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On Resolving -Set Domination In Graphs

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ABSTRACT

A Resolving Set $R \subset V$ is called Resolving Dominating Set of a connected graph G if every node in R'(i.e $V - R$)is adjacent to some other node in R. A Resolving Dominating Set is called Resolving-Set Dominating set (R-SD set) if every set $U' \subset R'$ there exists a non-empty set $Z \subset R$ such that the subgraph of $\langle U' \cup Z \rangle$ is connected in G. The Resolving -Set Domination number γ_{RSD}(G) is the minimum cardinality of overall Resolving -Set Dominating set in G. In this research paper, we introduce this parameter to some standard graphs and existing of bounds of the graph.

Keywords: Resolving Set, Resolving Dominating Set, Resolving -Set Dominating set, Resolving -Set Domination number, Graphs.

2020 Mathematics Subject Classification: 05C12, 05C69.

1 INTRODUCTION

The concept of Domination and Dominating Set has been studied in [5]. The idea of Resolving Set was discussed in [3]. The concept of Set Domination investigated in [4]. In [1], [3] and [5], the idea of Resolving Dominating Set was explored. The idea of this paper is introducing the new concept of Resolving -Set Domination, which is a Resolving Dominating Set satisfying the condition of Set Domination. This parameter can be applied and analyzed in path graphs, wheel graphs, cycle graphs and complete graphs. In this paper, we considered all the graphs to be simple, connected and undirected. [2] For any two finite nodes x and y in graph G, the distance $d(x, y)$ from x to y is the length of the shortest $x - y$ path in G. A $x - y$ path of length $d(x, y)$ is called a $x - y$ Geodesic. In order to $d(x, y)$ be defined for all pairs x, y of nodes in G, the graph G must be connected. [3] A node r of a connected graph G is said to Resolve two nodes x and y of G if $d_G(r, x) \neq d_G(r, x)$. For an ordered set $R = \{r_1, r_2, \dots \dots \dots \dots, r_m\} \subseteq V(G)$ and node y in G, then a representation of r with respect to R with m-vector, $r_G(y/R) = (d_G(y, r_1), d_G(y, r_2) \dots \dots . d_G(y, r_m))$. The set R is a Resolving Set for G if and only if $r_G(y/R) \neq r_G(x/R)$. The metric dimension of G denoted by $\dim(G)$, is the minimum cardinality overall sets of G. A Resolving Set of cardinality $\dim(G)$ is called a Basis. A Resolving Set $R \subset V$ is called Resolving Dominating Set of G if every node in R' is adjacent to some node in R. [4] A Resolving Dominating Set is called Resolving -Set Dominating set if every set $U' \subset R'$ there exist a non-empty set $Z \subset R$ such that the subgraph $\langle U' \cup Z \rangle$ is connected in G. The minimum cardinality of Resolving -Set Domination of a graph G is denoted by $\gamma_{RSD}(G)$. In this paper, we introduce this parameter to some standard graphs and the existence of bounds of the graph.

2 PRELIMINARIES

Let G=(V(G), E(G)) be a simple and undirected graph. Two vertices u,v of G are adjacent if uv ∈E(G). The open neighbourhood of u in G is the set $N_G(u)=\{v\in V(G):uv\in E(G)\}$. The closed neighbourhood of u in G is the set $N_G[u] = N_G(u) \cup \{u\}$. If U⊆V(G), the open neighbourhood of U in G is the set $N_G(U)= \bigcup_{u\in U} N_G(u)$. The closed neighbourhood of U in G is the set $N_G[U]=N_G(U)\cup U$.

A graph G is called connected if every pair of nodes can be joined by a path; otherwise, it is disconnected.

If every pair of distinct nodes is adjacent in a graph G, is called complete graph K_n.

A node r of a connected graph G is said to Resolve two nodes x and y of G if $d_G(r,x) \neq d_G(r,x)$. For an ordered set $R = \{r_1, r_2, \dots \dots \dots \dots, r_m\} \subseteq V(G)$ and node y in G, then a representation of r with respect to R with m-vector, $r_G(y/R) = (d_G(y,r_1), d_G(y,r_2) \dots \dots . d_G(y,r_m))$. The set R is a Resolving Set for G if and only if $r_G(y/R) \neq r_G(x/R)$.

A Resolving Set $R \subset V$ is called Resolving Dominating Set of G if every node in R' is adjacent to some node in R. A Resolving Dominating Set is called Resolving -Set Dominating set if every set $U' \subset R'$ there exist a non-empty set $Z \subset R$ such that the subgraph $\langle U' \cup Z \rangle$ is connected in G. The minimum cardinality of Resolving -Set Domination of a graph G is denoted by γ_{RSD}(G).

Example:

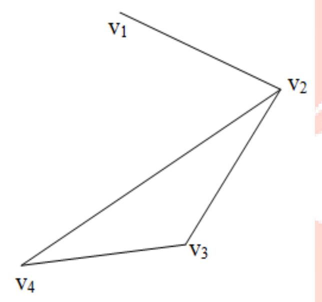

Fig.1:G= (V, E) Resolving -Set Dominating set

Let us consider the above connected graph G= (V, E) and $R_1 = \{v_1, v_3, v_4\} \subset V(G)$ be the Resolving Dominating Set of G with cardinality 3. But $R_2 = \{v_1, v_3\}$ is the Resolving Dominating Set with minimum cardinality 2. The set R_2 is called Resolving -Set Dominating set since it satisfies the condition of Set Domination i.e., the set $U' = \{v_2\} \subset R'$ where $R' = V - R_2$ and $Z = \{v_3\} \subset R_2$, then $\langle U' \cup Z \rangle = \{v_2, v_3\}$ is connected by the original graph. Another possible combination of the Resolving -Set Dominating set is $R_3 = \{v_2, v_4\}$. Hence, we conclude that the sets R_2 and R_3 are Resolving -Set Dominating set with $\gamma_{RSD}(G) = 2$.

A Resolving -Set Dominating set R is called a Minimal Resolving -Set Dominating set if no proper subset of R is Resolving -Set Dominating set.

Theorem 2.1 [5] A Resolving -Set Dominating set R is Minimal Resolving -Set Dominating set iff for each node $u \subset R$, one of the following two conditions holds:

a) u is an isolated node of R
b) There exists a node $v \in R'$ for which N $(v) \cap R = u$.

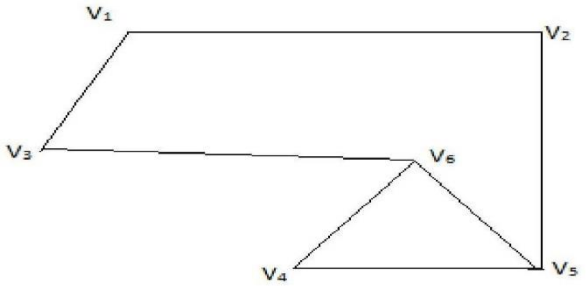

Fig.2: H (V, E)Minimal Resolving -Set Dominating set

Consider the above connected graph H (V, E). The set $R = \{v_1, v_5\}$ is the Resolving -Set Dominating set with Resolving -Set Domination number γ_{RSD}(H) $= 2$. It is also called Minimal Resolving -Set Dominating set since it satisfies the given condition (i.e) the complement of R of H (V, E) is $R' = \{v_2, v_3, v_4, v_6\}$ and let us assume $v_3 \in R'$ and $N(v_3) \cap R = \{v_1\}$ exists a unique node. Another possible Minimal Resolving -Set Dominating set of H (V, E) is $R_1 = \{v_1, v_4, v_6\}$ by assuming $v_2 \in V'$, then $N(v_2) \cap R = \{v_1\}$ exists a unique node. Hence, the condition holds.

Theorem 2.2 [5] For every non-trivial connected graph G, there exists a Resolving -Set Dominating set R and its complement R' also possesses Resolving -Set Dominating set.

Proof: Since G is a non-trivial connected graph, it has atleast one Resolving Dominating set and one Set Dominating set. So, it cannot be a null set. It is both the Resolving Dominating set and the Set Dominating set. Suppose it is a null set, add one node until it satisfies the properties of the Resolving Dominating set and the Set Dominating set. If R is a Resolving Dominating set, the nodes in R' are uniquely determined by their distances to the node in R. Construct a set $U' \subset R'$ such that U' is a Resolving -Set Dominating set for R'. This can be done by selecting nodes in V' that uniquely determine the distances to the nodes in R. Hence, R is also Resolving -Set Dominating set for R'. Since every node in $R'$is either in R or adjacent to a node in R, the result.

Theorem 2.3 Consider a graph G that has no isolated vertices, R' can be considered a Minimal Resolving -Set Dominating set if R is a Minimal Resolving -Set Dominating set.

Proof: Let R be the Minimal Resolving -Set Dominating set of a graph G, which means that every node in R' is adjacent to one node in R. If we remove any node from R, it is not exactly adjacent to all other nodes of a graph G. So, it fails to meet the property of Resolving -Set Dominating set. Since V' has no isolated nodes, every node in $R'$is adjacent to atleast one node in R, and if it is adjacent, it connects with the property of Set Dominating set. This implies that R' is a Resolving -Set Dominating set when R is a Minimal Resolving -Set Dominating set. Hence the proof.

3 EXACT VALUES OF SOME STANDARD GRAPHS IN RSD

Here, we discuss the Resolving -Set Dominating set with minimum cardinality of some standard graphs, such as Path, Cycle, Wheel and Complete graphs.

Observation 3.1 A connected graph G with n nodes, n≥2, then $\gamma_{RSD}(\mathrm{P_n})$ is $\mathrm{n} - k$, where k is defined as the cardinality of R' of $\mathrm{P_n}$.

The result is $$\gamma_{RSD}(P_{\mathrm{n}}) = \begin{cases} 1 \; if \; \mathrm{n} = 2,3 \\ 2 \; if \; \mathrm{n} = 4,56 \\ 3 \; if \; \mathrm{n} = 7,8,9 \\ \mathrm{n} - k \; if \; \mathrm{n} \geq 10 \end{cases}$$

Observation 3.2 A connected graph G with n nodes n≥3, then $\gamma_{RSD}(\mathrm{C_n})$ is $\mathrm{n}-k$, where k is defined as the cardinality of R' of $\mathrm{C_n}$.

The result is $\gamma_{RSD}(C_\mathrm{n}) = \begin{cases} 2 \; if \; \mathrm{n} = 3,4,5 \\ 3 \; if \; \mathrm{n} = 6,7,8 \\ \mathrm{n} - k \; if \; \mathrm{n} \geq 9 \end{cases}$

Observation 3.3 A connected graph G with n nodes n≥5, then $\gamma_{RSD}(\mathrm{W_n})$ is $\mathrm{n}-k$, where k is defined as the cardinality of R' of $\mathrm{W_n}$.

The result is $\gamma_{RSD}(W_\mathrm{n}) = \begin{cases} 2 \; if \; \mathrm{n} = 5,6 \\ 3 \; if \; \mathrm{n} = 7,8 \\ \mathrm{n} - k \; if \; \mathrm{n} \geq 9 \end{cases}$

Observation 3.4 For a Complete graph K_n with $\mathrm{n} \geq 4$, then $\gamma_{RSD}(K_\mathrm{n}) = \mathrm{n} - 1$.

Proof: Let K_n be the Complete graph, then the degree of the nodes is equal $\deg(v_i) = \deg(v_j)$ when $i \neq j$. The Resolving Set R_i of K_n with $\mathrm{n}-2$ and $\mathrm{n}-3$ nodes cannot be generated since it cannot give the distinct representation of R_i with respect to v_i that is, $r_{K_\mathrm{n}}(v_i/R_i) = r_{K_\mathrm{n}}(v_j/R_i)$. So, the only possible way is to increase the $|R_i|$, we get $r_{K_\mathrm{n}}(v_i/R_i) \neq r_{K_\mathrm{n}}(v_j/R_i)$. Obviously, R_i is easy to say that K_n is Resolving -Set Dominating set by the induced subgraph is connected, that is, $\langle U' \cup Z \rangle$ is connected, for $U' \subset R_i'$ and $Z \subset R_i$, all the nodes are adjacent to each other. Hence the result.

We also observe that a connected graph G with $\mathrm{n} = 3$, $C_3 = K_3$ if and only if

$\gamma_{RSD}(\mathrm{G}) = \mathrm{n} - 1$.

Now we obtain some bounds for the Domination number $\gamma_{RSD}(\mathrm{G})$ of a graph G.

4 BOUNDS ON RESOLVING -SET DOMINATION

Theorem 4.1 If G is graph with no isolated nodes, then $\gamma_{RSD}(\mathrm{G}) \leq \left\lfloor \frac{\mathrm{p}}{2} \right\rfloor$.

Proof: Let R be the Minimal Resolving -Set Dominating set of a graph G with p nodes. Since G has no isolated nodes, every node in G must have atleast one neighbour. Then R' is a Resolving -Set Dominating set. Both R and R' cover the nodes of G and hold the condition of Resolving -Set Dominating set of G. Hence $\gamma_{RSD}(\mathrm{G}) \leq \min\{|R|, |R'|\} \leq \left\lfloor \frac{\mathrm{p}}{2} \right\rfloor$.

Theorem 4.2 [5] For a graph G with p nodes and maximum degree $\Delta(\mathrm{G})$, then

$\gamma_{RSD}(\mathrm{G}) \leq \mathrm{p} - \Delta(\mathrm{G})$.

Proof: Given a connected graph G of p nodes and maximum degree $\Delta(\mathrm{G})$. Let R be the Resolving -Set Dominating set with $\gamma_{RSD}(\mathrm{G})$. Assume $v \in R$ has a maximum degree $\Delta(\mathrm{G})$. Then v is adjacent to $\mathrm{N}(v)$ such that $\Delta(\mathrm{G}) = |\mathrm{N}(v)|$. Hence, $v - \mathrm{N}(v)$ is also Resolving -Set Dominating set, then $\gamma_{RSD}(\mathrm{G}) \leq |v - \mathrm{N}(v)|$. Hence $\gamma_{RSD}(\mathrm{G}) \leq \mathrm{p} - \Delta(\mathrm{G})$.

Theorem 4.3 If a graph L has $\gamma_{RSD}(\mathrm{L}) \geq 2$, then $q \leq \left\lfloor \frac{1}{2}\left(p-\gamma_{RSD}(\mathrm{L})\right)\left(p-\gamma_{RSD}(\mathrm{L})+2\right) \right\rfloor$.

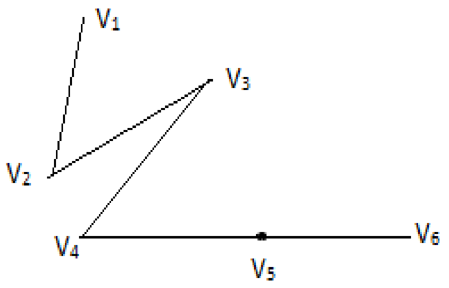

Fig.3: Bounds on Resolving -Set Dominating set

From the graph L, $R_1 = \{v_2, v_5\}$, and $R_2 = \{v_1, v_3, v_5\}$ are Resolving -Set Dominating set then the inequality (upper bound) holds for $5 < 12$ and $5 < 7$ with $\gamma_{RSD}(\mathrm{L}) = 2$ and $\gamma_{RSD}(\mathrm{L}) = 3$ respectively.

Theorem 4.4 [5] For a graph G with p nodes and maximum degree $\Delta(\mathrm{G})$ then

$\lceil \mathrm{p}/(1+\Delta(\mathrm{G})) \rceil \leq \gamma_{RSD}(\mathrm{G})$.

Proof: Let R be the Resolving -Set Dominating set with minimum cardinality $\gamma_{RSD}(\mathrm{G})$ of a graph G. Every node in R can dominates atmost itself (and neighbours) of $1+\Delta(\mathrm{G})$ nodes of G. Since R is a Resolving -Set Dominating set, it covers all nodes in G. Then $\mathrm{p} \leq |R|.|1+\Delta(\mathrm{G})|$. Hence the result.

Theorem 4.5 [5] If a connected graph G, then $\lceil (diam(\mathrm{G})+1)/3 \rceil \leq \gamma_{RSD}(G)$.

Proof: Given G is a connected graph. Assume R is Resolving -Set Dominating set of $\gamma_{RSD}(\mathrm{G})$. Taking an arbitrary path of length $diam(\mathrm{G})$. The diametral path induces atmost 2 edges from $\langle N[v] \rangle$ for each $v \in R$. Since R is a Resolving -Set Dominating set with $\gamma_{RSD}(\mathrm{G})$, the diametral path includes atmost $\gamma_{RSD}(\mathrm{G}) - 1$ edges joining the nearest nodes R. Then $diam(\mathrm{G}) \leq 2\gamma_{RSD}(\mathrm{G}) + \gamma_{RSD}(\mathrm{G}) - 1$. Hence $\lceil (diam(\mathrm{G})+1)/3 \rceil \leq \gamma_{RSD}(G)$.

CONCLUSION

The concept of Resolving -Set Domination has been introduced and investigated in this study. The investigation can be made with some simple graphs like Path graphs, Wheel graphs, Cycle graphs, and Complete graphs, and analyzing these results with a minimum cardinality of Resolving-Set Domination number. Existence of upper and lower bounds of this parameter for the attainment of bounds. For future investigation, this parameter can be used in standard graphs and its application can be led into the field of network analysis and robotics.

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