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SEIR MODEL FOR TRANSMISSION OF VECTOR BORNE DISEASE MALARIA INCORPORATING INFECTED MIGRANTS: A CASE STUDY OF GOA

[1]DR.NEETA MAZUMDAR, [2]KIM CHELSEA CAEIRO
[1]Coordinator, [2]Post graduate Student
[1]Department of Mathematics,
[1]Government College of Arts, Science and Commerce, Quepem, India

Abstract: Malaria continues to pose a serious threat to public health challenge, especially in tropical and subtropical areas. The paper deals with understanding Malaria transmission dynamics and evaluating interventional strategies using SEIR (Susceptible-Exposed-Infected-Recovered) model incorporating infected migrants with an infectivity factor θ which is well-suited for studying infectious diseases with latent periods, such as Malaria. The study area is the state of Goa, a popular tourist destination, where domestic travelers act as potential carriers of Malaria, influencing transmission dynamics and disease control efforts. A system of non-linear Ordinary Differential Equations is used and stability is analysed using the Jacobian matrix. It is seen that increasing the treatment rate and reducing transmission can significantly mitigate the disease burden. The data is then validated by using graphical method.

***Index Terms* – SEIR model, Jacobian matrix, Stability, Eigen Values, Infectivity factor, migrants**

I. Introduction

Malaria is still a major public health issue, especially in tropical and subtropical areas where Anopheles mosquitoes can spread the Plasmodium parasite due to favorable climatic circumstances. The SEIR (Susceptible-Exposed-Infectious-Recovered) framework is one example of a mathematical model that is essential for comprehending the dynamics of malaria transmission and developing successful intervention tactics. However, traditional SEIR models frequently make the assumption that the populace is closed off, ignoring how migration and human mobility affect the transmission of disease.

In many Malaria-endemic regions, population movement-whether due to economic migration, seasonal labor, tourism, or displacement-introduces infected individuals into otherwise low-risk areas, altering transmission dynamics. Incorporating infected migrants into the SEIR model provides a more realistic representation of Malaria spread, accounting for the introduction of new infections, changes in local disease prevalence, and potential challenges to Malaria elimination efforts.

This extended SEIR model includes an additional compartment for infected migrants, allowing for the assessment of their impact on local transmission. By considering migration rates, infection prevalence among incoming individuals, and interactions with the resident population, the model can help predict outbreaks, evaluate control measures, and inform public health policies aimed at minimizing the risk of Malaria resurgence.

2 Mathematical Model

The modelling equations are given as follows:

$$\frac{dS}{dt} = b - \mu S - \beta SI + \lambda E - \theta m_I \beta S$$

$$\frac{dE}{dt} = \beta SI + \theta m_I \beta S - (\lambda + \mu + \alpha)E$$

$$\frac{dI}{dt} = \alpha E - (\mu_1 + \mu + \gamma)I + m_I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

where,
s(t) : susceptible population
e(t) : exposed populations
i(t) : infected populations
r(t) : recovered populations
β : rate with which susceptible population goes to the exposed compartment
α : rate with which exposed population goes to the infected compartment
γ : rate at which infectious population goes to recovered compartment
λ : rate at which exposed population gets some treatment and again becomes susceptible
b: birth rate
μ : death rate
μ_1 : death rate due to malaria
θ : infectivity factor
m_I : infected migrants

Consider the above system of equations as (1)
The boundary values for the model are given in the table below:

Table 1: The Epidemiological parameters of Malaria for Goa including infectivity factor θ and infected migrants m_I

N(0)	15,83,000
s (Susceptible)	15,10,000
e (Exposed)	63,000
i (Infected)	5,200
r (Recovered)	4800
S	0.9546
E	0.0398
I	0.275
R	0.0030
β	0.275
α	0.083
γ	0.158
λ	0.04

b	0.014
μ	0.008
μ_1	0.00025
θ	0.2
m_I	287

3 Equilibrium Points:

We now have points for both endemic and disease-free equilibrium.

$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$

$b - \mu S - \beta SI + \lambda E - \theta m_I \beta S = 0 \ldots..(2)$

$\beta SI + \theta m_I \beta S - (\lambda + \mu + \alpha)E = 0 \ldots..(3)$

$\alpha E - (\mu_1 + \mu + \gamma)I + m_I = 0 \ldots \ldots 4)$

$\gamma I - \mu R = 0 \ldots..(5)$

3.1 Disease Free Equilibrium Points:

From Equation (2),

$S = \frac{b + \lambda E}{\beta I + \mu + \theta m_I \beta}$

But at the Disease-Free State,

$\beta = 0, E = 0, m_I = 0$

$\Longrightarrow S = \frac{b}{\mu}$

From equation (3):

$E = \frac{\beta SI + \theta m_I \beta S}{\lambda + \mu + \alpha}$

But $\beta = 0$

$\Longrightarrow E = 0$

From equation (4):

$I = \frac{\alpha E - m_I}{\mu_1 + \mu + \gamma}$

But $E = 0, m_I = 0$

$\Longrightarrow I = 0$

From equation (5):

$R = \frac{\gamma I}{\mu}$

Since $I = 0 \ \Longrightarrow R = 0$

Thus we have Disease Free Equilibrium Point is $E_0 = \left(\frac{b}{\mu}, 0,0,0\right)$.

3.2 Endemic Equilibrium Point

Now rewriting equations to find endemic equilibrium point:

$$0 = b - \mu S - \beta SI + \lambda E - \theta m_I \beta S \quad (2)$$

$$0 = \beta SI - (\lambda + \mu + \alpha)E + \theta m_I \beta S \quad (3)$$

$$0 = \alpha E - (\mu_1 + \mu + \gamma)I + m_I \quad (4)$$

$$0 = \gamma I - \mu R \quad (5)$$

From equation (5):
$\gamma I^* = \mu R^*$
From equation (4):
$E^* = \frac{(\mu_1 + \mu + \gamma)I^* - m_I}{\alpha}$
From equation (3):
$\beta S^* I^* + \theta m_I \beta S^* - (\lambda + \mu + \alpha)E^* = 0$
Rearrange for S^* :
$S^* = \frac{(\lambda + \mu + \alpha)E^*}{\beta I^* + \theta m_I \beta}$
Substituting E^* we get:
$S^* = \frac{(\lambda + \mu + \alpha)(\mu_1 + \mu + \gamma)I^* - (\lambda + \mu + \alpha)m_I}{\alpha(\beta I^* + \theta m_I \beta)}$
Now solving for I^*,

$$I^* = \frac{b\alpha(\beta + \theta m_I \beta)}{\mu(\lambda + \mu + \alpha)(\mu_1 + \mu + \gamma) + \beta(\lambda + \mu + \alpha)(\mu_1 + \mu + \gamma) + \theta m_I \beta(\lambda + \mu + \alpha)}$$

$$S^* = \frac{(\lambda + \mu + \alpha)(\mu_1 + \mu + \gamma)I^* - (\lambda + \mu + \alpha)m_I}{\alpha(\beta I^* + \theta m_I \beta)}$$

$$E^* = \frac{(\mu_1 + \mu + \gamma)I^* - m_I}{\alpha}$$

$$R^* = \frac{\gamma I^*}{\mu}$$

These give the endemic equilibrium points of the system.

4 Stability of Equilibrium points

4.1 Disease Free Equilibrium point and its Stability:

$S = \frac{b}{\mu}, I = 0$ then we get,

$$J(E_0) = \begin{bmatrix} -\mu & \lambda & -\beta\frac{b}{\mu} & 0 \\ 0 & -(\lambda + \mu + \alpha) & \beta\frac{b}{\mu} & 0 \\ 0 & \alpha & -(\mu_1 + \mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Then the Characteristic equation is given by $\det[J(E_0) - \lambda' I_4] = 0$

$$\Rightarrow \begin{vmatrix} -\mu - \lambda' & \lambda & -\beta\frac{b}{\mu} & 0 \\ 0 & -(\lambda + \mu + \alpha) - \lambda' & \beta\frac{b}{\mu} & 0 \\ 0 & \alpha & -(\mu_1 + \mu + \gamma) - \lambda' & 0 \\ 0 & 0 & \gamma & -\mu - \lambda' \end{vmatrix} = 0$$

Let $\mu_1 + 2\mu + \gamma + \lambda + \alpha = a_1$

$\mu_1\lambda + \mu_1\mu + \mu_1\alpha + \mu\lambda + \mu^2 + \alpha\mu + \lambda\gamma + \mu\gamma + \alpha\gamma - \alpha\beta\frac{b}{\mu} = a_2$

Then the above equation becomes,
$(\lambda')^2 + a_1\lambda' + a_2 = 0$

If it has two negative real roots, then $\det[J(E_0) - \lambda' I_4] = 0$ has all four eigenvalues are negative, then the Disease-Free Equilibrium point is Locally Asymptotically State by Stability Criterion.

4.2 Endemic Equilibrium point and its Stability:

Now finding Jacobian and Eigen values for Endemic Equilibrium state:

$$J(E^{**}) = \begin{bmatrix} -\beta I^* - \mu - \theta m_I \beta & \lambda & -\beta S^* & 0 \\ \beta I^* + \theta m_I \beta & -(\lambda + \mu + \alpha) & \beta S^* & 0 \\ 0 & \alpha & -(\mu_1 + \mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Since,

$\det[J(E^{**}) - \lambda' I_4] = 0$

$$\Rightarrow \begin{vmatrix} -\beta I^* - \mu - \theta m_I \beta - \lambda' & \lambda & -\beta S^* & 0 \\ \beta I^* + \theta m_I \beta & -(\lambda + \mu + \alpha) - \lambda' & \beta S^* & 0 \\ 0 & \alpha & -(\mu_1 + \mu + \gamma) - \lambda' & 0 \\ 0 & 0 & \gamma & -\mu - \lambda' \end{vmatrix} = 0$$

Then $\det[J(E^{**} - \lambda' I_4)]$ becomes $\lambda'^3 + A\lambda'^2 + B\lambda' + C = 0$

where $A = \mu + \beta I^* + \lambda + \theta m_I \beta + \mu + \alpha + \mu_1 + \mu + \gamma$

$B = \lambda\mu + \mu^2 + \alpha\mu + \mu_1\mu + \mu^2 + \gamma\mu + \beta I^*\lambda + \beta I^*\mu + \beta I^*\alpha + \beta I^*\mu_1 + \beta I^*\mu + \beta I^*\gamma + \theta m_I\beta\lambda + \theta m_I\beta\mu + \theta m_I\beta\alpha + \lambda\mu_1 + \lambda\mu + \lambda\gamma + \mu_1\mu + \mu^2 + \theta m_I\beta\mu_1 + \theta m_I\beta\mu + \theta m_I\beta\gamma + \mu\gamma + \alpha\mu_1 + \alpha\mu + \alpha\gamma - \beta S^*\alpha - \beta I^*\lambda - \theta m_I\beta\lambda$

$C = \lambda\mu_1\mu + \lambda\mu^2 + \lambda\gamma\mu + \mu_1\mu^2 + \mu^3 + \mu^2\gamma + \gamma\mu_1\mu + \alpha\mu^2 + \alpha\gamma\mu - \beta S^*\alpha\mu + \beta I^*\lambda\mu_1 + \beta I^*\lambda\mu + \beta I^*\lambda\gamma + \beta I^*\mu\mu_1 + \beta I^*\mu^2 + \beta I^*\mu\gamma + \beta I^*\alpha\mu_1 + \beta I^*\alpha\mu + \beta I^*\alpha\gamma - \beta^2 I^* S^*\alpha + \theta m_I\beta\lambda\mu_1 + \theta m_I\beta\lambda\mu + \theta m_I\beta\lambda\gamma + \theta m_I\beta\mu\mu_1 + \theta m_I\beta\mu^2 + \theta m_I\beta\mu\gamma + \theta m_I\beta\alpha\mu_1 + \theta m_I\beta\alpha\mu + \theta m_I\beta\alpha\gamma - \theta m_I\beta^2 S^*\alpha - \beta\lambda I^*\mu_1 - \beta I^*\mu\lambda - \beta I^*\gamma\lambda - \theta m_I\beta\mu_1\lambda - \theta m_I\beta\mu\lambda - \theta m_I\beta\gamma\lambda + \beta^2 S^*\alpha I^* + \theta m_I\beta^2 S^*\alpha$

5 Numerical Solutions:

5.1 Stability for Disease free Equilibrium points:

Substituting the parameter values and boundary conditions given in the table and solving the Quadratic equation using the Quadratic formula we obtain the two eigen values from the characteristic equation:

$\lambda_1 = 0.0520, \lambda_2 = -0.3493$

Since one of the eigen values $\lambda_1 = 0.0520$ is positive, the disease-free equilibrium is unstable. Confirming that a Malaria outbreak will occur if the disease is introduced into the population as small perturbations near the disease-free equilibrium will grow over time rather than decay.

This indicates the need to prevent further Malaria transmission and work toward disease elimination. Hence the authorities ought to implement intervention strategies like vector control or treatment in order to reduce transmission.

5.2 Stability for Endemic Equilibrium Points:

The eigen values obtained upon numerical calculations are $-15.79, -0.233, -0.066$, -0.008.

Since all eigenvalue's posses' negative real components, the endemic equilibrium is asymptotically stable locally. This implies that small deviations from equilibrium will decay over time, ensuring a stable endemic state for Malaria transmission.

A stable endemic equilibrium indicates that Malaria will remain present in the population unless proactive measures are taken. Continuous strategies such as mosquito control, widespread treatment initiatives, and health awareness campaigns are crucial to shifting the system toward disease elimination. Additionally, the role of infected migrants cannot be overlooked, as they contribute to the continuous introduction of new cases, reinforcing the need for surveillance and targeted health measures to prevent further transmission.

6 Graphical Solution:

To trace the solution of the proposed model, Table 2 gives the following estimations of the coefficients. The model fitting is done using the Python program. Here $t = 0$ refers to the starting date of the timeline i.e 1 March 2025.

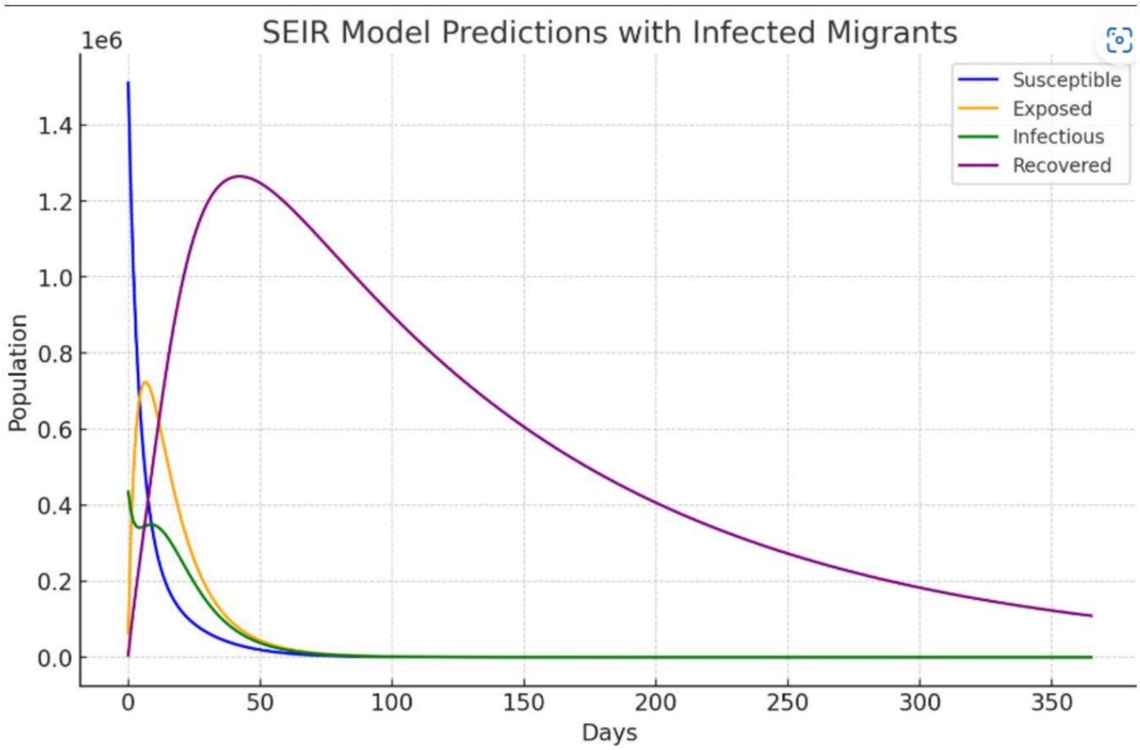

Figure 1: SEIR model predictions for the transmission of Malaria incorporating infected migrants

The graph gives us the predictions for the number of susceptible, exposed, infectious and recovered population for the state of Goa.

Results and Discussion:

The SEIR model simulation incorporating infected migrants provides key in- sights into the transmission dynamics of Malaria in Goa. The susceptible population initially starts at a high value and rapidly declines, indicating a high transmission rate likely due to the introduction of infected migrants. The exposed population rises sharply, peaking around 20 30 days, reflecting the latent phase before individuals becomes infectious. The infectious population follows with a peak within 30 50 days, marking the period of highest disease burden, before gradually declining as individuals recover. The recovered population steadily increases and surpasses all other categories, showing that a significant portion of the population eventually develops immunity or exits the infectious cycle. The sharp initial increase in cases highlights the role of infected migrants in driving transmission, emphasizing the need for early intervention strategies such as screening and vector control. The peak transmission period suggests that immediate public health initiatives are crucial within the first 50 days to curb the spread. Over time, the declining susceptible population and the rising recovered population indicate a gradual reduction in cases, assuming no new significant sources of infection emerge. Overall, the model predicts a temporary but notable Malaria outbreak, underscoring the importance of timely detection, effective health policies, and preventive strategies to mitigate the impact in Goa.

Conclusion:

This study highlights the significant role of infected migrants in sustaining Malaria transmission and the challenges in achieving disease elimination. The SEIR model, extended to include migration, confirms that Malaria will persist at an endemic level unless proactive intervention measures are implemented. While traditional control measures such as vector management, improved healthcare access, and widespread treatment programs are vital, the impact of migration-related transmission cannot be overlooked. Hence addressing this issue through targeted screenings and preventive strategies is equally important. A comprehensive, data-driven approach integrating disease surveillance, healthcare policies, and mathematical modeling will be essential in steering the system toward Malaria eradication.

REFERENCES:

1. Osman, Mojeeb and Kwasi Adu, Isaac and Yang, Cuihong. (2017). A Simple SEIR Mathematical Model of Malaria Transmission. Asian Re- search Journal of Mathematics. 7. 1-22. 10.9734/ARJOM/2017/37471.

2. Khatua, A. (1970, January 1). Mathematical modelling of some infec- tious diseases and their control strategies. Shodhganga@INFLIBNET: Mathematical modelling of some infectious diseases and their control strategies. http://hdl.handle.net/10603/451451

3. Mandal S, Sarkar RR, Sinha S. Mathematical models of malaria a re- view. Malar J. 2011 Jul 21;10:202. doi: 10.1186/1475-2875-10-202. PMID: 21777413; PMCID: PMC3162588.

4. Al Basir, F., and Abraha, T. (2023). Mathematical modelling and optimal control of malaria using awareness-based interventions. Math- ematics, 11(7), 1687.

5. Ministry of Health & Family Welfare-Government of India. (n.d.-a). Malaria. National Center for Vector Borne Diseases Control (NCVBDC).

6. Mojeeb AL-Rahman EL-Nor Osman. (n.d.). Simple Mathematical Model for Malaria Transmission. Journal of Advances in Mathematics and Computer Science.

7. Gupta, A. (1970, January 1). Mathematical Modeling and Analysis of Communicable Diseases and their control. Shodhganga@INFLIBNET: Mathematical Modeling And Analysis Of Communicable Diseases And Their Control. http://hdl.handle.net/10603/357132

8. Anikat. (n.d.). A study of infectious diseases through Mathemati- cal Models. Shodhganga@INFLIBNET: A study of infectious diseases through mathematical models. http://hdl.handle.net/10603/593417

9. Ndamuzi, E., & Gahungu, P. (2021, September 29). Mathematical Modeling of Malaria Transmission Dynamics Case of Burundi. SCIRP. https://www.scirp.org/journal/paperinformation?paperid=112452

10. Centers for Disease Control and Prevention. (n.d.). How malaria spreads. Centers for Disease Control and Prevention. https://www.cdc.gov/malaria/causes/index.html