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# On Locally (1,2)Q-Sets In Bitopological Space

[1] Archana Devi S, [2] Abinaya A
[1]Assistant Professor, [2] Student, M.Sc., Mathematics
[1]Department of Mathematics,
Nadar Saraswathi College of Arts and Science, (Autonomous),Theni.

***Abstract***

Bourbaki studied the concept of locally closed sets in topological spaces. Following this topologists introduced the new notions of locally closed sets by replacing open sets by nearly open sets and generalized open sets and/or by replacing the closed sets by nearly closed sets and generalized closed sets. Levine studied the concept of Q-sets in 1961 and Thangavelu and Rao introduced the notion of q-sets in 2002. Recently, the authors further investigated the properties of q-sets and Q-sets in topology. A. Gowri studied the combined study on the concept Locally Q-sets and its properties in 2019. The aim of this paper is to study new concept of sets in Locally (1,2) Q-sets in Bitopological space and its properties.

## I. INTRODUCTION

Kelly introduced the concept of Bitopological space in 1961. Levine introduced the concept of Q-sets in 1061. The concept of q-set was introduced in 2002. The author aimed to explained about the further properties of q-sets and Q-sets. This paper have been studied about the concept of the Locally (1,2) Q-sets in Bitopological space and its properties.

## II. PRELIMINARIES

Throughout this paper $(X,\tau_1,\tau_2)$ and $(Y,\sigma_1,\sigma_2)$ denote the bitopological spaces. $X\backslash A$ is the complement of a subset $A$ of $X$. The interior and closure operators are respectively denoted by $int_1 A$ $and$ $cl_2 A$. The following definitions and lemmas will be useful in sequel. The phrase "iff" is used for the phrase "if and only if". The following definitions and lemmas will be useful in this paper.

### 2.1 DEFINITION

A subset A of a topological space X is called $(1,2)\alpha -$ open if $A \subseteq (int_1\,(cl_2\,(int_1\,A)))and$ $(1,2)\alpha -$ closed if $(int_1(cl_2\,(int_1\,A))) \subseteq A$

### 2.2 DEFINITION

A subset B of a topological space X is called Regular open if $A = int_1(cl_2(A))$ and Regular closed if $A = cl_2(int_1(A))$

### 2.3 DEFINITION

A subset A of a topological space X is called A q-set if $int_1(cl_2(A)) \subseteq cl_2(int_1(A))$

**2.4 DEFINITION**

A subset B of a topological space X is called A Q-set if $int_1(cl_2(A)) = cl_2(int_1(A))$

## III. LOCALLY (1,2)Q-SETS

**3.1 DEFINITION**

Let $(X, \tau_1, \tau_2)$ be a topological space. A subset of X is called a locally (1,2) Q-set if it is the intersection of an open set with a (1,2) Q-set. Also, a subset of $X$ is called a locally (1,2) Q-set if it is the intersection of an open set with a (1,2) Q-set.

**3.2 DEFINITION**

Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function. $f$ is locally (1,2) Q-continuous from $(X, \tau_1, \tau_2)$ to $(Y, \sigma_1, \sigma_2)$ if the inverse image of every open set in $\sigma_1$ is a locally (1,2) Q-set in $(X, \tau_1, \tau_2)$.

**3.3 LEMMA**

$A$ is a $(1,2)Q$-set if and only if $X \setminus A$ is a $(1,2)Q$-set.

**3.4 LEMMA**

If $B$ is open and $A$ is a $(1,2)q$-set then $A \cap B$ and $B \setminus A$ are $(1,2)q$-sets.

**3.5 THEOREM**

Every (1,2) Q-set is a locally (1,2) Q-set and every open set is locally (1,2) Q-set.

**Proof**
**Case(i)**

Every (1,2) Q-set is a locally (1,2) Q-set. Let $A$ be a (1,2) Q-set. Then, by definition, $int_1(cl_2(A)) = cl_2(int_1(A))$Since $A$ is a (1,2) Q-set, we can write $A = A \cap X$, where $X$ is the entire space. Now, $X$ is an open set in $\tau_1$, and $A$ is a (1,2) Q-set. Therefore, $A = A \cap X$ satisfies the definition of a locally (1,2) Q-set. Hence, every (1,2) Q-set A is a locally (1,2) Q-set.

**Case(ii)**

Every open set is a locally (1,2) Q-set. Let $U$ be an open set in $\tau_1$. Then, we can write $U = U \cap X$, where $X$ is the entire space. Since X is a (1,2) Q-set $as\ int_1(cl_2(X)) = cl_2(int_1(X)) = X$we have $U = U \cap X$, where $U$ is an open set and $X$ is a (1,2) Q-set. Therefore, $U$ satisfies the definition of a locally (1,2) Q-set. Hence, every open set $U$ is a locally (1,2) Q-set.

**3.6 THEOREM**

A subset $B$ of a space $X$ is a locally (1,2) Q-set if and only if $X \setminus B$ is a union of a closed set with a (1,2) Q-set.

**Proof**
**Case(i)**

Suppose $B$ is a locally (1,2) Q-set. Then, by definition, there exists an open set $G$ in $X$ and a (1,2) Q-set $A$ in $X$ such that $B = G \cap A$. $X \setminus B = (X \setminus G) \cup (X \setminus A)$, Where, $B = G \cap A$, and taking the complement of both sides. $X \setminus G$ is closed. Since $G$ is open in $X$, its complement $X \setminus G$ is closed. $X \setminus A$ is a (1,2) Q-set. Since $A$ is a (1,2) Q-set in $X$, its complement $X \setminus A$ is also a (1,2) Q-set. $X \setminus B$ is a union of a closed set $(X \setminus G)$ with a (1,2) Q-set $(X \setminus A)$.

**Case(ii)**

Suppose $X\backslash B$ is a union of a closed set $F$ with a (1,2) Q-set $C$. Then $X\backslash B = F \cup C$. $B = X(F \cup C)$. $X\backslash B = F \cup C$, and taking the complement of both sides. $B = (X\backslash F) \cap (X\backslash C)$where $B = X(F \cup C)$, and using the distributive law. $X\backslash F$ is open. Since $F$ is closed in $X$, its complement $X\backslash F$ is open. $X\backslash C$ is a (1,2) Q-set. Since $C$ is a (1,2) Q-set in $X$, its complement $X\backslash C$ is also a (1,2) Q-set. $B$ is a locally (1,2) Q-set, since it is the intersection of an open set $(X\backslash F)$ with a (1,2) Q-set $(X\backslash C)$.

**3.7 THEOREM**

In an extremally disconnected space, every q-set is a (1,2) Q-set and hence every (1,2) Q-set is a locally (1,2) Q-set.

**Proof**

By the definition of (1,2) Q-set and (1,2) Q-set and by the definition of extremally disconnected bitopological space: A bitopological space $(X, \tau_1, \tau_2)$is said to be extremally disconnected if the closure of every open set in $\tau_1$ is open in $\tau_2$.

Firstly; every (1,2) Q-set is a (1,2) Q-set. Let A be a (1,2) Q-set in X. Then, by definition, $int_1(cl_2(A)) \subseteq cl_2(int_1(A))$. Since $X$ is extremally disconnected, $cl_2(A)\ is\ open\ in\ \tau_2, we\ have\ int_1(cl_2(A)) = cl_2(A)$. Also, Since $int_1(A)$ is open in $\tau_1$, it follows that $cl_2(int_1(A))$ is open in $\tau_2$. Given that $int_1(cl_2(A)) \subseteq cl_2(int_1(A))$, we deduce that $cl_2(A) \subseteq cl_2(int_1(A))$. As $cl_2(A)$ is open in $\tau_2$, we have $cl_2(A) = int_1(cl_2(A))$. Therefore, $int_1(cl_2(A)) = cl_2(int_1(A))$, demonstrating that $A$ is a (1,2) Q-set. Now, To prove, every (1,2) Q-set is a locally (1,2) Q-set: Since every (1,2) Q-set is a (1,2) Q-set, and every (1,2) Q-set is a locally (1,2) Q-set (by definition), it follows that every (1,2) Q-set is a locally (1,2) Q-set.

**3.8 THEOREM**

Let $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be two bitopological spaces. Let $(X_1 \times Y_1, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ be the product bitopological space. Let $U$ and $V$ be subsets of $X$ and $Y$, respectively. Then $U \times V$ is a locally (1,2) Q-set in $(X_1 \times Y_1, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ if and only if $U$ is a locally (1,2) Q-set in $(X, \tau_1, \tau_2)$ and $V$ is a locally (1,2) Q-set in $(Y, \sigma_1, \sigma_2)$

**Proof**

**Case(i)**

Suppose $U \times V$ is a locally (1,2) Q-set in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$. Then there exists an open set $G$ in $\tau_1 \times \sigma_1$ and a (1,2) Q-set $A$ in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$such that $U \times V = G \cap A$. Let $p_1: X \times Y \rightarrow X$ and $p_2: X \times Y \rightarrow Y$ be the projection maps. Then $p_1(G)$ is open in $\tau_1$ and $p_2(G)$ is open in $\sigma_1$. $U$ is a locally (1,2) Q-set in $(X, \tau_1, \tau_2)$To see this, note that $p_1(A)$ is a (1,2) Q-set in $(X, \tau_1, \tau_2)$since $A$ is a (1,2) Q-set in$(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$. . Then $U = p_1(G \cap A) = p_1(G) \cap p_1(A)$shows that $U$ is a locally (1,2) Q-set in $X, \tau_1, \tau_2)$. Similarly, $V$ is a locally (1,2) Q-set in $(Y, \sigma_1, \sigma_2)$.

**Case(ii)**

Conversely, suppose $U$ is a locally (1,2) Q-set in $X, \tau_1, \tau_2)$ and $V$ is a locally (1,2) Q-set in $(Y, \sigma_1, \sigma_2)$. Then there exist open sets $G_1$ in $\tau_1$ and $G_2$ in $\sigma_1$, and (1,2) Q-sets $A_1$ in $(X, \tau_1, \tau_2)$ and $A_2$ in $(Y, \sigma_1, \sigma_2)$ such that $U = G_1 \cap A_1$ and $V = G_2 \cap A_2$. Then $G_1 \times G_2$ is open in $\tau_1 \times \sigma_1$, and $A_1 \times A_2$ is a (1,2) Q-set in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$. Moreover, $U \times V = (G_1 \cap A_1) \times (G_2 \cap A_2) = (G_1 \times G_2) \cap (A_1 \times A_2)$ shows that $U \times V$ is a locally (1,2) Q-set in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$.

**3.9 THEOREM**

Suppose $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a bijection. Then $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is locally (1,2) Q-continuous if and only if $f^{-1}(H)$ is a union of a closed set in $(X, \tau_1)$ with a (1,2) Q-set in $(X, \tau_1, \tau_2)$for every closed set $H$ in $(Y, \sigma_1)$ .

**Proof**

**case(i)**

Suppose $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous. Let $H$ be a closed set in $(Y,\ \sigma_1)$. Then $Y\backslash H$ is open in $(Y,\sigma_1\ )$. Since f is locally (1,2) Q-continuous, $f^{-1}(Y\backslash H)$is a locally (1,2) Q-set in $(X,\tau_1,\tau_2)$. By definition of locally (1,2) Q-set, there exists an open set $G$ in $(X,\tau_1)$ and a (1,2) Q-set $A$ in $(X,\tau_1,\tau_2)$ such that $f^{-1}(Y\backslash H)=G\ \cap\ A$. Taking the complement of both sides, $f^{-1}(H)=\ X(G\ \cap\ A)=\ (X\backslash G)\cup\ (X\backslash A)$. Since $G$ is open in $(X,\tau_1), X\backslash G$ is closed in $(X,\tau_1)$. Since $A$ is a (1,2) Q-set in $(X,\tau_1,\tau_2), X\backslash A$ is also a (1,2) Q-set in $(X,\tau_1,\tau_2)$.

**Case(ii)**

Conversely, suppose $f^{-1}(H)$ is a union of a closed set in $(X,\tau_1)$with a (1,2) Q-set in $(X,\tau_1,\tau_2)$ for every closed set $H$ in $(Y,\sigma_1)$. Let $U$ be an open set in $(Y,\sigma_1)$. Then $Y\backslash U$ is closed in $(Y,\sigma_1)$

By assumption, $f^{-1}(Y\backslash U)$ is a union of a closed set in $(X,\tau_1)$ with a (1,2) Q-set in $(X,\tau_1,\tau_2)$.

Taking the complement of both sides, we get $f^{-1}(U)=X\big(f^{-1}(Y\backslash U)\big)$ is a locally (1,2) Q-set in $(X,\tau_1,\tau_2)$Therefore, $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous.

**3.10 THEOREM**

If $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous, then for every x∈X and for every open set $V$ in $(Y,\sigma^1)$ with $f(x)\in V$, there is a locally (1,2) Q-set $G\ \cap\ A$ in $(X,\tau_1,\tau_2)$ such that $x\ \in\ G\ \cap\ A$ and $f(G\ \cap\ A)\subseteq V$.

**Proof**

Since $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous, for every open set $V$ in $(Y,\sigma^1)$ , $f^{-1}(V)$ is a locally (1,2) Q-set in : $(X,\tau_1,\tau_2)$. By definition of locally (1,2) Q-set, there exists an open set $G$ in $(X,\ \tau_1)$ and a (1,2) Q-set $A$ in : $(X,\tau_1,\tau_2)$such that $f^{-1}(V)=\ G\ \cap\ A$. Since $f(x)\in\ V, x\ \in\ f^{-1}(V)=\ G\ \cap\ A$. Since $f^{-1}(V)=\ G\ \cap\ A$, we have $f(G\ \cap\ A)\subseteq\ V$. Therefore, for every $x\ \in\ X$ and for every open set $V$ in $(Y,\sigma_1)$with $f(x)\in\ V$, there is a locally (1,2) Q-set $G\ \cap\ A$ in $(X,\tau_1,\tau_2)$ such that $x\ \in\ G\ \cap\ A$ and $f(G\ \cap\ A)\subseteq\ V$.

**3.11 THEOREM**

Let $(X,\tau_1,\tau_2)$ be a bitopological space such that the class of locally (1,2) Q-sets is closed under arbitrary union. Then for any function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$, the following are equivalent:

(a) $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous.

(b) $f^{-1}(H)$ is a union of a closed set in $(X,\tau_1)$ with a (1,2) Q-set in $(X,\tau_1,\tau_2)$for every closed set $H$ in $(Y,\sigma_1)$.

(c) For every $x\ \in\ X$ and for every open set $V$ in $(Y,\sigma_1)$ with $f(x)\in\ V$, there is a locally (1,2) Q-set $G\ \cap\ A$ in $(X,\tau_1,\tau_2)$such that $x\ \in\ G\ \cap\ A$ and $f(G\ \cap\ A)\ \subseteq\ V$.

**Proof**

**Case(i)** (a) → (b)

Suppose $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is locally (1,2) Q-continuous. Let $H$ be a closed set in $(Y,\sigma_1)$. Then $Y\backslash H$ is open in $(Y,\sigma_1)$. Since $f$ is locally (1,2) Q-continuous, $f^{-1}(Y\backslash H)$is a locally (1,2) Q-set in $(X,\tau_1,\tau_2)$. By definition of locally (1,2) Q-set, there exists an open set $G$ in$(X,\tau_1)$ and a (1,2) Q-set $A$ in $(X,\tau_1,\tau_2)$ such that $f^{-1}(Y\backslash H)=\ G\ \cap\ A$. Taking the complement of both sides, we get $f^{-1}(H)=\ X(G\ \cap\ A)=\ (X\backslash G)\cup\ (X\backslash A)$. Since $G$ is open in $(X,\tau_1)$X\G is closed in $(X,\tau_1)$. Since $A$ is a (1,2) Q-set in $(X,\tau_1,\tau_2)$, $X\backslash A$ is also a (1,2) Q-set in $(X,\tau_1,\tau_2)$.

**Case(ii)** (b) → (c)

Suppose $f^{-1}(H)$ is a union of a closed set in $(X,\tau_1)$with a (1,2) Q-set in $(X,\tau_1,\tau_2)$ for every closed set $H$ in $(Y,\sigma_1)$. Let $x\ \in\ X$ and $V$ be an open set in $(Y,\sigma_1)$. with $f(x)\in\ V$. Then $f^{-1}(V)$ is a locally (1,2) Q-set in $(X,\tau_1,\tau_2)$. By definition of locally (1,2) Q-set, there exists an open set $G$ in $(X,\tau_1)$and a (1,2) Q-set $A$ in $(X,\tau_1,\tau_2)$such that $f^{-1}(V)=\ G\ \cap\ A$. Since $x\ \in\ f^{-1}(V)$, we have $x\ \in\ G\ \cap\ A$. Since $f^{-1}(V)=\ G\ \cap\ A$, we have $f(G\ \cap\ A)\subseteq\ V$.

**Case(iii)** (c) → (a)

Suppose for every $x \in X$ and for every open set $V$ in $(Y, \sigma_1)$ with $f(x) \in V$, there is a locally (1,2) Q-set $G \cap A$ in $(X, \tau_1, \tau_2)$ such that $x \in G \cap A$ and $f(G \cap A) \subseteq V$. Then $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is locally (1,2) Q-continuous.

**3.12 THEOREM**

Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a locally (1,2) Q-continuous function. If $H$ is a closed set in $(Y, \sigma_1)$, then $f^{-1}(H)$ is a union of a closed set in $(X, \tau_1)$ with a (1,2) Q-set in $(X, \tau_1, \tau_2)$.

**Proof**

Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a locally (1,2) Q-continuous function. Then $Y \setminus H$ is open in $(Y, \sigma_1)$. Since $f$ is locally (1,2) Q-continuous, $f^{-1}(Y \setminus H)$ is a locally (1,2) Q-set in $(X, \tau_1, \tau_2)$. By definition of locally (1,2) Q-set, there exists an open set $G$ in $(X, \tau_1)$and a (1,2) Q-set $A$ in $(X, \tau_1, \tau_2)$ such that $f^{-1}(Y \setminus H) = G \cap A$. Taking the complement of both sides, we get $f^{-1}(H) = X(G \cap A) = (X \setminus G) \cup (X \setminus A)$. Since $G$ is open in $(X, \tau_1, ), X \setminus G$ is closed in $(X, \tau_1)$. Since $A$ is a (1,2) Q-set in $(X, \tau_1, \tau_2)$, $X \setminus A$ is also a (1,2) Q-set in $(X, \tau_1, \tau_2)$. Therefore, $f^{-1}(H)$ is a union of a closed set $(X \setminus G)$ in $(X, \tau_1)$ with a (1,2) Q-set $(X \setminus A)$ in $(X, \tau_1, \tau_2)$.

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