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Optimizing Ordered Quantity With Deterministic Service Rate Before Selling The Items With Constant Demand Rate In The Inventory System When The Amount Received Is Uncertain.

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Abstract:

Sometimes it happens that the arrival of the quantity of items differ from the ordered quantity and this quantity is needed to give some time for service to each item instead of a fixed time of service for all items in the inventory before allowing it for sell. After servicing all items with the rate of service greater than the rate of demand items stars to be depleted from the inventory up to the zero level of inventory. This service time is cost of holding on all received items in the inventory. The optimal order quantity depends only on the mean and standard deviation of the amount arrived and if we can reduce the service time by increasing service persons or facilities the total cost goes near to cost of EOQ model with uncertain arrival of quantity.

Key words Inventory system, Demand rate, Service rate, Replenishment rate, Lead time, Queuing system Service time, re-order point.

1. Introduction

It is seen that when the quantity arrives to the shop and it some fixed time is allotted for servicing all received items with the condition that the quantity arrives is uncertain, the best or minimum order quantity turns out to depends only on the mean and standard deviation of the amount received. Here it was assumed that after servicing all items within a fixed time, all will put for sell together. It is also interesting to know that if instead of service time for all items we have service rate what could be the best order quantity? In this paper we have assumed the same criteria. That is quantity arrives in uncertain amount with deterministic service rate and after completion of service for all items we will put items together for selling. It is also assumed that the demand rate is constant and known to us. Also to avoid queue of customers we have assumed that service rate is more than demand rate. That is the waiting time of customer is that much small so that it can be negligible.

2. Literature review

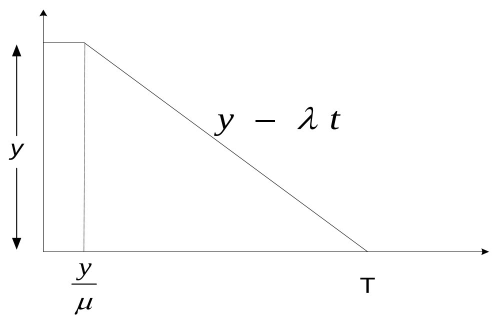
Edward A. Silver (1976) worked on the quantity received from a supplier may not match the quantity ordered—due to defects, shortages, shipping issues, etc. Silver's extended Economic Order Quantity (EOQ) model tackles this by considering that only the mean and standard deviation of the received amount matter in determining the optimal order quantity. Traditional models generally assume one-sided randomness but A. Hamid Noori and Gerald Keller ((1986)) extended the classic continuous-review (Q, r) inventory model to account for uncertainty in both demand during lead time and supply availability. Chirag Trivedi, Y.K. Shan, Nita H. Shah. (1994), present an EOQ model in which a temporary price discount is offered, but the supply received is random rather than equal to the order. Nita H. Shah, Chirag J. Trivedi (1996), extended the classic Economic Order Quantity (EOQ) model is by considering both random lead times and random demand instead of fixed values. Karush (1957) showed that inventory with random demand and lead times can be modelled as a queuing system. Berman O. Kaplan E.H., and Shimshak D.G. (1993) considered the inventory during the provision of service and inventory depleted according to the demand rate when there are no customers waiting in the queue and when customers are waiting in a queue it is depleted according to service rate. Ha (1997) worked on single-item make-to-stock production system and considered poison demand and exponential production times and used an M/M/1/S queuing system for modeling the system. Arda and Hennet (2006) analyzed inventory control of a multi-supplier strategy in a two-level supply chain with random arrival for customers and random delivery time for suppliers, the system was represented as a queuing network. Jung Woo Baek and Seung Ki Moon (2014), provides a queueing-theoretic model for production-inventory systems with lost sales, develops exact analytical results for performance evaluation, and offers managerial insights into balancing production, inventory levels, and customer service. Seyedhoseini et al. (2015) applied queuing theory to propose a mathematical model for inventory systems with substitute flexibility.

3. Notations and Assumptions for the model:

The model is developed under the very stringent assumption and the notations used for the derivation are as under:

- q = the order quantity
- q* = Economic order quantity to be determined.
- C_3 = Replenishment cost per order which is known and constant
- C_1 = Holding cost per unit per unit time which is known and constant
- L = Lead time which is zero.
- The stockout cost is zero.
- b = the bias factor
- $\lambda =$ Demand rate is constant and known
- $\frac{y}{\mu}$ = Total Service time to serve y items.
- T = Total Cycle time.
- $EOQ = \sqrt{\frac{2\lambda C_3}{C_1}}$ = The economic quantity in units.
- $E(y|q) = bq$
- $ECPUT(q)$ = Expected costs per unit time if a requisition quantity q is used
- $ECWS(Q)$ = Expected cost without service
- $q_{ws}{}^*$ = Economic order quantity without service
- The time horizon if infinite.
- $TC(q)$ = Average total cost per cycle
- The replenishment rate is infinite

Consider the following figure. Where inventory has taken time $\frac{y}{\mu}$ for the process to be done on all items before it starts for depletion. After the time $\frac{y}{\mu}$ items are depleted at the rate of λ and the inventory becomes zero at time T.



4. The Mathematical Model:

For developing the expected costs per unit time, we consider the time as being made up of cycles and the new cycle begins each time when the q quantity is ordered. As the lead time is zero the order will be placed when the inventory level drops to zero.

When the ordered quantity arrives at the inventory it takes some time for packing each item or labelling each item or do some required process on each item before put all for selling. In each order the quantity q is ordered and it is assumed that this processing or service rate is μ. That is total time taken for service will be $\frac{q}{\mu}$ and we don't want customer to wait for item for neglecting the waiting cost we have also assumed that $\frac{q}{\mu} < \frac{1}{\lambda}$. In real life it may happen that instead of ordered quantity some other quantity arrives at shop. Suppose that the quantity y is received instead of q. That is y is a random variable and its maximum value is q. Because when the quantity y arrives, and all y items are labelled or packed or assembled one by one with the service rate μ, it stays for some time duration in the inventory for service before the selling. This stay is delayed by the service time $\frac{y}{\mu}$, which affects the inventory depletion rate.

So $\frac{y}{\mu} < \frac{1}{\lambda}$ means the whole batch is served in less time than it typically takes to consume one unit. That is, service is extremely fast compared to demand — we finish service of q items before, on an average, a single item is demanded.

Therefore, the Inventory level at any time t, $0 \leq t \leq T$ will be,

$$I(t) = \begin{cases} y & if\ 0 \leq t \leq \frac{y}{\mu} \\ y - \lambda\left(t - \frac{y}{\mu}\right) & if\ \frac{y}{\mu} < t \leq T \end{cases}$$

Where the total cycle time is

$$T(y) = \frac{y}{\mu} + \frac{y}{\lambda} = \frac{y(\lambda + \mu)}{\lambda\mu}$$

The expected value of T(y) is given by

$$E(T|q) = \frac{(\lambda + \mu)}{\lambda\mu} E(y|q) = \frac{bq(\lambda + \mu)}{\lambda\mu}$$

Let C(y) be the costs in the current cycle. And hence it will be,

$$C(y) = C_1 \int_0^T I(t)dt + C_3$$

$$= C_1 \int_0^{y/\mu} y\, dt + C_1 \int_{y/\mu}^{T} \left[y - \lambda\left(t - \frac{y}{\mu}\right)\right] dt + C_3$$

Letting $\left(t - \frac{y}{\mu}\right) = u$ we have

$$= C_1 \int_0^{1/\mu} y\, dt + C_1 \int_0^{y/\lambda} [y - \lambda u]\, du + C_3$$

Thus, we have,

$$C(y) = \left(\frac{y^2}{\mu} + \frac{y^2}{2\lambda}\right) C_1 + C_3$$

$$C(y) = y^2 \left(\frac{2\lambda + \mu}{2\lambda\mu}\right) C_1 + C_3$$

The expected value of C(y) is given by

$$E(C|q) = C_1 \cdot \left(\frac{2\lambda + \mu}{2\lambda\mu}\right) \cdot E(y^2|q) + C_3$$

As $E(y|q) = bq$ and $E(y^2|q) = {\sigma_{y|q}}^2 + [E(y|q)]^2$

$$E(C|q) = C_1 \cdot \left(\frac{2\lambda + \mu}{2\lambda\mu}\right) \cdot \left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3$$

By the result of Renewal Reward Process, if a cycle is completed every time a renewal occurs and the long-run average reward per unit time is equal to the expected reward earned during a cycle divided by the expected length of a cycle. Thus, we have, the expected cost per unit time is

$$ECPUT(q) = \frac{E(C|q)}{E(T|q)}$$

Substituting values of $E(C|q)$ and $E(T|q)$ in the above equation we have,

$$ECPUT(q) = \frac{C_1 \cdot \left(\frac{2\lambda + \mu}{2\lambda\mu}\right) \cdot \left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3}{\frac{bq(\lambda + \mu)}{\lambda\mu}}$$

$$= \frac{C_1 \cdot (2\lambda + \mu) \cdot \left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3 \cdot 2\lambda\mu}{2bq(\lambda + \mu)}$$

$$= \frac{C_1 \cdot (2\lambda + \mu) \cdot \left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3 \cdot 2\lambda\mu}{bq(\lambda + \mu)}$$

$$= \frac{2\lambda C_1 {\sigma_{y|q}}^2 + 2\lambda C_1 (bq)^2 + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 (bq)^2 + C_3 \cdot 2\lambda\mu}{2bq(\lambda + \mu)}$$

$ECPUT(q)$ will be minimum if $\frac{d(ECPUT(q))}{dq} = 0$ and for its solution $\frac{d^2(ECPUT(q))}{dq^2} > 0$

Let $\frac{d(ECPUT(q))}{dq} = 0$, then we have,

$$\frac{1}{2b(\lambda + \mu)}\left[\frac{q(4\lambda C_1 b^2 q + 2\mu C_1 b^2 q) - (2\lambda C_1 {\sigma_{y|q}}^2 + 2\lambda C_1 (bq)^2 + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 (bq)^2 + C_3 \cdot 2\lambda\mu)}{q^2}\right] = 0$$

$$q(4\lambda C_1 b^2 q + 2\mu C_1 b^2 q) - (2\lambda C_1 {\sigma_{y|q}}^2 + 2\lambda C_1 (bq)^2 + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 (bq)^2 + C_3 \cdot 2\lambda\mu) = 0$$

$$2\lambda C_1 (bq)^2 + \mu C_1 (bq)^2 - C_1 {\sigma_{y|q}}^2 (2\lambda + \mu) - C_3 \cdot 2\lambda\mu = 0$$

$$C_1 (bq)^2 (2\lambda + \mu) - C_1 {\sigma_{y|q}}^2 (2\lambda + \mu) - 2\lambda\mu C_3 = 0$$

$$C_1 (bq)^2 (2\lambda + \mu) = C_1 {\sigma_{y|q}}^2 (2\lambda + \mu) + 2\lambda\mu C_3$$

$$q^2 = \frac{C_1 {\sigma_{y|q}}^2 (2\lambda + \mu) + 2\lambda\mu C_3}{C_1 b^2 (2\lambda + \mu)}$$

$$q = \frac{1}{b}\sqrt{\frac{C_1 {\sigma_{y|q}}^2 (2\lambda + \mu) + 2\lambda\mu C_3}{C_1 (2\lambda + \mu)}}$$

$$q = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}}$$

Provided $b \neq 0, C_1 \neq 0$.

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECPUT(q))}{dq^2} > 0$

$$q^* = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}}$$

Provided $b \neq 0, C_1 \neq 0$.

$$q^* = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{EOQ^2}{\left(\frac{2\lambda}{\mu}+1\right)}} \qquad ------(1)$$

If the service time is taken zero there is no need for service for any item and we have the EOQ model with uncertain arrival y when the ordered quantity is q and hence, we have,

$$T(y) = \frac{y}{\lambda} \text{ and } C(y) = \frac{y^2}{2\lambda}C_1 + C_3$$

$$E(T|q) = \frac{E(y|q)}{\lambda} = \frac{bq}{\lambda}$$

$$E(C|q) = \frac{C_1}{2\lambda}\cdot E(y^2|q) = \frac{C_1}{2\lambda}\left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3$$

And expected cost without service will be,

$$ECWS(q) = \frac{E(C|q)}{E(T|q)}$$

$$= \frac{\frac{C_1}{2\lambda}\left({\sigma_{y|q}}^2 + (bq)^2\right) + C_3}{\frac{bq}{\lambda}}$$

$$= \frac{C_1{\sigma_{y|q}}^2}{2bq} + \frac{C_1 bq}{2} + \frac{\lambda C_3}{bq}$$

$ECWS(q)$ will be minimum if $\frac{d(ECWS(q))}{dq} = 0$ and its solution $\frac{d^2(ECWS(q))}{dq^2} > 0$

Let $\frac{d(ECWS(q))}{dq} = 0$, then we have,

$$q^*_{ws} = \frac{1}{b}\sqrt{\frac{C_1{\sigma_{y|q}}^2 + 2\lambda C_3}{C_1}} = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{2\lambda C_3}{C_1}} = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + EOQ^2}$$

$$q^*_{ws} = \sqrt{{\sigma_{y|q}}^2 + EOQ^2} \qquad ----(2)$$

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECWS(Q))}{dq^2} > 0$

5. Sensitive Analysis

In our model customers arrive as the demand rate λ and there will be no waiting for any customer and hence we must have the condition,

$$\frac{q}{\mu} < \frac{1}{\lambda} \Rightarrow \lambda q < \mu$$

That is,

$$q^* = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}} < \frac{\mu}{\lambda}$$

If $\sigma_{y|q} = 0$ we have

$$q^* = \frac{1}{b}\sqrt{\frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}} < \frac{\mu}{\lambda}$$

Even it can be seen that $q^* \propto \sqrt{C_3}$. That is, smaller the C_3 we can prefer small batches and hence more frequent orders can be placed. Smaller batches, each batch doesn't need very high service rate. But if b is small q* becomes large and hence service rate.

To check the effectiveness and utility of the current model along with the theoretical theory when we take the ratio of the quantity obtained by the current model to the quantity obtained by the basic EOQ model with uncertain arrival of items tends to 1 as the service time tends to zero. That is, if we ignore the service time, it becomes the EOQ mode with uncertain arrival of items.

when $\frac{1}{\mu} \longrightarrow 0$,

$$q^* = \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + \frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}} \rightarrow \frac{1}{b}\sqrt{{\sigma_{y|q}}^2 + EOQ^2} = q^*_{ws}$$

$$\lim_{\frac{1}{\mu}\to 0} \frac{q^*}{q^*_{ws}} = 1$$

Inventory with service will have more total variable cost than the total variable cost in classical inventory because of service time and hence holding cost. If we reduce the service time by increasing service persons or facilities, we will have no difference between our model and classical inventory model.

Case 1: Let $\sigma_{y|q} = \sigma$: If the standard deviation of the quantity received is independent of the quantity requisitioned:

Thus, we have

$$ECPUT(q) = \frac{2\lambda C_1\sigma^2 + 2\lambda C_1(bq)^2 + \mu C_1\sigma + \mu C_1(bq)^2 + C_3 \cdot 2\lambda\mu}{2bq(\lambda+\mu)}$$

If $\sigma = 0$, received quantity will be certain, that is the quantity actually received will be the EOQ.

And

$$q^* = \frac{1}{b}\sqrt{0 + \frac{2\lambda C_3}{C_1\left(\frac{2\lambda}{\mu}+1\right)}}$$

$$q^* = \frac{1}{b}\frac{EOQ}{\sqrt{\left(\frac{2\lambda}{\mu}+1\right)}}$$

when $\frac{1}{\mu} \longrightarrow 0$,

$$q^*_{ws} = \frac{1}{b}\sqrt{EOQ^2} = \frac{EOQ}{b}$$

Which is the optimal quantity when arrival is random and standard deviation of quantity arrived is independent of the quantity q requisitioned. That is $\sigma_{y|q} = \sigma = 0$.

$$q_{ws}^* = \frac{EOQ}{b}$$

Case 2: Let $\sigma_{y|q} = \sigma_1 q$: If the standard deviation of the quantity received is proportional to the quantity requisitioned:

Thus, we have

$$ECPUT(q) = \frac{2\lambda C_1 {\sigma_1}^2 q^2 + 2\lambda C_1 (bq)^2 + \mu C_1 {\sigma_1}^2 q^2 + \mu C_1 (bq)^2 + C_3 \cdot 2\lambda\mu}{2bq(\lambda + \mu)}$$

$$= \frac{1}{2b(\lambda + \mu)}\left[2\lambda C_1 {\sigma_1}^2 q + 2\lambda C_1 b^2 q + \mu C_1 {\sigma_1}^2 q + \mu C_1 b^2 q + \frac{2\lambda\mu C_3}{q}\right]$$

$ECPUT(q)$ will be minimum if $\frac{d(ECPUT(q))}{dq} = 0$ and its solution $\frac{d^2(ECPUT(q))}{dq^2} > 0$

Let $\frac{d(ECPUT(q))}{dq} = 0$, then we have,

$$\frac{1}{2b(\lambda + \mu)}\left[2\lambda C_1 {\sigma_1}^2 + 2\lambda C_1 b^2 + \mu C_1 {\sigma_1}^2 + \mu C_1 b^2 - \frac{2\lambda\mu C_3}{q^2}\right] = 0$$

$$2\lambda C_1 {\sigma_1}^2 + 2\lambda C_1 b^2 + \mu C_1 {\sigma_1}^2 + \mu C_1 b^2 - \frac{2\lambda\mu C_3}{q^2} = 0$$

$$2\lambda C_1 {\sigma_1}^2 + 2\lambda C_1 b^2 + \mu C_1 {\sigma_1}^2 + \mu C_1 b^2 = \frac{2\lambda\mu C_3}{q^2}$$

$$2\lambda C_1 {\sigma_1}^2 + 2\lambda C_1 b^2 + \mu C_1 {\sigma_1}^2 + \mu C_1 b^2 = \frac{2\lambda\mu C_3}{q^2}$$

$$q^* = \sqrt{\frac{2\lambda C_3}{C_1} \times \frac{1}{\left(\frac{2\lambda}{\mu} + 1\right)({\sigma_1}^2 + b^2)}}$$

Provided $b \neq 0, C_1 \neq 0$

$$q^* = \sqrt{\frac{EOQ^2}{\left(\frac{2\lambda}{\mu} + 1\right)({\sigma_1}^2 + b^2)}}$$

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECPUT(q))}{dq^2} > 0$

when $\frac{1}{\mu} \longrightarrow 0$,

$$q_{ws}^* = \frac{EOQ}{\sqrt{({\sigma_1}^2 + b^2)}}$$

Which is the exact formula for the uncertain amount of received quantity when the standard deviation of the quantity received is proportional to the quantity requisitioned:

6. Hypothetical Numerical Example:

Consider the following example: Annual demand D = 120000, Ordering Cost per order = 10, Holding cost per item per day =100 and the demand rate per month will be R= 333.33

$$EOQ = \sqrt{\frac{2RC_3}{C_1}} = 8.16497\ units\ per\ order$$

Case 1: Let $\sigma_{y|q} = \sigma = 0$ and for b =0.6, 0.8, 1, 1.2, 1.4, 1.6

$$q^* = \frac{1}{b}\frac{EOQ}{\sqrt{\left(\frac{2\lambda}{\mu}+1\right)}} = \frac{1}{b} \times \frac{8.16497}{\sqrt{\left(\frac{2\lambda}{\mu}+1\right)}}$$

$$q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{b}$$

For b= 0.6, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{0.6} = 13.608276$

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	12.70001	0.933257
5000	0.066667	12.78275	0.939336
5500	0.060606	12.85166	0.9444
6000	0.055556	12.90994	0.948683
6500	0.051282	12.95989	0.952353
7000	0.047619	13.00316	0.955533
7500	0.044444	13.04101	0.958315
8000	0.041667	13.07441	0.960769
8500	0.039216	13.10409	0.96295
9000	0.037037	13.13064	0.964901

For b= 0.8, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{0.8} = 10.206207$

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	9.52501	0.933257
5000	0.066667	9.587062	0.939336
5500	0.060606	9.638745	0.9444
6000	0.055556	9.682458	0.948683
6500	0.051282	9.719915	0.952353
7000	0.047619	9.752369	0.955533
7500	0.044444	9.78076	0.958315
8000	0.041667	9.805807	0.960769
8500	0.039216	9.828067	0.96295
9000	0.037037	9.847982	0.964901

For b= 1, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{1} = 8.16497$

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	7.620008	0.933257
5000	0.066667	7.66965	0.939336
5500	0.060606	7.710996	0.9444
6000	0.055556	7.745967	0.948683
6500	0.051282	7.775932	0.952353
7000	0.047619	7.801895	0.955533
7500	0.044444	7.824608	0.958315
8000	0.041667	7.844645	0.960769
8500	0.039216	7.862454	0.96295
9000	0.037037	7.878386	0.964901

For b= 1.2, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{1.2} = 6.804138$

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	6.350006	0.933257
5000	0.066667	6.391375	0.939336
5500	0.060606	6.42583	0.9444
6000	0.055556	6.454972	0.948683
6500	0.051282	6.479943	0.952353
7000	0.047619	6.501579	0.955533
7500	0.044444	6.520507	0.958315
8000	0.041667	6.537205	0.960769
8500	0.039216	6.552045	0.96295
9000	0.037037	6.565322	0.964901

For b= 1.4, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{1.4} = 5.832118$

For b= 1.6, $q_{ws}^* = \frac{EOQ}{b} = \frac{8.16497}{1.6} = 5.103104$

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	5.442863	0.933257
5000	0.066667	5.478321	0.939336
5500	0.060606	5.507854	0.9444
6000	0.055556	5.532833	0.948683
6500	0.051282	5.554237	0.952353
7000	0.047619	5.572782	0.955533
7500	0.044444	5.589006	0.958315
8000	0.041667	5.603318	0.960769
8500	0.039216	5.616039	0.96295
9000	0.037037	5.627419	0.964901

μ	λ/μ	q*	q*/q_{ws}^*
4500	0.074074	4.762505	0.933257
5000	0.066667	4.793531	0.939336
5500	0.060606	4.819373	0.9444
6000	0.055556	4.841229	0.948683
6500	0.051282	4.859957	0.952353
7000	0.047619	4.876184	0.955533
7500	0.044444	4.89038	0.958315
8000	0.041667	4.902903	0.960769
8500	0.039216	4.914034	0.96295
9000	0.037037	4.923991	0.964901

From the above table it can be seen that the optimal quantity q* with service time tends to the optimal quantity q_{ws}* without service time when arrival is uncertain and standard deviation of quantity arrived is independent of the quantity q requisitioned when the service time tends to zero.

Case 2: Let $\sigma_{y|q} = \sigma_1 \cdot EOQ$ and for b =0.6, 0.8, 1, 1.2, 1.4, 1.6

Assume that y follows uniform distribution with q=EOQ and the y is in the range from 0.9EOQ to 1.1EOQ. That is

$y \sim U(0.9EOQ, 1.1EOQ)$ and $\sigma_{y|q} = \frac{1.1EOQ-0.9EOQ}{\sqrt{12}} = 0.0577 \times EOQ$ and hence $\sigma_1 = 0.0577$

For above example,

$$\sigma_{y|q} = 0.0577 \times 8.16497 = 0.471119$$

$$q_{ws}{}^* = \frac{EOQ}{\sqrt{\sigma_1{}^2 + b^2}} = \frac{8.16497}{\sqrt{0.00333 + b^2}}$$

$$q^* = \frac{EOQ}{\sqrt{\left(\frac{2\lambda}{\mu}+1\right)(\sigma_1{}^2 + b^2)}} = \frac{8.16497}{\sqrt{\left(\frac{2\lambda}{\mu}+1\right)(0.00333 + b^2)}}$$

For b= 0.6, $q_{ws}{}^* = \frac{8.16497}{\sqrt{0.00333+b^2}} = 13.545771$

For b= 0.8, $q_{ws}{}^* = \frac{8.16497}{\sqrt{0.00333+b^2}} = 10.179758$

μ	λ/μ	q*	q*/$q_{ws}{}^*$
4500	0.074074	12.64168	0.933257
5000	0.066667	12.72404	0.939336
5500	0.060606	12.79263	0.9444
6000	0.055556	12.85065	0.948683
6500	0.051282	12.90036	0.952353
7000	0.047619	12.94343	0.955533
7500	0.044444	12.98111	0.958315
8000	0.041667	13.01436	0.960769
8500	0.039216	13.0439	0.96295
9000	0.037037	13.07033	0.964901

μ	λ/μ	q*	q*/$q_{ws}{}^*$
4500	0.074074	9.500326	0.933257
5000	0.066667	9.562218	0.939336
5500	0.060606	9.613767	0.9444
6000	0.055556	9.657367	0.948683
6500	0.051282	9.694726	0.952353
7000	0.047619	9.727096	0.955533
7500	0.044444	9.755414	0.958315
8000	0.041667	9.780395	0.960769
8500	0.039216	9.802598	0.96295
9000	0.037037	9.822462	0.964901

For b= 1, $q_{ws}{}^* = \frac{8.16497}{\sqrt{0.00333+b^2}} = 8.16497$

For b= 1.2, $q_{ws}{}^* = \frac{8.16497}{\sqrt{0.00333+b^2}} = 6.796285$

μ	λ/μ	q*	q*/q_{ws}*
4500	0.074074	7.607352	0.933257
5000	0.066667	7.656912	0.939336
5500	0.060606	7.698189	0.9444
6000	0.055556	7.733102	0.948683
6500	0.051282	7.763017	0.952353
7000	0.047619	7.788937	0.955533
7500	0.044444	7.811612	0.958315
8000	0.041667	7.831617	0.960769
8500	0.039216	7.849396	0.96295
9000	0.037037	7.865301	0.964901

μ	λ/μ	q*	q*/q_{ws}*
4500	0.074074	6.342677	0.933257
5000	0.066667	6.383998	0.939336
5500	0.060606	6.418413	0.9444
6000	0.055556	6.447522	0.948683
6500	0.051282	6.472464	0.952353
7000	0.047619	6.494075	0.955533
7500	0.044444	6.51298	0.958315
8000	0.041667	6.529659	0.960769
8500	0.039216	6.544482	0.96295
9000	0.037037	6.557744	0.964901

For b= 1.4, $q_{ws}{}^{*} = \frac{8.16497}{\sqrt{0.00333+b^2}} = 5.827170$

μ	λ/μ	q*	q*/q_{ws}*
4500	0.074074	5.438245	0.933257
5000	0.066667	5.473673	0.939336
5500	0.060606	5.503181	0.9444
6000	0.055556	5.528139	0.948683
6500	0.051282	5.549525	0.952353
7000	0.047619	5.568054	0.955533
7500	0.044444	5.584264	0.958315
8000	0.041667	5.598564	0.960769
8500	0.039216	5.611274	0.96295
9000	0.037037	5.622644	0.964901

For b= 1.6, $q_{ws}{}^{*} = \frac{8.16497}{\sqrt{0.00333+b^2}} = 5.099788$

μ	λ/μ	q*	q*/q_{ws}*
4500	0.074074	4.75941	0.933257
5000	0.066667	4.790417	0.939336
5500	0.060606	4.816241	0.9444
6000	0.055556	4.838084	0.948683
6500	0.051282	4.8568	0.952353
7000	0.047619	4.873016	0.955533
7500	0.044444	4.887202	0.958315
8000	0.041667	4.899718	0.960769
8500	0.039216	4.910841	0.96295
9000	0.037037	4.920792	0.964901

From the above table it can be seen that the optimal quantity q* with service time tends to the optimal quantity $q_{ws}{}^{*}$ without service time when arrival is uncertain and the standard deviation is proportional to the q amount requisitioned when the service time tends to zero.

7. Conclusion

When the arrival of quantities is uncertain, the best order quantity depends on the mean and standard deviation of the amount actually received. In the situation, when the replenished items cannot be sold immediately, and they must undergo a processing stage one by one, the processing time increases the average customer waiting time and therefore raises the holding cost of all items in the system. Two cases of supply uncertainty are considered: (i) when the standard deviation of the received quantity is independent of the ordered amount, and (ii) when the standard deviation is proportional to the ordered amount. If we can reduce the process time on the product — that is, decrease the waiting time as much as possible — the new model simplifies to the classical case where there is no service time and the arrival of quantity is uncertain.

8. References

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