IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Euclidean Geometry

NISHANT KUMAR#1 AND ABHISHEK KUMAR#2

1.Asst.Professor, Univ.Dept. of Mathematics, B.R.A.Bihar University ,Muzaffarpur.

2.Research Scholar, Univ.Dept. of Mathematics, B.R.A.Bihar University,Muzaffarpur.

Abstract

In Greek Geometry, we would further go through the Deductive method, Regular Polyhedra, Ruler and compass construction, conic sections, Higher degree curves. Book I to VI and further book X to XIII of Euclid's Element have been studied in depth. We have seen how Euclid's Elements remained not only the core of mathematical education, but also at the heart of western culture, we have looked into scholarly and personal discovery of Euclid's text. Book I to IV and VI discuss plane geometry. In this sections many results about plane geometry are proved e.g. if a triangle has two equal angles, then the angles subtended by the sides are equal. Book V and VII to X deal with number theory, with numbers treated geometrically via their representation as line segments with various lengths. Book XI- XIII concern solid geometry. A typical result is the 1:3 ratio between the volume of a cone and cylinder with the same height and base. This result has been found here.

Introduction : -

Euclidean Geometry is constructive. Euclid first used proof by contradiction. Euclidean geometry also allows the method of superposition, in which a figure is transferred to another point in space. Euclid treated the conics as sections of the three different types of cones (right angled, acute-angled, and obtuse-angled). The ellipse we also obtained as sections of any cones and of a circular cylinder.

Euclid lived in Alexandria about 300 B.C. and trained students there, through his own education was probably acquired in Plato's Academy. Euclid's work is actually an organisation of the separate discoveries of the classical Greeks; this is clear from a comparison of its contents with what is known of the earlier work.

Euclid's most famous work is the Elements. The major sources of the material in this work can generally be identified despite our slim knowledge of the classical period. Euclid owes much of his material to the Plantonists with whom he studied. The particular choices of axioms, the arrangement of the theorems, and some proofs, are his, as are the polish and the rigor of the demonstrations. The form of presentation of proof has, however, already been noted in Autolycus and was pretty surely used by others who preceded Euclid. Euclid was unquestionable a great mathematician. His other writings support this judgement despite any question as to how much of the Elements is original with Euclid. There are no extant manuscripts written by Euclid himself. Hence his writings have had to be reconstructed from numerous recensions, commentaries and remarks by the other writers.

The Background of Euclid's Elements : -

Proclus says that Euclid put into his Elements many of Eudoxus theorems, perfected theorems of Theaetetus, and made irrefragable demonstrations of results loosely proved by his predecessors. Among the most important must have been those by Heron (C. 100 B.C. – C.A.D. 100), Porphyry (3rd cent A.D.) and Pappus (end of 3rd cent A.D.). Presumably Euclid's book was so good that it superseded the ones supposed to have been written by Hippocrates of Chios and by the Platonists Leon and Theudius. All of the English and Latin editions of Euclid's Elements stem originally from Greek manuscripts. These were Theon of Alexandria's recension of Euclid's Elements (end of the 4th Cent A.D.) , copies of Theon's recension, written versions of lectures by Theon, and one Greek manuscript found by Francois Peyrard (1760-1822) in the Vatican library. This tenth-century manuscript is a copy of an edition of Euclid that precedes Theon's. Hence the historians J.L. Heiberg and Thomas L. Heath have used principally this manuscript for their study of Euclid, comparing it of course with the other available manuscripts and commentaries. These are also Arabic translations of Greek works and Arabic commentaries presumably based on Greek manuscripts no longer available. These, two, have been used to decide what was in Euclid's Elements. But the Arabic translation and revisions are on the whole inferior to the Greek manuscripts. Ofcourse, the reconstruction, since it is based on so many sources, leaves some matters in doubt. The purpose of Euclids Elements is in question. It is considered by some as a treatise for learned mathematicians and by others as a text for students. Proclus gives some weight to the latter belief.

In view of the length and incomparable historical importance of this work, we shall devote several sections of this chapter to a review of and comment on the contents. Since we still learn Euclidean geometry, we may be some what surprised by the contents of the Elements. The high school versions most widely used during our century are patterned on Legendre's modification of Euclid's work. Some algebra used by legendre is not in the Elements, though, as we shall see, the equivalent geometrical material is.

Euclidean Geometry : -

Euclidean geometry is a mathematical system attributed to the Alexandrian Greek mathematician Euclid, which be described in his text book on geometry; the elements. Euclid's method consists in assuming a small set of intuitively appealing axioms, and deducing many other propositions (theorem) from these. Although many of Euclid's results had been stated by earlier mathematicians. Euclid was the first to show how these propositions could fit into a comprehensive deductive and logical system. The Elements begins with the plane geometry still taught in secondary school as the first axiomatic system and the first examples of formal proof. It goes on to the solid geometry of three dimensions. Much of the elements states results of what are now called algebra and number theory, explained in geometrical language.

For more than two thousand years, the adjective "Euclidean" was unnecessary because no other sort of geometry has been conceived. Euclid's axioms seemed so intuitively obvious (with the possible exception of the parallel postulate) that any theorem proved from them was deemed true in an absolute, often metaphysical, sense. Today, however, many other self consistent non-Euclidean geometries are known, the first ones having been discovered in the early, 19^{th} century. An implication of Albert Einstein's theory of general relativity is that physical space itself is not Euclidean, and Euclidean space is a good approximation for it only where the gravitational field is weak.

The Elements are mainly a systematization of earlier knowledge of geometry. It superiority over earlier treatments was rapidly recognized, with the result that there was little interest in preserving the earlier ones, and they are now nearly all lost.

There are 13 total books in the Elements : -

Book I – IV and VI discuss plane geometry ; Many results about plane figures are proved eg. If a triangle has two equal angles, then the sides subtended by the angles are equal. The Pythagorean theorem is proved.

Book V and VII – X deal with number theory, with numbers treated geometrically via their representation as line segments with various lengths. Notations such as prime numbers and rational and irrational numbers are introduced. The infinitude of prime numbers is proved.

Book XI-XIII concern solid geometry. A typical result is the 1:3 ratio between the volume of a cone and a cylinder with the same height and base.

Book I begins with the definitions of the concepts to be used in the first part of the work. We shall note only the most important ones; these are numbered as in Heath's edition.

Axioms : -

The basic facts which are taken for granted, without proof are called axioms.

Sometimes axioms are intuitively evident as it clear from the following examples.

(i) Halves of equals are equal
(ii) $a > b$ and $b > c \Rightarrow a > c$
(iii) The whole is equal to the sum of its parts and greater than any of its parts. In the development of plane geometry, we make some axioms and then deduce results by logical reasoning.

Theorems : -

The conclusions obtained through logical reasoning based on previously proved results and some axioms constitute a statement which is known as a theorem or a proposition.

Defintions: -

(i) A point is that which has no part

(ii) A line is breadthless length. The word line means curve

(iii) The extremities of a line are points. This definition makes clear that a line or curve is always finite in length. A curve extending to infinity doesnot occur in the Elements.

(iv) A straight line is a line which lies evenly with the points on itself. In keeping with definition 3 the straight line of Euclid is our line segment. The definition is believed to be suggested by the mason's level or an eye looking along a line.

(v) A surface is that which has length and breadth only.

(vi) The extremities of a surface are lines. Hence, a surface too is a bounded figure.

(vii) A plane surface is a surface which lies evenly with the straight lines on itself.

(viii) A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.

(ix) And the end point is called the centre of the circle

(x) A diameter of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of a circle and such a straight line also bisects the circle

(xi) Parallel straight lines are straight lines which being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

The opening definitions are framed in terms of concepts that are not defined, and hence serve no logical purpose Euclid might no, have realized that the initial concepts must be undefined and was naively explaining their meaning in terms of physical concepts. Some commentators say he appreciated that the definitions were not logically helpful but wanted to explain what his terms represented intuitively so that his readers would be convinced that the axioms and postulates were applicable to these concept.

Euclid next lays down five postulates and fine common notions. He adopts the distinction already made by Aristotle, namely that the common notions are truths applicable to all sciences whereas the postulates apply only to geometry. As we have noted, Aristotle said that the postulates need not be known to be true but that their truth would be tested by whether the results deduced from them agreed with reality. Proclus even speaks of all of mathematics an hypothetical; that is it merely deduces what must follow from the assumption, whether or not the latter are true. Presumably Euclid accepted Aristotle's views concerning the truth of the postulates. However, in the subsequent history of mathematics, both the postulates and the common notions were accepted as unquestionable truths at least until the advent of non-Euclidean Geometry.

References :

1. Howard Eves; An introduction to the History of Mathematics, Saunders, 1990, ISBN- 0-03-029558-0 p. 141
2. Historical background published by Neugebauer and Sachs (1945)
3. Vander Waerden (1983), p.2
4. Heath (1925) in edition of Euclid's elements vol. 1, p.354
5. Misner, Thorne and Wheeler (1973), p.47
6. Euclid book 1;
7. Deniel Shanks (2002), Solved and Unsolved problems in Number Theory, American Mathematical Society.