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Graceful Labelling For Radio Geometric Mean And Radio Harmonic Mean

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Abstract:

Topological indices and graph labelling are essential tools in mathematical chemistry and graph theory, providing valuable insights into the structure and properties of molecular graphs and complex networks. In this paper, we explore the computation of Graceful Lebelling, Radio Geometric Mean, and Radio Harmonic Mean indices for prominent graph families, including butterfly graphs, complete bipartite graphs, and Friendship graphs.

1. Introduction:

Imagine a world where complex systems and networks are woven together like intricate webs, each with its own unique structure and properties. Graph theory, a branch of mathematics, helps us unravel these complexities, providing insights into the very fabric of our interconnected world. From the design of efficient communication networks to the analysis of molecular structures, graph theory has far-reaching implications.

Graph labelling, like graceful labelling, are a crucial aspect of graph theory. They help us understand the intricate relationships between nodes and edges, revealing patterns and structures that might otherwise remain hidden. By exploring these labelling, researchers can unlock new secrets about the behaviour of complex systems and networks.

Recent studies have introduced innovative concepts, such as radio geometric mean and radio harmonic mean indices, which offer fresh perspectives on graph structures. These indices have the potential to revolutionize our understanding of complex networks, enabling us to analyse and predict their behaviour with greater accuracy.

In this paper, we would wish into the computation of graceful labelling of radio geometric mean, and radio harmonic mean indices for specific graph families. By exploring these graph invariants, we aim to uncover new insights into the structural properties of these graphs and their potential applications in various fields.

Our research contributes to the ever-growing field of graph theory, pushing the boundaries of human knowledge and understanding. By exploring the intricacies of complex systems and networks, we can unlock new possibilities for innovation and discovery.

2. Preliminaries

Definition 2.2: A Radio Geometric Mean labelling is a one to one mapping $f:V(G)\to\mathbb{N}$ Satisfying the condition

$$d(x,y)+\left|\sqrt{h(x_i)h(x_j)}\right|\geq 1+diam(G) \text{ every } u\in V(G).$$

The Radio Geometric Mean Number of graph G is Denoted by *rgmn(G).*

Definition 2.3: A Radio Harmonic Mean Labelling of a Connected Graph G is one to one map $f:V(G)\to\mathbb{N}$ such that for two distinct vertices u and v of G satisfies the condition

$$d(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil\geq 1+diam(G).$$

Definition 2.4: The graph obtained by inserting vertices into every wing, assuming that the sum of the inserted vertices in every wing is the same is called Butterfly Graph

Definition 2.5: The friendship graph is obtained by joining the copies of C_3 with a common vertex known as the central vertex is called Friendship Graph.

Definition 2.6: It is a type of bipartite graph where every vertex in one set is connected to every vertex in another set is called Complete Bipartite Graph.

3. Main Results

Theorem 3.1: Butterfly Graph BF_n for $n\geq 2$ admits Radio Geometric Mean Graceful Labeling and Radio Harmonic Mean Graceful Labeling.

Proof: Let G be the butterfly Graph BF_n.

The number of vertices $|V(BF_n)|=2n+1$ where $n\geq 2$ and the number of edges $|E(BF_n)|=2(2n-1)$.

Consider the vertex v as the central vertex, u as a left wing and w as a right wing

Let us define the vertex set $V(BF_n)=\{u_i:i=1,2,...,n\}\cup\{w_i:i=1,2,...,n\}\cup\{v=w_{n-1}+1\}$

Let the edge $E(BF_n)$ set of the graph G be , $E(BF_n)=\{(v,u_i):1\leq i\leq n\}\cup\{(v,w_i):1\leq i\leq n\}\cup\{(u_i,u_{i+1}):1\leq i\leq n-1\}\cup\{(w_i,w_{i+1}):1\leq i\leq n-1\}$ so, bijective function from $f:V(BF_n)\to\{1,2,...,2n+1\}$ as

$u_1=1;\ u_n=2;\ w_1=3;\ w_n=4;$

$u_{i+1}=w_n+i;\quad 1\leq i\leq n-1$

$w_{i+1}=u_{n-1}+i;\quad 1\leq i\leq n-1$

$v=n$

Hence admits graceful labelling.

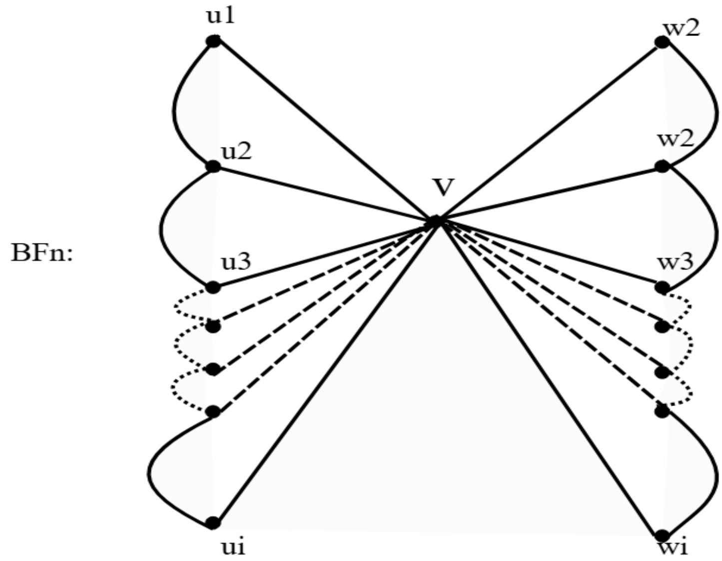

Fig 1

Theorem 3.2: Friendship Graph F_n^m for $n \geq 7$ and $m \geq 3$ admits Radio Geometric Mean Graceful Labeling and Radio Harmonic Mean Labeling.

Proof: Let G be the Friendship Graph F_n^m.

The number of vertices $\left|V\left(F_n^m\right)\right| = 2n+1$ where $n \geq 7$ and the number of edges $\left|E\left(F_n^m\right)\right| = nm$ where m is the number of triangles and n is the number of vertices.

Consider the central vertex as v_n.

Let us define the vertex set $V\left(F_n^m\right) = \{v_i : i = 1, 2, ..., 2n+1, n = 1, 2, ...\}$ and the edge set $E\left(F_n^m\right) = \{(v_n, v_i) : i = 2, 3, ..., n-1\} \cup \{(v_i, v_{i+1}) : i = 2, 4, ..., n-2\}$.

So the bijective mapping $f : V\left(F_n^m\right) \rightarrow \{1, 2, ..., n\}$ as

$f(v_1) = 1;\ \ f(v_n) = n;\ \ f(v_{n-2}) = m;\ \ f(v_2) = m+1;$

For odd vertices,

$f(v_{i+1}) = v_{i+1} + 1;$ for $i = 2, 4, ..., 2m-1;$

For even vertices

$f(v_{2+i}) = m + \left\lceil \frac{2+i}{2} \right\rceil;$ for $i = 2, 4, ..., n-1;$

Hence admits graceful labelling.

F_n^m :

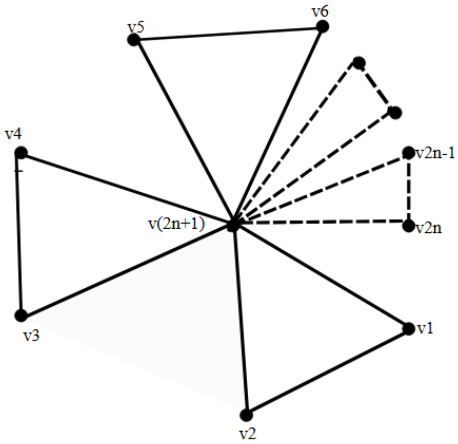

Fig 2

Theorem 3.3: Complete Bipartite Graph $K_{m,n}$ admits Radio Geometric Mean Graceful Labeling and Radio Harmonic Mean Graceful Labeling.

Proof: Let G be the Complete Bipartite Graph with the vertex set $V = V_1 \cup V_2$, where $V_1 = v_1, v_2, ..., v_m$ and $V_2 = u_1, u_2, ..., u_n$.

The number of vertices $|V(K_{m,n})| = m+n$ and the number of edges $|E(K_{m,n})| = m*n$.

Let us define the vertex set as $V(K_{m,n}) = \{v_i : i = 1, 2, ..., m+n\}$ and the edge set $E(K_{m,n}) = \{(v_i, u_j) : 1 \le i \le \&1 \le j \le n\}$.

so the bijective function from $f : V(K_{m,n}) \to \{1, 2, ..., m+n\}$ as

$f(v_i) = i$ where $1 \le i \le m$ and

$f(u_j) = v_m + j$ where $1 \le j \le n$.

Hence admits graceful labelling.

$K_{m,n}$:

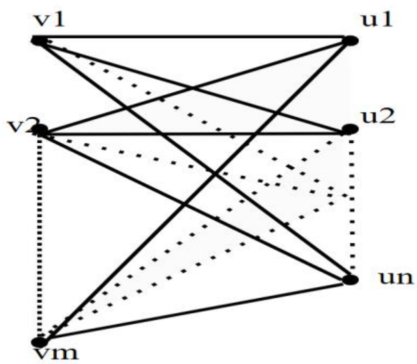

Fig 3

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