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SYSTEMATIC DEVELOPMENT OF GEOMETRICAL KNOWLEDGE IN EGYPT

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Abstract

We have seen that the Egyptians approximated the area of a circle. They observed that the area of a circle of diameter. 9 units was very close to the area of a square with sides of 8 units, so that the area of circles of other diameters could be obtained by multiplying the diameter by 8/9 and then squairing it. This gives an effective approximation of π (in accuracy being with in less than one percent).

The Pyramids themselves are another indication of the sophistication of Egyptian mathematics. They knew the formula for the volume of a pyramid 1/3 times the height times the length times the width as well as of a truncated or clipped pyramid. They were also aware long before Pythagoras, of the rule that a triangle with sides 3,4 and 5 units yields a perfect right angle and Egyptian builders used ropes knotted at intervals of 3,4 and 5 units in order to ensure exact right angles for their stone work. All the above discussed achievement of the Indus valley people lead us to conclude that the knowledge of geometry and mensuration must had been in state of developed stage in the vicinity of 3000 BC to 300 BC.

Introduction

The earliest recorded beginnings of geometry can be traced to early peoples, who discovered obtuse triangles in the ancient Indus valley (see Harappan Mathematics), and ancient Babylonia (see Babylonian Mathematics) from around 3000 B.C. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas and volumes, which were developed to meet some practical need in surveying, construction, astronomy and various crafts. Among these were some surprisingly sophisticated principles, and a modern mathematician might be hard put to derive some of them without the use of calculus. For example, both the Egyptians and the Babylonians were aware of versions of the Pythagorean theorem about 1500 years before Pythagoras; the Egyptians had a correct formula for the volume of a frustum of a square pyramid.

Egyptian geometry refers to geometry as it was developed and used in Ancient Egypt in the period ranging from 3000 BC to 300 BC.

We only have a limited number of problems from ancient Egypt that concern geometry Geometrical problems appears in both the Moscow Mathematical Papyrus (MMP) and in the Rhind Mathematical Papyrus (RMP). The examples demonstrate that the Ancient Egyptians knew how to compute areas of several geometric shapes and the volumes of cylinders and pyramids. Also the Egyptians used many sacred geometric shapes such as squares and triangles on temples and obelisks.

The pyramids themselves are another indication of the sophistication of Egyptian mathematics. Setting aside claims that the pyramids are first known structures to observe the golden ratio of 1:1.618 (which may have occured for purely aesthetic and not mathematical, reasons), there is certainly evidence that they knew the formula for the volume of a pyramid -$\frac{1}{3}$ times the height times, the length times the width- as well as of a truncated or clipped pyramid. They were also aware, long before Pythagoras, of the rule that a triangle with sides 3,4 and 5 units yields a perfect right angle, and Egyptian builders used ropes knotted at intervals of 3, 4 and 5 units in order to ensure exact right angles for their stonework (in fact, the 3,4,5 right triangle is often called "Egyptian").

Value of π in ancient Egyptian Geometry :

The ancient Egyptians knew that they could approximate the area of a circle as follows :

$$\text{Area of circle} = \left(Diameter \times \frac{8}{9}\right)^2$$

Problem 30 of the Ahmes papyrus uses these methods to calculate the area of a circle, according to a rule that the area is equal to the squre of 8/9 of the circles diameter. This assumes that π is $4 \times \left(\frac{8}{9}\right)^2$ (or 3.160493----) with an error of slightly over 0.63 percent.

This value was slightly less accurate than the calculations of the Babyonians (25/8=3.125, with in 0.53 percent), but was not otherwise surpassed until Anchimedes approximation of 211875/67441=3.14163, which had an error of just over 1 in 10,000.

Interestingly, Ahmes knew of the modern 22/7 as an approximation for pi, and used it to split a hekat, hekat $\times\frac{22}{7}\times\frac{7}{22}$ = hekat; however, Ahmes continued to use the traditional 256/81 value for pi for computing his hekat volume found in a cylinder.

Problem 48 involved using a square with side 9 units. This square was cut into a 3×3 grid. The diagonal of the corner squares were used to make an irregular octagon with an area of 63 units.

This gave a second value of π = 3.111 .... The two problems together indicate a range of values for pi between 3.11 and 3.16.

Problems 14, in the Moscow Mathematical Papyrus gives only ancient example finding the volume of a frustum of a pyramid, describing the correct formula.

$$V = \frac{1}{3}h\,(x_1^2 + x_1x_2 + x_2^2)$$

where x_1 = radius of top of frustum

x_2 = radius of base of frustum

The Egyptians approximated the area of a circle by using shapes whose area they did know. They observed that the area of a circle of diameter 9 units, for examples, was very close to the area of a square with sides of 8 units so that the area of circles of other diameters could be obtained by multiplying the diameter by 8/9 and then squairing it. This gives an effective approximation of π accurate to within less than one percent.

Area of different Geometrical shapes in Ancient Egyptian Geometry :

The Ancient Egyptians wrote out their problems in multiple parts. They gave the title and the data for the given problem, in some of the texts they would show how to solve the problem, and as the last step they verified that the problem was correct. The scribes did not use any variables and the problems were written in prose form. The solutions were written out in steps, outlining the process.

Triangles :

The ancient Egyptians knew that the area of a triangle is $A = \frac{1}{2}\,bh$, when b =base and h = height.

Calculations of the area of a triangle appear in both the RMP and the MMP[1].

Rectangles :

Problem 49 from the RMP finds the area of a rectangular plot of land.[1] Problem 6 of MMP finds the lengths of the sides of a rectangular area given the ratio of the lengths of the sides of a rectangular area given the ratio of the lengths of the sides. This problem seems to be identical to one of the Lahun Mathematical Papyrus in London. The problem is also interesting because it is clear that the Egyptians were familiar with square roots. They even had a special hieroglyph for finding a square root. It looks like a corner and appears in the fifth line of the problem. We suspect that they had tables giving the square roots of some often used numbers. No such tables have been found however.[2] Problem 18 of MMP computes the area of a length of garment cloth.[3]

The Lahun Papyrus Problem 1 in LV. 4 is given as : An Area of 40 "mH" by 3 "mH" shall be divided in 10 areas each of which shall have a width that is $\frac{1}{2}$ of $\frac{1}{4}$ of their length.[4] A translation of the problem and its solution as it appears on the fragment is given on the website maintained by University College London.[4]

Circles

Problem 48 of the RMP compares the area of a circle (approximated by an octagon) and its circumscribing square. This problem's result is used in problem 50.

Trisect each side. Remove the corner triangles. The resulting octagonal figure approximate the circle. The area of the octagonal figure is : $9^2 - 4\frac{1}{2}\,(3)(3)$.

Next we approximate 63 to be 64 and note that 64 = 8^2.

Thus the number $\mathbf{4\left(\frac{8}{9}\right)^2 = 3.16049}$**----plays the role of** $\boldsymbol{\pi}$ **= 3.14159-**

That this octagonal figure, whose area is easily calculated, so accurately approximate the area of the circle is just plain good luck. Obtaining a better approximation to the area using finer divisions of a square and a similar argument is not simple.[1]

Problem 50 of the RMP finds the area of a round field of diameter 9 khet.[1] This is solved by using the approximation that circular field of diameters 9 has the same area as a square of side8. Problem 52 finds the area of a trapazium with apparently equally slanting sides. The lengths of the parallel sides and the distances between them being the given number.[2]

Hemisphere :

Problem 10 of the MMP computes the area of a hemisphere[2].

Volumes of different Geometrical shapes in Ancient Egyption Geometry :

Several problems compute the volume of cylindrical granaries (41, 42 and 43 of the RMP) while problem 60 RMP seems to concern a pillar or a cone instead of a pyramid. It is rather small and steep, with a seked (slope) of four palms (Pen cubit)[1].

A problem appearing in section IV.3 of the Lahun Mathematical Papyrus computes the volume of a granary with a circular base. A similar problem and procedure can be found in the Rhind papyrus (Problem 43). Several problems in the Moscow Mathematical Papyrus (problem 14) and in the Rhind Mathematical Papyrus (numbers 44,45,46) compute the volume of a rectangular granary.[1,2]

Problem 14 of the Mascow Mathematical Papyrus computes the volume of a truncated pyramid.

Sequed in Ancient Egyptian Geomtry :

Problem 56 of the RMP indicates an understanding of the idea of geometric similarity. This problem discussies the ratio run/rise, also known as the sequed. Such a formula would be need for building pyramids. In the next problem (Problem 57), the height of a pyramid is calculated from the base length and the seked (Egyption for slope), while problem 58 gives the length of the base and the height and uses these measurements to compute the seqed.

In problem 59 part I computes the seqed, while the second part may be a computation to check the answer. If you constant a pyrmid with base side 12 (cubits) and with a seqed of 5 plans 1 finger, what is its attitude ?[1]

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