IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Isotropic Dark Energy Cosmological Model In $f(R)$ Theory Of Gravitation.

K.V. Somwanshi[1]

S.S.S.K.R. Innani.MahavidyalayaS, Karanja (Lad)-444105

Abstract

In this investigation, we examine the Bianchi Type III Universe within the context of the $f(R)$ theory of gravity. To derive meaningful insights, the Field Equations are resolved by using power law relationship between F and 'a' and by establishing the relationship between the metric potential B^n and C. Furthermore, we advance our understanding by developing important cosmological parameters such as the deceleration parameter and the Dark energy density parameter, total energydensity parameter. Through this analysis, we study the physical characteristics and behaviour of this model.

Keywords: - Holographic Dark Energy (HDE), $f(R)$ Gravity, Bianchi Type III, Cosmological Parameter.

1. Introduction

Einstein's general theory of relativity has provided a complex explanation for the phenomenon of gravitation, proving to be highly effective in its description. Additionally, it has served as the basis for various models of the universe. However, Einstein himself acknowledged that the general relativity falls short in adequately accounting for the fundamental properties of matter. As a result, there have been intriguing efforts in recent years to expand upon the general theory of relativity by incorporating specific desired characteristics that were absent in the original theory. Notably, recent observations in cosmology [1] have played a crucial role in enhancing our comprehension of the late-time cosmic acceleration. This accelerated expansion of the universe strongly indicates the existence of a negative pressure originating from an exotic form of dark energy. The origin of dark energy holds significant importance in modern cosmology, and several proposed solutions aim to unravel this perplexing puzzle. One such solution is the holographic dark energy (HDE) model [2-3], which suggests that the dark energy density can be represented by the equation,$\rho_{DE}= 3d^2M_P{}^2L^{-2}$ where d is a numerical factor approximately equal to unity,M_P represents the reduced Planck mass $M_P{}^2 = 1/8\pi G$, and L denotes the event horizon of the universe.

Recent advancements in science, particularly in the fields of astronomy, astrophysics, cosmology, data science, and space science, have significantly contributed to our understanding of the universe. Observations have provided strong evidence that our universe is expanding at an accelerated rate [4-11]. This phenomenon is believed to be influenced by 'Dark Matter' (DM) and 'Dark Energy' (DE), which are yet to be fully understood. The exploration of alternative gravitational theories beyond Einstein's General Relativity (GR) has become crucial in addressing fundamental challenges related to quantum gravity, dark energy, and dark matter. Although GR has been successful in many aspects, it still faces unresolved issues such as singularities and the nature of DE and DM. These challenges have prompted researchers to seek modifications or extensions of GR that can address its limitations across both ultraviolet (UV) and infrared (IR) scales [12]. One such approach is the $f(R)$ gravity theory, which offers an intriguing generalization of the Einstein-Hilbert action by replacing the Ricci scalar, R, with an analytic differentiable function. This extension is motivated by the need to formulate quantum field theories on curved spacetime [13]. The $f(R)$ gravity theory has found successful applications in cosmology and astrophysics [14-16] It is important to note that modified gravity theories do not necessarily have to recover GR entirely but can provide equivalent formulations. For example, the teleparallel equivalent of general relativity is a formulation that is equivalent to GR in certain conditions. Conversely; extended gravity theories can exhibit GR as a li miting case, depending on specific choices or conditions [17].

Researchers have been closely examining a class of $f(R)$ gravity models to gain further insights into the nature of the universe [18-22]. These models belong to a class of derivative gravity theories of higher order that address higher-order curvature invariants expressed as functions of the Ricci scalar, R. The importance of these models lies in their ability to circumvent Ostrogradski's instability [23], which is a potential issue in general higher derivative theories [24]. The construction of phenomenologically viable f(R) gravity theories faces various challenges, including instabilities within and beyond matter [25], vacuum stability [26], and constraints derived from well-established gravity properties in our solar system [27–28]. Moreover, determining the specific functional form of $f(R)$ from cosmological observations is also challenging due to the fact that the background expansion does not uniquely determine it [29]. The search for accurate solutions in the $f(R)$theory of gravity is of vital importance and presents a formidable challenge due to the intricate nature of the equations of motion involving higher-order terms. Despite this complexity, a wide range of exact and numerical solutions have been successfully derived using various methodologies. Beginning with the simplest scenario, a specific class of $f(R)$ gravity with constant curvature has been extensively studied. The solutions within this class, such as Schwarzschild-like [30], Reissner-Nordström-like [31], and Kerr-Newman-like solutions [32], deviate from the solutions of General Relativity (GR) only through a constant coefficient that can be incorporated into Newton's constant. Moreover, static spherically symmetric solutions incorporating perfect fluid [33], Yang-Mills field [34], non-linear Yang-Mills field [31, 35], Maxwell field, and non-linear electromagnetic fields [36] have been obtained. By employing Noether symmetries, axially symmetric solutions have been derived from exact spherically symmetric solutions [37]. Furthermore, intriguing connections have been established between solutions in Einstein's conformally invariant Maxwell theory and those in $f(R)$ gravity without matter fields in arbitrary dimensions [38]. The investigation also extends to spherically symmetric vacuum solutions in $f(R)$ gravity within higher dimensions, as detailed in [39]. Therefore, considerable attention has been focused on spherically symmetric black hole solutions within the framework of $f(R)$ gravity, including the generator method [40] and the Lagrange multiplier method [41].

The DE problem has been approached from various angles, and one alternative solution is known as the holographic principle. This principle was initially proposed by 't Hooft [42] in the realm of black hole physics. According to this principle, the entropy of a system is not determined by its volume, but rather by its surface area [43]. Expanding on this concept, Fischler and Susskind [44] introduced a modified version of the holographic principle in the cosmological context. They suggested that the gravitational entropy within a closed surface should not always exceed the particle entropy passing through the past light-cone of that surface during cosmological evolution. In relation to the DE problem, the holographic principle establishes a connection between the holographic DE (HDE) density ρ_Λ and the Hubble parameter H, expressed as $\rho_\Lambda = H^2$. However, it does not account for the current accelerated expansion of the universe. To address this limitation, Granda and Oliveros [45] proposed a holographic density model, denoted as $\rho_\Lambda \approx \mu h2 + \lambda H$, where H represents the Hubble parameter and μ and λ are constants subject to constraints imposed by observational data. Their model successfully explains the accelerated expansion of the universe and aligns with current observational findings. In recent times, several authors have conducted significant studies on HDE within various theories of gravitation, further exploring its implications and potential applications [46-49].

The paper is organised as follows. The first section 1, introduced to the topic and background. The second sect.2, focuses on the $f(R)$ gravity Formalism and space time. The sect.3 contains metric and field equation The sect.4, deals with the solution of the field equation. In sect.5, the geometrical and physical aspect of the models are explored. A discussion and conclusion are included in the concluding part.

2 $f(R)$ Gravity Formalism and Metric

$f(R)$theories of gravity are extended theories of gravity which simplify to general relativity in the most elementary case of the function $f(R)$. The idea of understanding this theory stemmed in two ways – one by considering a variation of the Einstein-Hilbert action with the metric (the metric formalism), and the other by varying the metric and an independent connection (called the Palatini formalism) [50]. Each of these methods describe a form of an extended theory of gravity by considering some function of the Ricci scalar – the field equations in each case (i.e. the field equations for an $f(R)$ theory of gravity for some form of the function) can be derived by either of these two formalisms. The nature of the actions of $f(R)$ theories of gravity was first studied extensively in [51].

We consider a class of modified gravity in which modifies Einstein-Hilbert action by replacing Ricci curvature scalar R by an arbitrary function of curvature $f(R)$ as follows

$$S = \int \sqrt{-g}\left(\frac{f(R)}{2} + kL_m\right) d^4x \qquad (1)$$

Where g is the metric determinant, L_m is the matter Lagrangian

The field equation of $f(R)$is given by

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}F(R) = k^2T_{ij} \qquad (2)$$

Where $F = \frac{df}{dR}$,∇ *is a covariant derivative*,$\Box = \nabla^\alpha\nabla_\alpha$is the d'Alembertian, $T_{ij}$is the matter energy-momentum tensor

The standard FRW model is in good agreement with the present-day universe. However, it does not give a clear description of the early stage of evolution of the universe. Bianchi type models provide a physically realistic description of the initial universe. It is well known that, near the Big Bang, the universe is neither isotropic nor spherically symmetric. Thus, anisotropic models play an important role in cosmology.

The energy momentum tensor for matter and holographic dark energy are defined as,

$$T_{ij} = \rho_m u_i u_j \qquad (3)$$

$$\overline{T_{ij}} = (\rho_\lambda + p_\lambda) u_i u_j + p_\lambda g_{ij} \qquad (4)$$

Whereρ_m and $\rho_\lambda$are the energy densities of matter and holographic dark energy and P_λ is the pressure of holographic dark energy. u^i =(0,0,0,1) is the four-velocity vector in co-moving co-ordinates which satisfies the condition $u^i u_j = 1$. (5)

In co-moving coordinate system from equation (4) and (5)

$T_1^1 = T_2^2 = T_3^3 = p_\lambda$ and $T_4^4 = \rho_m + \rho_\lambda$, $T_j^i = 0$,for $i \neq j$

3. Metric and Field Equation

The spatially homogeneous and anisotropic Four-dimensional Bianchi Type -III metric is given by,

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2hx}dy^2 - C^2\,dz^2 \qquad \text{---(6)}$$

where A, B, C are the metric potentials and functions of cosmic time t only.

From equation (2), (3), (4) (5) and (6) the field equations are given by,

$$F\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{h^2}{A^2}\right] - \frac{f}{2} + \frac{B_4}{B}F_4 + \frac{C_4}{C}F_4 + F_{44} = K^2P_\lambda \qquad (7)$$

$$F\left[\frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} - \frac{h^2}{A^2}\right] - \frac{f}{2} + \frac{A_4}{A}F_4 + \frac{C_4}{C}F_4 + F_{44} = K^2P_\lambda \qquad (8)$$

$$F\left[\frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC}\right] - \frac{f}{2} + \frac{A_4}{A}F_4 + \frac{B_4}{B}F_4 + F_{44} = K^2P_\lambda \qquad (9)$$

$$\mathrm{F}\left[\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}\right]-\frac{f}{2}+\left[\frac{A_4}{A}F_4+\frac{B_4}{B}F_4+\frac{C_4}{C}F_4\right]=\mathrm{K}^2(\rho_\mathrm{m}+\rho_\lambda) \qquad (10)$$

And $\frac{A_4}{A}-\frac{B_4}{B} = 0$ (11)

Where the derivative with respect to cosmic time t is denoted by the subscript '4'.

The continuity equation for the matter and holographic dark energy from equation (4) can be written as

$$\rho_m+\rho_m\left(\frac{2A_4}{A}+\frac{C_4}{C}\right)+\rho_\lambda+(1+\omega_\lambda)\rho_\lambda\left(\frac{2A_4}{A}+\frac{C_4}{C}\right)=0 \qquad (12)$$

But we have considering minimally interacting matter and holographic dark energy components, both the components conserve separately, hence can be written as,

$$\rho_m+\rho_m\left(\frac{2A_4}{A}+\frac{C_4}{C}\right)=0 \qquad (13)$$

$$\rho_\lambda+(1+\omega_\lambda)\rho_\lambda\left(\frac{2A_4}{A}+\frac{C_4}{C}\right)=0 \qquad (14)$$

Where $\omega_\lambda=\frac{P_\lambda}{\rho_\lambda}$ is the EoS Parameter for holographic dark energy trough which we analyse different universe eras.

Solving equation (11) we get.

$$\mathrm{A} = \mathrm{B} \qquad (15)$$

Using eq^n (15), eq^n (7), (8), (9) and (10) are given by,

$$\mathrm{F}\left\{\frac{B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{B_4C_4}{BC}-\frac{h^2}{B^2}\right\}-\frac{1}{2}f+\frac{B_4}{B}F_4+\frac{C_4}{C}F_4+F_{44}=\mathrm{K}^2\mathrm{P}_\lambda \qquad (16)$$

$$\mathrm{F}\left\{\frac{C_{44}}{C}+2\frac{B_4C_4}{BC}\right\}-\frac{1}{2}f+2\frac{B_4}{B}F_4+F_{44}=\mathrm{K}^2\mathrm{P}_\lambda \qquad (17)$$

And $\mathrm{F}\left\{2\frac{B_{44}}{B}+\frac{C_{44}}{C}\right\}-\frac{1}{2}f+\left\{2\frac{B_4}{B}F_4+\frac{C_4}{C}F_4\right\}==\mathrm{K}^2(\rho_\mathrm{m}+\rho_\lambda)$ (18)

4.Solution and the Model

We have three independent field equation 10-12 with six unknown parameter B, C, F, ρ_m, $\rho_\lambda$and p_λ.s Hence, to obtain explicit solution of this system, we require three additional constraints relating to these parameters.

At first, we assume that, the expansion scalar (Θ) in the model is proportional to shear scalar (σ) which gives the relationship between the metric potential as,

$$\mathrm{B}^\mathrm{n} = \mathrm{C} \qquad (19)$$

Where n ≠ 1, is a positive constant which takes core of the anisotropy of the space time.

Uddin et al. [52] using power law relationship between f and a i.e.$\mathrm{F(R)} \propto [\mathrm{a(t)}]^\mathrm{m}$, where m is constant => $\mathrm{F(R)} = \mathrm{F_0}[\mathrm{a(t)}]^\mathrm{m}$, where $\mathrm{F_0}$ is proportionality constant. (20)

The Hubble parameter H proposed by Berman [53] defined as

$$\mathrm{H} = \beta[\mathrm{a(t)}]^{-\alpha} \qquad (21)$$

Where $\beta > 0, \alpha \geq 0$ are constant and a(t) is average scale factor.

This relation gives a constant value of deceleration parameter

Solving equation (21), we get

$$a(t) = (\alpha\beta t + C)^{\frac{1}{\alpha}}, \alpha \neq 0$$

and $a(t) = C_1 e^t, \alpha = 0$

for suitable value of $\dot{\beta} = 1$ s

$a(t) = (\alpha t + C)^{\frac{1}{\alpha}}, \alpha \neq 0$ (22)

and $a(t) = C_1 e^t, \alpha = 0$ (23)

The spatial volume $V = ABC = a^3$

Using equation (15), (19) and (22), we get

$$V = B^{n+2} = (\alpha t + C)^{\frac{3}{\alpha}}$$

$\therefore B = (\alpha t + C)^{\frac{3}{\alpha(n+2)}}$

=> $A = B = (\alpha t + C)^{\frac{3}{\alpha(n+2)}}$ (24)

And $C = (\alpha t + C)^{\frac{3n}{\alpha(n+2)}}$ (25)

Using equation (24) and (25), equation (6) becomes

$$ds^2 = dt^2 - (\alpha t + C)^{\frac{6}{\alpha(n+2)}} dx^2 - (\alpha t + C)^{\frac{6}{\alpha(n+2)}} e^{-2hx} dy^2 - (\alpha t + C)^{\frac{6n}{\alpha(n+2)}} dz^2 \quad (26)$$

5.Physical Properties of the Model

The Physical and kinematical parameters which are important to discuss the physics of the cosmological model (20) are,

The spatial volume $V = ABC = a^3$

$V = (\alpha t + C)^{\frac{3}{\alpha}}$

* Hubble Parameter $= \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$

$= \frac{1}{\alpha t + C}$

* Deceleration parameter $= \frac{d}{dt}\left(\frac{1}{H}\right) - 1$

$= \alpha - 1$

* Expansion scalar $\theta = 3H = \frac{3}{\alpha t + C}$

The anisotropy parameter $\Delta = 0$

Shear scalar $\sigma^2 = 0$

Jerk Parameter $J = q + 2q^2 - \frac{\dot{q}}{H} = 2\alpha^2 - 3\alpha + 1S$

The Physical Parameters, matter energy density, holographic dark energy density and Holographic dark energy pressure in the model are given by following equation.

$$\rho_m = \frac{\rho_0}{A^2 C} = \frac{\rho_0}{(\alpha t + C)^{\frac{6}{(n+2)\alpha}} (\alpha t + C)^{\frac{3n}{(n+2)\alpha}}} = \frac{\rho_0}{(\alpha t + C)^{\frac{3}{\alpha}}}$$

$$\therefore \rho_\lambda = \frac{F_0}{K^2}\left\{\frac{9(m-2\alpha)(n^2+2)+18\alpha(n^2+2n+3)-3m\alpha(n+2)^2}{(n+2)^2(m-2\alpha)} - 3m - 2m(m-\alpha)\right\}(\alpha t + C)^{\frac{m}{\alpha}-2} - \frac{6h^2F_0}{K^2[m(n+2)-6]}(\alpha t + C)^{\frac{m(n+2)-6}{(n+2)\alpha}} - \frac{\rho_0}{(\alpha t + C)^{\frac{3}{\alpha}}}$$

$$P_\lambda = \frac{F_0}{K^2}\left\{\frac{3(\alpha 3)}{n+2} + \frac{6\alpha[3(n^2+2n+3)+\alpha(n+2)^2]}{(n+2)^2(m-2\alpha)}\right\}(\alpha t + C)^{\frac{m}{\alpha}-2} + \frac{F_0}{K^2}\left\{h^2 + \frac{6h^2}{m(n+2)-6}\right\}(\alpha t + C)^{\frac{m(n+2)-6}{(n+2)\alpha}} + \frac{3m(n+3)F_0}{(n+2)K^2}(\alpha t + C)^{\frac{m}{\alpha}}$$

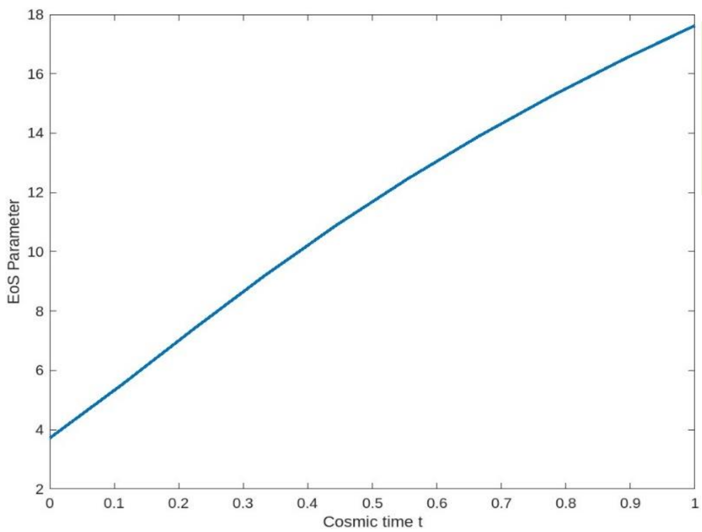
Matter density parameter $\Omega_M = \frac{\rho_0}{3H^2} = \frac{\rho_0}{3(\alpha t+C)^{\frac{3}{\alpha}} \mathrm{X} \frac{1}{(\alpha t+C)^2}} = \frac{\rho_0}{3(\alpha t+C)^{\frac{3}{\alpha}-2}}$

Dark energy density parameter $\Omega_\lambda = \frac{\rho_\lambda}{3H^2}$

$$= \frac{F_0}{3K^2}\left\{\frac{9(m-2\alpha)(n^2+2)+18\alpha(n^2+2n+3)-3m\alpha(n+2)^2}{(n+2)^2(m-2\alpha)} - 3m - 2m(m-\alpha)\right\}(\alpha t + C)^{\frac{m}{\alpha}} - \frac{6hF_0}{K^2[m(n+2)-6]}(\alpha t + C)^{2+\frac{m}{\alpha}-\frac{6}{(n+2)\alpha}} - \frac{\rho_0}{(\alpha t + C)^{\frac{3}{\alpha}-2}}$$

Total Energy density parameter $\Omega = \Omega_M + \Omega_\lambda$

$$= \frac{F_0}{3K^2}\left\{\frac{9(m-2\alpha)(n^2+2)+18\alpha(n^2+2n+3)-3m\alpha(n+2)^2}{(n+2)^2(m-2\alpha)} - 3m - 2m(m-\alpha)\right\}(\alpha t + C)^{\frac{m}{\alpha}} - \frac{6hF_0}{K^2[m(n+2)-6]}(\alpha t + C)^{2+\frac{m}{\alpha}-\frac{6}{(n+2)\alpha}}$$



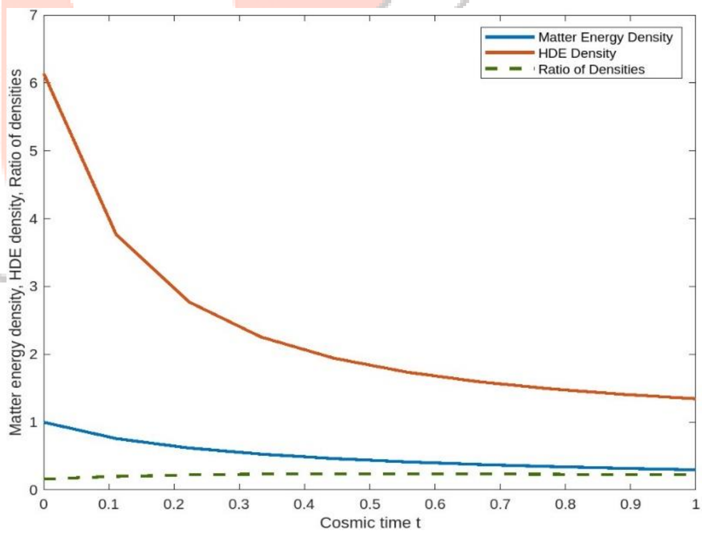
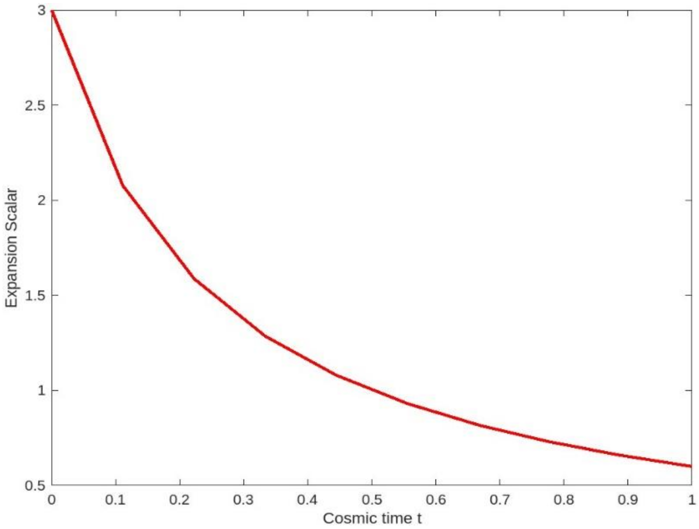


Fig. 1 Plot of matter energy density (ρ_m), HDE density (ρ_λ) Fig. 2 Plot of EoS
parameter ω_λ versus cosmic time t (Gyr)
and the ratio of densities $\frac{\rho_m}{\rho_\lambda}$ versus cosmic time t(Gyr)



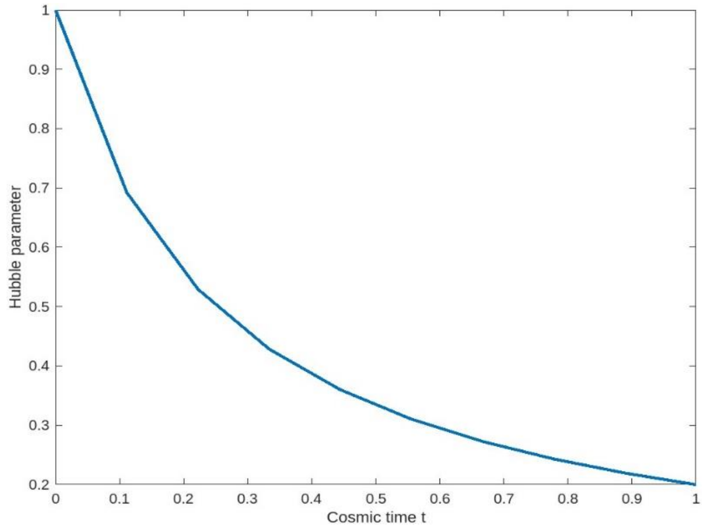
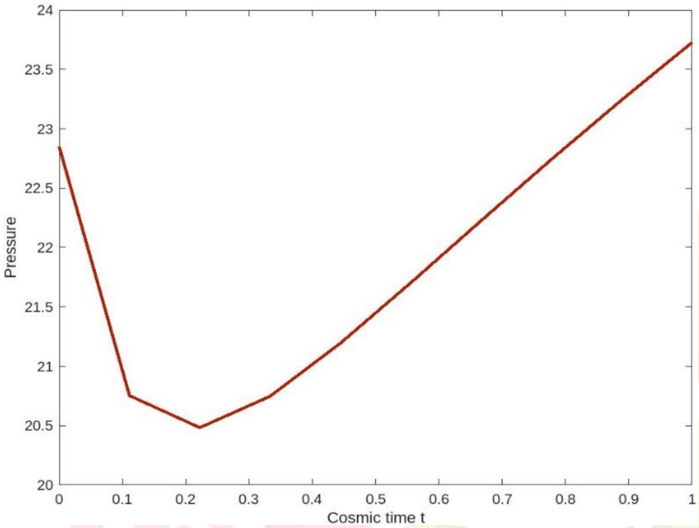


Fig. 3 Plot of Expansion scalar versus cosmic time t (Gyr) Fig. 4 Plot of Hubble parameter versus cosmic time t (Gyr)



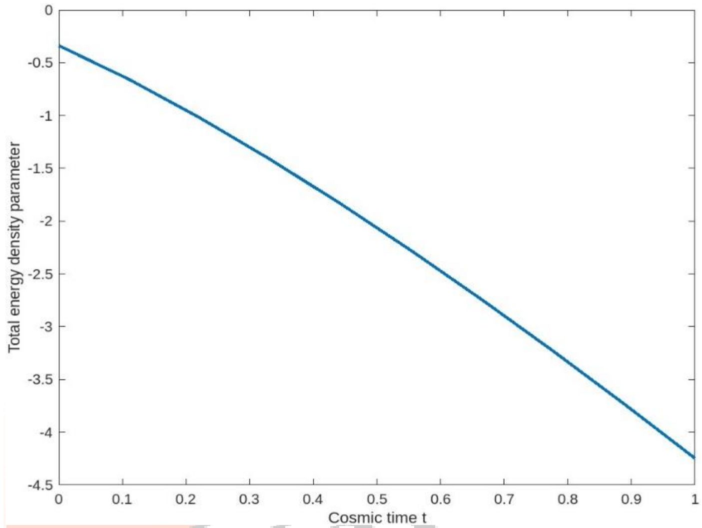


fig. 5 Plot of Pressure versus cosmic time t (Gyr) Fig. 6 Plot of total energy density parameter versus cosmic time t (Gyr)

6.Conclusion

Here, we have studied Bianchi type-III universe filled with matter and Holographic dark energy cosmological model in $f(R)$Theory of gravity. The sign of DP (q) shows whether the model either accelerates or decelerates. For $\alpha > 1$, deceleration parameter $q > 0$, therefore the model represents a decelerating model. For $0 < \alpha < 1$, we get $-1 \leq q < 0$ which describes an accelerating model of the universe. The behaviour of matter energy density ρ_m,dark energy density ρ_λ are shown in fig.1.The fig. shows that the matter energy density and dark energy density approaches to a non zero constant.

The volume V becomes infinite as $t \rightarrow \infty$ i. e. volume increases with time and vice-versa. From Fig.2we can conclude that EoS Parameter approaches to ∞ with the increasing value of t. Fig. 3 and Fig.4 reveals that the scalar expansion θ and Hubble's parameter H assume constant value as $t \rightarrow \infty$ and hence universe expands endlessly with the influence of dark energy. Fig.5 and Fig.6 shows the behaviour of the pressure and total energy density parameter in this case lim $t \rightarrow \infty$ $(\frac{\sigma}{\theta}) = 0$, hence derived model approaches to isotropy.

References

[1]S Capozziello, A Stabile, and A Troisi, Class Quantum Grav. 24, 2153 (2007) https://doi.org/10.1088 0264-9381/24/8/013

[2] S. Thomas, Phys. Rev. Lett. 89 (2002) 081301.

[3] S. D. Hsu, Phys. Lett. B 594 (2004).

[4] H. V. Peiris et al., First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Inflation, Astrophys. J. Suppl. Ser. 148, 213 (2003).

[5] M. Tegmark et al., Cosmological parameters from SDSS and WMAP, Phys. Rev. D 69, 103501 (2004).

[6] V. Springel et al., The large-scale structure of the Universe, Nature 440, 1137 (2006).

[7] P. Astier et al., The Supernova Legacy Survey: measurement of ΩΛ and ω from the first-year data set, Astron. Astrophys. 447, 31 (2006).

[8] A. G. Riess et al., New Hubble Space Telescope Discoveries of Type Ia Supernovae at z ≥ 1: Narrowing Constraints on the Early Behavior of Dark Energy*, Astrophys. J. 659, 98 (2007).

[9] D. N. Spergel et al., Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology, Astrophys. J. Suppl. Ser. 170, 377 (2007).

[10] P. A. R. Ade et al., Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. 571, A16 (2014).

[11] E. Komatsu et al., Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, Astrophys. J. Suppl. Ser. 180 330 (2009).

[12] S. Capozziello and M. De Laurentis, Extended theories of gravity, Phys. Rep. 509, 167 (2011).

[13] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, – 24 – 1984).

[14] S. Nojiri and S. D. Odintsov, Introduction to modified gravity and gravitational alternative for dark energy, Int. J. Geom. Methods Mod. Phys. 04, 115 (2007).

[15] M.V.Santhi,V.U.M.Rao and Y.Aditya Can.J.Phys.96: 55-61 (2018)

[16] Thomas P. Sotiriou and V. Faraoni, f(R) theories of gravity, Rev. Mod. Phys. 82, 451 (2010).

[17] Y. F. Cai, S. Capozziello, M. De Laurentis, and E. N. Saridakis, f(T) teleparallel gravity and cosmology, Rep. Prog. Phys. 79, 106901 (2016).

[18] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Is cosmic speed-up due to new gravitational physics?, Phys. Rev. D 70, 043528 (2004).

[19] G. Allemandi, A. Borowiec, and M. Francaviglia, Accelerated cosmological models in Ricci squared gravity, Phys. Rev. D 70, 103503 (2004).

[20] X. Meng and P. Wang, Cosmological evolution in 1/R-gravity theory, Class. Quantum Gravit. 21, 951 (2004).

[21] S. M. Carroll, A. De Felice, V. Duvvuri, D. A. Easson, M. Trodden, and M. S. Turner, Cosmology of generalized modified gravity models, Phys. Rev. D 71, 063513 (2005).

[22] S.D.Katore and R.J.Baxi Indian J.Phys. (2019)

[23] M. Ostrogradski, Mem. Ac. St. Petersbourg VI 4, 385 (1850).

[24] R. P. Woodard, Avoiding dark energy with 1/R modifications of gravity, The Invisible Universe: Dark Matter and Dark Energy. Lecture Notes in Physics, 720, Springer, astro-ph/0601672 (2007).

[25] M. E. Soussa and R. P. Woodard, The force of gravity from a Lagrangian containing inverse powers of the Ricci scalar, Gen. Relativ. Gravit. 36, 855 (2004).

[26] V. Faraoni, Stability of modified gravity models, Phys. Rev. D 72, 124005 (2005).

[27] E. E. Flanagan, The conformal frame freedom in theories of gravitation, Class. Quantum Gravit. 21, 3817 (2004)

[28] T. Clifton and J. D. Barrow, The power of general relativity, Phys. Rev. D 72, 103005 (2005).

[29] T. Multamaki and I. Vilja, Cosmological expansion and the uniqueness of the gravitational action, Phys. Rev. D 73, 024018 (2006).

[30] T. Multam¨aki and I. Vilja, spherically symmetric solutions of modified field equations in f(R) theories of gravity, Phys. Rev. D 74, 064022 (2006).

[31] T. Moon, Y. S. Myung, E. J. Son, f(R) black holes, Gen. Relativ. Gravit. 43, 3079 (2011).

[32] J. A. R. Cembranos, A. de la Cruz-Dombriz, P. Jimeno Romero, Kerr–Newman black holes in f(R) theories, Int. J. Geom. Methods Mod. Phys. 11, 1450001 (2014).
[33] T. Multam¨aki and I. Vilja, Static spherically symmetric perfect fluid solutions in f(R) theories of gravity, Phys. Rev. D 76, 064021 (2007).
[34] S. H. Mazharimousavi, M. Halilsoy, T. Tahamtan, Constant curvature f(R) gravity minimally coupled with Yang–Mills field, Eur. Phys. J. C 72, 1958 (2012).
[35] S. H. Mazharimousavi, M. Halilsoy, Black hole solutions in f(R) gravity coupled with non-linear Yang–Mills field, Phys. Rev. D 84, 064032 (2011).
[36] M. E. Rodrigues, E.L. Junior, G.T. Marques, V.T. Zanchin, Regular black holes in f(R) gravity coupled to nonlinear electrodynamics, Phys. Rev. D 94(2), 024062 (2016).
[37] R. A. Hurtado, J. R. Arenas, Spherically symmetric and static solutions in f(R) gravity coupled with EM fields, Phys. Rev. D 102, 104019, (2020).
[38] S. Capozziello, M. De laurentis, A. Stabile, Axially symmetric solutions in f(R)-gravity, Class. Quantum Gravity 27, 165008 (2010).
[39] S. H. Hendi, B. E. Panah, S. M. Mousavi, Some exact solutions of F(R) gravity with – 26 – charged (a)dS black hole interpretation, Gen. Relativ. Gravit. 44, 835 (2012).
[40] Z. Amirabi, M. Halilsoy, S. H. Mazharimousavi, Generation of spherically symmetric metrics in f(R) gravity, Eur. Phys. J. C 76(6), 338 (2016).
[41] L. Sebastiani and S. Zerbini, Static spherically symmetric solutions in F(R) gravity, Eur. Phys. J. C 71, 1591 (2011).
[42] G. ’t Hooft, Dimensional reduction in quantum gravity, preprint (2009), arXiv: grqc/9310026.
[43] M. Li, A model of holographic dark energy, Phys. Lett. B 603 (2004) 1.
[44] W. Fischler and L. Susskind, Holography and cosmology, preprint (1998), arXiv: hepth/9806039.
[45] L. N. Granda and A. Oliveros, Infrared cut-off proposal for the holographic density, Phys. Lett. B 669 (2008) 275–277.
[46] D. R. K. Reddy et al., Bianchi type-V bulk viscous string cosmological model in Saez-Ballester scalar–tensor theory of gravitation, Astrophys. Space Sci. 349 (2014) 473–477.
[47] V. U. M. Rao and U. Y. D. Prasanthi, Bianchi type-I and -III modified holographic Ricci Dark energy models in Saez-Ballester theory, Eur. Phys. J. Plus 132 (2017) 64.
[48] M. V. Santhi et al., Anisotropic magnetized holographic Ricci dark energy cosmological models, Can. J. Phys. 95 (2017) 381–392.
[49] M. V. Santhi et al., Bianchi type-V I0 modified holographic Ricci dark energy model in a scalar–tensor theory, Can. J. Phys. 95 (2017) 179–183.
[50] M. P. Hobson, G. P. Efstathiou, A. N. Lasenby,” General Relativity, An Introduction for Physicists”, Cambridge University Press, 2006.
[51] H.A. Buchdahl,” Non-linear Lagrangians and cosmological theory”, 1970, Mon. Not. Roy. Astr. Soc. 150, 1
[52]K.Uddin.J.E.Lidsey, and R.Tavakol.Class.QuantumGrav.24,3951(2007)
[53] M.S.Berman.NuovoCimento B,74.182(1983).