\tilde{C}}/w_1 + \gamma_2 w_{\tilde{C}}/w_2$$

and $w_{\tilde{C}} = \min(w_1, w_2)$. (1.6)

Proof: The proof is similar to Theorem 1.2

1.8 Subtraction of two GFNs

Let us consider two generalized trapezoidal fuzzy number $\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1)$ *and* $\tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2)$. In order to compute the subtraction operation between $\tilde{A}_1$ *and* $\tilde{A}_2$, the value of $\tilde{A}_1(-)\tilde{A}_2$ may be defined as $\tilde{A}_1(-)\tilde{A}_2 = \tilde{A}_1(+)(-\tilde{A}_2)$. Now as discussed in Theorem 1.3, the addition operation on $\tilde{A}_1$ *and* $(-\tilde{A}_2)$ can be easily performed. Hence, we can deduce the following formula:

$$\tilde{C} = \tilde{A}_1(-)\tilde{A}_2 = (\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}; \beta_{\tilde{C}}, \gamma_{\tilde{C}}; w_{\tilde{C}})$$

where

$$\underline{m}_{\tilde{C}} = (m_1 - \beta_1) - (m_4 + \gamma_2) + (\beta_1 w_{\tilde{C}}/w_1 + \gamma_2 w_{\tilde{C}}/w_2)$$

$$\overline{m}_{\tilde{C}} = (m_2 + \gamma_1) - (m_3 - \beta_2) - (\gamma_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2)$$

$$\beta_{\tilde{C}} = \beta_1 w_{\tilde{C}}/w_1 + \gamma_2 w_{\tilde{C}}/w_2$$

$\gamma_{\tilde{C}} = \gamma_1 w_{\tilde{C}}/w_1 + \beta_2 w_{\tilde{C}}/w_2$ (1.7)

and $w_{\tilde{C}} = \min(w_1, w_2)$.

1.9 Numerical illustration

Example 1.1 Let $\tilde{A}_1 = (\mathbf{2,4;1,1;0.5})$ ***and*** $\tilde{A}_2 = (\mathbf{6,8;1,1;1})$ be two generalized trapezoidal fuzzy numbers. Find $\tilde{A}_1(+)\tilde{A}_2$.

Result: With the help of (1.6), we obtain $\tilde{A}_1(+)\tilde{A}_2 = (7.5,12.5; 1.5,1.5; 0.5)$.

Example 1.2 Let $\tilde{A}_1 = (\mathbf{0.6;0.2,0.1;0.8})$ ***and*** $\tilde{A}_2 = (\mathbf{0.2;0.3,0.2;1.0})$ be two GTFNs. Find $\tilde{A}_1(+)\tilde{A}_2$.

Result: Utilizing (1.1), the result may be found as: $\tilde{A}_1(+)\tilde{A}_2 = (0.74,0.84; 0.44,0.26; 0.8)$.

Example 1.3 Let $\tilde{A}_1 = (\mathbf{0.6;0.1,0.1;0.5})$ ***and*** $\tilde{A}_2 = (\mathbf{0.4;0.1,0.1;1.0})$ be two GTFNs. Find $\tilde{A}_1(-)\tilde{A}_2$.

Result: Utilizing (1.7), we obtain $\tilde{A}_1(-)\tilde{A}_2 = (0.15,0.25; 0.15,0.15; 0.5)$.

1.10 Conclusion

In this paper, addition and subtraction of generalized trapezoidal and also triangular fuzzy numbers have been analyzed. For this purpose, the existing operations on GFNs have been studied and the drawbacks have been pointed out. In order to overcome the drawbacks, new arithmetic operators on generalized trapezoidal and triangular fuzzy numbers have been proposed based on *extension principle*. Furthermore, the proposed operators can deal with both the non-normalized and normalized fuzzy numbers and also eventually causes less loss of information than the existing ones.

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