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An Approach Of Multi-Criteria Decision-Making Problems With Bipolar Vague Sets Vikor Method Using Matlab

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Abstract: In this paper we describe the concept of bipolarity on vague sets. We develop a bipolar vague VIKOR Method with multi-criteria decision-making problems in which the ranking order of the alternatives are engaged as bipolar vague sets. Lastly a mathematical illustration is given to demonstrate the purpose and efficiency of the established bipolar vague VIKOR method using MATLAB.

Index Terms - Bipolar vague set, VIKOR method and multi-criteria decision-making.

I. INTRODUCTION

Zadeh.L.A[9] announced the conception of fuzzy set and stable by himself and other as create numerous applications in the area of math and in other zones of science and technology. Gau.W.L. and Buehrar.D.J[5] presented the belief of vague sets. Lee.K.M[7] familiarized the bipolar vague fuzzy sets which are a delay of fuzzy sets. TOPSIS method was established by Hwang.C.L. and Yoon.K [6] is a famous in multi criteria decision making method. Chen, Hong.D.H and Choi.C.H[2,3,4] done their studies in multiple attribute decision making problems in fuzzy and vague set theory. Venkata Kalyani.U, Eswarlal.T and Bhargavi.Y[8] established a bipolar vague-valued TOPSIS to explain MCDM problems. In this paper we present a bipolar vague VIKOR method using MATLAB to crack multi criteria decision making problems in which the performance evaluation values as fine as the weights of the conditions are taken as bipolar vague sets correspondingly.

II. PRELIMINARIES

Here in this paper the bipolar vague topological spaces are denoted by (X, BV_τ). Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set A are denoted by BVInt(A) and BVCl(A). The complement of a bipolar vague set A is denoted by A^c and the empty set and whole sets are denoted by $0_\sim$ and $1_\sim$ respectively.

Definition 2.1: [7] Let X be the universe. Then a bipolar valued fuzzy set A on X is defined by positive membership function μ_A^+, that is μ_A^+: X→ [0,1], and a negative membership function μ_A^-, that is μ_A^-: X→ [-1,0]. For the sake of simplicity, we shall use the symbol A = $\{\langle x, \mu_A^+(x), \mu_A^-(\mathrm{x})\rangle : x \in X\}$.

Definition 2.2: [7] Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+$= max $\{\mu_A^+(x), \mu_B^+ x)\}$
(ii) $\mu_{A\cup B}^-$= min $\{\mu_A^-(x), \mu_B^- x)\}$
(iii) $\mu_{A\cap B}^+$= min $\{\mu_A^+(x), \mu_B^+ x)\}$
(iv) $\mu_{A\cap B}^-$= max $\{\mu_A^-(x), \mu_B^- x)\}$
(v) $\mu_{A^c}^+$(x) = 1-$\mu_A^+(x)$ and $\mu_{A^c}^-$(x) = -1-$\mu_A^-(x)$ for all $x \in X$.

Definition 2.3: [5] A vague set A in the universe of discourse U is a pair of (t_A, f_A) where t_A: U→[0,1], f_A: U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$. The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the vague value of u in A, and denoted by $\nu_A(u)$, that is $\nu_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.4: [5] Let A be a non-empty set and the vague set A and B in the form A = $\{\langle x, t_A(x), 1 - f_A(x)\rangle : x \in X\}$, B = $\{\langle x, t_B(x), 1 - f_B(x)\rangle : x \in X\}$. Then

(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$

(ii) A ∪ B = $\{\langle \max(t_A(x), t_B(x)), \max(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$.

(iii) A ∩ B = $\{\langle \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$.

(iv) $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle : x \in X\}$.

Definition 2.5: [1] Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A = $\{\langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : x \in X\}$ where $[t_A^+, 1 - f_A^+]$: X→[0,1] and $[-1 - f_A^-, t_A^-]$: X→[-1,0] are the mapping such that $t_A^+(x) + f_A^+(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $\nu_A^+ = [t_A^+, 1 - f_A^+]$ and $\nu_A^- = [-1 - f_A^-, t_A^-]$.

III. An Approach of Multi-criteria Decision-Making Problems with Bipolar Vague Sets VIKOR Method Using MATLAB

The VIKOR method is based on covering ranking method that is used in multi-criteria decision making. It benefits in ranking and choosing alternatives by considering criteria and if comprise solutions that maximize group utility and minimize individual regret. We first give the proper mathematical expression of a multi-criteria decision-making problem with bipolar vague sets. Then we present bipolar vague VIKOR method procedures to solve it. The basic attitude to VIKOR method is particularly useful when decision makers cannot direct their preferences and it focuses on outcome of the solution to the ideal solution.

Let Y = $\{y_1, \ldots\ldots y_n\}$ and Z = $\{z_1, \ldots\ldots z_m\}$ be the groups of alternatives and criteria that are determined by a decision maker correspondingly. A specific weight $\beta_j \in [0,1]$ is allocated for apiece of criterion z_j and represents the position of z_j. we assume that the weights of criteria satisfy the normalized condition $\sum_{j=1}^{n} \beta_j = 1$. The ratings of any alternative y_i in Y on all criterion z_j's are given by a bipolar vague set are $V_{\sigma i} = \{(y_i, (V_{\pi ij}, V_{\mu ij})), j=1,2,\ldots.m\}$ where $V_{\pi ij} \in [0,1]$ and $V_{\mu ij} \in [-1,0]$ are respectively characterize the benefit and the cost that are determined for y_j with respect to z_j. we direct a bipolar vague multi criterion decision making problem in matrices format as follows where the rows represent $\{y_1, \ldots\ldots y_n\}$ the set of alternatives and columns represent the set of criteria $\{z_1, \ldots\ldots z_m\}$.

$$\begin{bmatrix} (V_{\pi 11}, V_{\mu 11}) & (V_{\pi 12}, V_{\mu 12}) & \ldots\ldots\ldots & (V_{\pi 1m}, V_{\mu 1m}) \\ (V_{\pi 21}, V_{\mu 21}) & (V_{\pi 22}, V_{\mu 22}) & \ldots\ldots\ldots & (V_{\pi 2m}, V_{\mu 2m}) \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\ \ldots\ldots\ldots & \ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\ (V_{\pi n1}, V_{\mu n1}) & (V_{\pi n2}, V_{\mu n2}) & \ldots\ldots\ldots & (V_{\pi nm}, V_{\mu nm}) \end{bmatrix} \quad (3.1)$$

$\beta = [\beta_1, \ldots\ldots \beta_m]^T$ where $0 \leq \beta_j \leq 1$, $\sum_{j=1}^{n} \beta_j = 1$ as we considered above. Let $V_p = [V_{\pi ij}, V_{\mu ij}]_{nxm}$. The bipolar vague positive ideal solution and the bipolar vague negative ideal solution are expressed with bipolar vague vectors respectively as follows:

$$BVPIS = [(V_{\pi i1}^+, V_{\mu i1}^+), (V_{\pi i2}^+, V_{\mu i2}^+), \ldots\ldots\ldots (V_{\pi nm}^+, V_{\mu nm}^+)]^T \quad (3.2)$$ and

$$BVNIS = [(V_{\pi i1}^-, V_{\mu i1}^-), (V_{\pi i2}^-, V_{\mu i2}^-), \ldots\ldots\ldots (V_{\pi nm}^-, V_{\mu nm}^-)]^T \quad (3.3)$$

where $V_{\pi ij}^+ = \max_i\{V_{\pi ij}\}$, $V_{\mu ij}^+ = \max_i\{V_{\mu ij}\}$, $V_{\pi ij}^- = \min_i\{V_{\pi ij}\}$ and $V_{\mu ij}^- = \min_i\{V_{\mu ij}\}$, {i=1,2,….n} and {j=1,2,….m}.

we now describe the Euclidean distances of each alternative y_i (i = 1, 2, …. n) from BVPIS and BVNIS by:

$$D(BVPIS, (V_{\pi ij}, V_{\mu ij})) = \sqrt{\tfrac{1}{2}\sum_{j=1}^{m}((V_{\pi ij}^+ - V_{\pi ij})^2 + (V_{\mu ij}^+ - V_{\mu ij})^2)} \quad (3.4)$$ and

$$D(BVPIS, BVNIS) = \sqrt{\tfrac{1}{2}\sum_{j=1}^{m}((V_{\pi ij}^+ - V_{\pi ij}^-)^2 + (V_{\mu ij}^+ - V_{\mu ij}^-)^2)} \quad (3.5)$$

The alternative with minimum value is the best alternative in VIKOR method. According to the above argument the process of bipolar vague VIKOR method can be concise in algorithm as follows:

Algorithm:

Step 1: Construct the bipolar vague decision matrix

Step 2: Normalization of the bipolar vague decision matrix.

Step 3: Define the positive ideal solution and negative ideal solution.

Step 4: Define and compute the value of γ_i and δ_i (i=1, 2, …. n)

γ_i and δ_i characterize the average and worst group scores for the alternative y_i correspondingly with the relations

$$\gamma_i = \sum_{j=1}^{m} \frac{\beta_j \times D(BVPIS,(V_{\pi ij}, V_{\mu ij}))}{D(BVPIS,BVNIS)} \qquad (3.6)$$

$$\text{and } \delta_i = \max_j \left\{ \frac{\beta_j \times D(BVPIS,(V_{\pi ij}, V_{\mu ij}))}{D(BVPIS,BVNIS)} \right\} \quad (3.7)$$

Here, β_j is the weight of z_j.

The smaller values of γ_i and δ_i agree to the better average and worse group scores for alternative y_i correspondingly.

Step 5: Calculate the values of index VIKOR ω_i (i=1, 2, …. n) by the relation

$$\omega_i = \theta \frac{(\gamma_i - \gamma^-)}{(\gamma^+ - \gamma^-)} + (1-\theta) \frac{(\delta_i - \delta^-)}{(\delta^+ - \delta^-)} \qquad (3.8)$$

Here, γ^-= min (γ_i), γ^+ = max (γ_i)

δ^-= min (δ_i), γ^+ = max (δ_i)

and θ portrays the decision-making mechanism coefficient. If θ >0.5, it is for maximum group utility; If $\theta < 0.5$, it is minimum regret; it has been conditional that the decision-making mechanism coefficient is mostly taken as $\theta = 0.5$.

Step 6: Rank the priority of alternatives

We rank the alternatives by ω_i, γ_i and δ_i according to the rule of traditional VIKOR method. The smaller value indicates the better alternative.

Step 7: Compromise solutions

In comprise solution the alternative y(1) which is the best ranked by the measure ω is minimum if the following conditions are satisfied:

Condition 1: Acceptable Advantage:

ω(y(2)) - ω(y(1)) > = Dω where y(2) is the alternative with second position in the ranking list by ω; Dω = 1/(n-1).

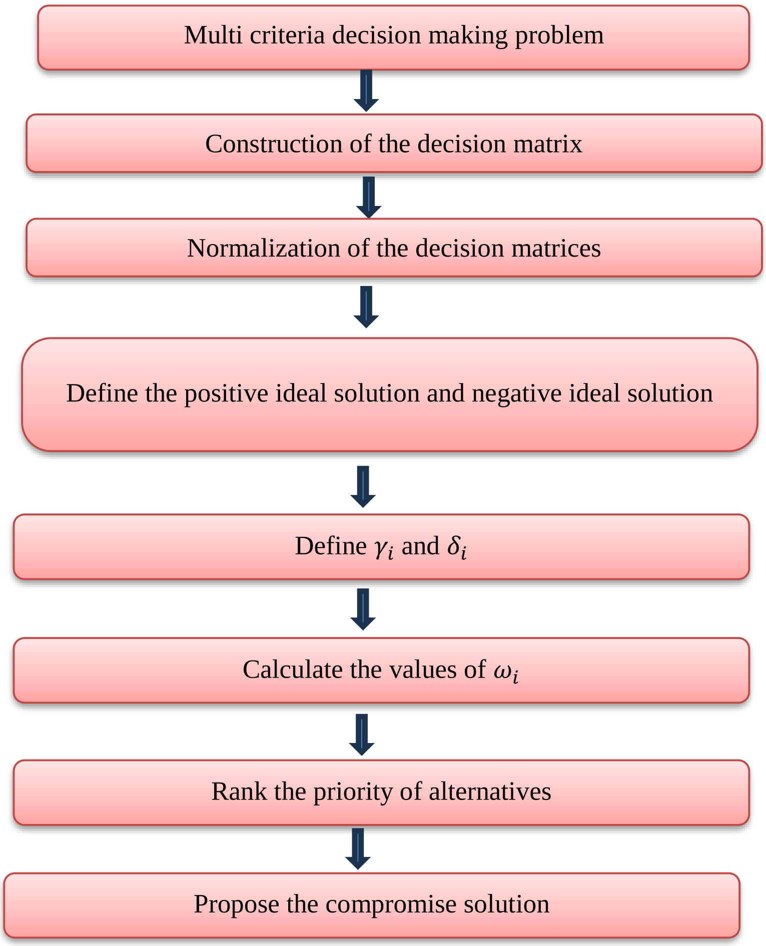
Condition 2: Acceptable stability in decision making:

The alternative y(1) must also be the best ranked by γ or/and δ. This compromise solution is stable within a decision-making process, which could be the method of maximum group utility (when θ >0.5 is needed), or by consensus θ about 0.5, or with $\theta < 0.5$.

If one of the conditions is not satisfied then a set of compromise solutions is proposed which consists of

- Alternative y(1) and y(2) if only the condition 2 is not satisfied or
- Alternative y(1), y(2),………y(n) if the condition 1 is not satisfied y(n) is determined by the relation ω(y(n))−ω(y(1)) < Dω for maximum n.

Figure 1. Decision making procedure of proposed multi criteria decision making method.



IV. APPLICATION OF BIPOLAR VAGUE VIKOR METHOD USING MATLAB

Here in this segment, we have given a mathematical example as an application of bipolar vague VIKOR method.

Example: Undertake that we have a decision maker who is disordered in selecting a smartphone among four smartphones like Vivo, Samsung, Xiaomi and Apple that are showed for sale in some electronics Stores. Suppose that he is think on the following features in order to own his finest smartphone:

(i) Internet access
(ii) Multimedia capabilities
(iii) Applications and software
(iv) Hardware and performance

Since it is known that every feature disturbs the cost and benefit of the planned smartphone. Therefore, these smartphones represent the alternatives and the mentioned features signify the criteria in our multi decision making problem.

Let us denote the smartphones y_1 be Vivo, y_2 be Samsung, y_3 be Xiaomi and y_4 be Apple and Let us denote the criterion by z_1, z_2, z_3 and z_4 be Internet Access, Multimedia capabilities, Applications and software and Hardware and performance respectively. Assume that the weight of the criteria is given by the decision maker in matrices format with bipolar vague values as $\beta = [0.3,0.25,0.2,0.25]^T$. Now we apply the planned multi criteria decision making method to solve the problem using the succeeding steps:

Step 1: Construction of the bipolar vague decision matrix

We construct the decision matrix by equation (3.1) information provided by the decision makers in terms of bipolar vague sets with respect to the criteria as follows:

$$\begin{array}{c} \\ \\ \end{array}\begin{bmatrix} & z_1 & z_2 & z_3 & z_4 \\ y_1 & [0.3,0.4][-0.5,-0.3] & [0.3,0.5][-0.4,-0.3] & [0.1,0.4][-0.3,-0.1] & [0.3,0.4][-0.2,-0.1] \\ y_2 & [0.6,0.8][-0.2,-0.1] & [0.3,0.6][-0.3,-0.2] & [0.5,0.6][-0.2,-0.2] & [0.1,0.4][-0.4,-0.4] \\ y_3 & [0.4,0.6][-0.3,-0.1] & [0.2,0.5][-0.1,-0.1] & [0.6,0.7][-0.4,-0.2] & [0.5,0.6][-0.1,-0.3] \\ y_4 & [0.4,0.6][-0.3,-0.2] & [0.3,0.5][-0.4,-0.2] & [0.2,0.4][-0.4,-0.1] & [0.3,0.6][-0.3,-0.1] \end{bmatrix}$$

Step 2: Normalization of the decision matrix

we do not need to normalize the decision matrix because all the criteria are considered as benefit type.

Step 3: Define the positive ideal solution and negative ideal solution

The positive ideal solution is constructed by equation (3.2) as follows:

BVPIS = [0.6, 0.8] [-0.5, -0.3], [0.3, 0.6] [-0.4, -0.3], [0.6,0.7] [-0.4, -0.2], [0.5, 0.6] [-0.4, -0.4] and

The negative ideal solution is constructed by equation (3) as follows:

BVNIS = [0.3, 0.4] [-0.2, -0.1], [0.2, 0.5] [-0.1, -0.1], [0.1,0.4] [-0.2, -0.1], [0.1, 0.4] [-0.1, -0.1]

Step 4: Calculate γ_i and δ_i

we have calculated the values of γ_i and δ_i by the equations (3.2), (3.3), (3.4) and (3.5). The values are corrected to two decimal places obtainable as follows:

γ_1 = 0.69, γ_2 = 0.53, γ_3 = 0.44, γ_4 = 0.36 and

δ_1 = 0.24, δ_2 = 0.18, δ_3 = 0.25, δ_4 = 0.13

Step 5: Calculate the values of ω_i

Assume θ = 0.5 in the equation (3.6), we get ω_1 = 0.97, ω_2 = 0.46, ω_3 = 0.62 and ω_4 = **0**.

Step 6: Rank the priority of alternatives

The favorite order of the alternatives based on the traditional guidelines of the VIKOR method is $y_4 > y_2 > y_3 > y_1$.

Step 7: Compromise solutions

Table 4.1: The values of γ_i, δ_i and ω_i for all alternatives

	y_1	y_2	y_3	y_4	**Ranking**	**Compromise Solutions**
γ_i	0.69	0.53	0.44	0.36	$y_4 > y_3 > y_2 > y_1$	y_4
δ_i	0.24	0.18	0.25	0.13	$y_4 > y_2 > y_1 > y_3$	y_4
$\omega_i(\theta= 0.5)$	0.97	0.46	0.62	0	$y_4 > y_2 > y_3 > y_1$	y_4

After rank the alternatives by sorting the values of γ_i, δ_i and ω_i in decreasing order. The results are three ranking lists, which is tabulated in Table 4.1. So, the compromise solution is y_4. The parameter θ in VIKOR technique normally considered as (θ = 0.5).

Therefore, the fourth smartphone is the best among the set of smartphones which is considered under study.

V. THE INFLUENCE OF PARAMETER θ

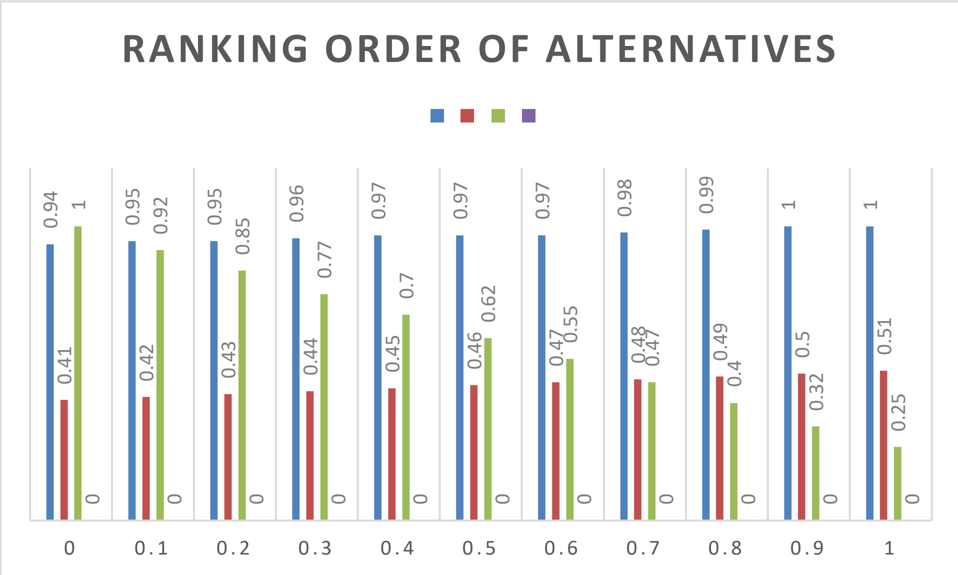
In this segment, we existing sensitivity analysis to demonstration the impact of dissimilar values of the decision-making mechanism coefficient on ranking order of the alternatives.

Fig 2 signifies the graphical representation of alternatives (y_i), i =1,2,3 and 4 for different values of θ. Table 5.1 signifies the ranking order of alternatives (y_i) with the value of θ changing from 0 to 1. Some deviations in ranking of alternatives are occurred but also y_4 has the best ranked alternative for different values of θ.

Table 5.1: Ranking orders of alternatives under different θ values

Values of θ	**Values of ω_i**	**Preference order of alternatives**	**Compromise Solutions**
θ = 0	ω_1 = 0.94, ω_2 = 0.41, ω_3 = 1.0 and ω_4 = 0	$y_4 > y_2 > y_1 > y_3$	y_4
θ = 0.1	ω_1 = 0.95, ω_2 = 0.42, ω_3 = 0.92 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.2	ω_1 = 0.95, ω_2 = 0.43, ω_3 = 0.85 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.3	ω_1 = 0.96, ω_2 = 0.44, ω_3 = 0.77 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.4	ω_1 = 0.97, ω_2 = 0.45, ω_3 = 0.70 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.5	ω_1 = 0.97, ω_2 = 0.46, ω_3 = 0.62 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.6	ω_1 = 0.97, ω_2 = 0.47, ω_3 = 0.55 and ω_4 = 0	$y_4 > y_2 > y_3 > y_1$	y_4
θ = 0.7	ω_1 = 0.98, ω_2 = 0.48, ω_3 = 0.47 and ω_4 = 0	$y_4 > y_3 > y_2 > y_1$	y_4

$\theta = 0.8$	$\omega_1 = 0.99, \omega_2 = 0.49, \omega_3 = 0.40$ and $\omega_4 = 0$	$y_4 > y_3 > y_2 > y_1$	y_4
$\theta = 0.9$	$\omega_1 = 1.0, \omega_2 = 0.50, \omega_3 = 0.32$ and $\omega_4 = 0$	$y_4 > y_3 > y_2 > y_1$	y_4
$\theta = 1$	$\omega_1 = 1.0, \omega_2 = 0.51, \omega_3 = 0.25$ and $\omega_4 = 0$	$y_4 > y_3 > y_2 > y_1$	y_4



VI CONCLUSION

In this paper we have considered an algorithm which is useful to decision makers in evaluating the multi-criteria decision-making problems. We have established bipolar vague VIKOR method with multi-criteria decision-making problem. Lastly, we have solved a multi-criteria decision-making problem to show the feasibility and efficiency of the planned method and also presented a sensitive analysis to demonstrate the impact of different values of the decision-making mechanism coefficient on ranking order of the alternatives.

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