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On Nano$(1,2)^*$Generalized Semi Continuous And Irresolute In Nano Bitopological Space

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Abstract: The main aspect is to introduced and study some new class of function called Nano $(1,2)^*$ generalized semi Continuous function (briefly $N(1,2)^*$ gscf) and Nano $(1,2)^*$ generalized semi Irresolute function(briefly $N(1,2)^*$ gsif) Also, introduce and study about Nano $(1,2)^*$ semi-interior , Nano $(1,2)^*$ semi-closure and Nano $(1,2)^*$ generalized semi Continuous and Irresolute. And also to investigate some properties of these sets and to study some properties, remarks related to these sets.

Index terms:N $(1,2)^*$ generalized semi closed sets, N $(1,2)^*$ generalized semi Continuous function, N$(1,2)^*$generalized semi Irresolute function.

I. Itroduction

This paper delves into the fundamental definitions and properties of **nanogeneralized semi-continuity** and **irresoluteness**, focusing on their role within the framework of **nano bitopological spaces**. By examining the intricate relationships between open sets, closures, and continuity in such spaces, we aim to extend the classical notions and provide new perspectives on their applications in areas like quantum computing, nanotechnology, and theoretical physics. The interplay between these topological concepts is expected to shed light on their importance in modeling phenomena at the nanoscale and could lead to novel applications in emerging technological fields.

II. Prelimnaries

2.1 Definition

Let U be a non empty set, finite universe of objects and R_1, R_2 are two an equivalence relations on U. Let $X_1, X_2 \subseteq U$. Let

$\tau_{R_1}(X_1) = \{\emptyset, U, L_{R_1}(X_1), U_{R_1}(X_1), B_{R_1}(X_1)\}$

$\tau_{R_2}(X_2) = \{\emptyset, U, L_{R_2}(X_2), U_{R_2}(X_2), B_{R_2}(X_2)\}$. Then $(U, \tau_{R_{1,2}}(X))$ is called the Nano- Bitopological Space.

2.2 Definition

Let U be a non-empty set, finite Universe of objects and $\mathrm{R}_1, \mathrm{R}_2$ are two equivalence relations on U. Let $X_1, X_2 \subseteq U$. Let S is said to be **Nano $(1,2)^*$ Open** if $S = A \cup B$ where $A \in \tau_{R_1}(X)$ and $B \in \tau_{R_2}(X)$.

2.3 Definition

The Complement of $(1,2)^*$Open set is called **Nano $(1,2)^*$Closed**.

2.4 Definition

Let A be subset of a Nano bitopological space $(U, \tau_{R_{1,2}}(X))$. Then

The **Nano$(1,2)^*$–Interior** of A, denoted by $N\,\tau_{R_{1,2}}Int(A)$ is defined as

$$\cup\,\{\boldsymbol{F} : \boldsymbol{F} \subseteq \boldsymbol{A}\ \boldsymbol{and}\ \boldsymbol{F}\ \boldsymbol{is}\ \boldsymbol{N\tau_{R_{1,2}}}(\boldsymbol{X})\boldsymbol{open}\}$$

The **Nano$(1,2)^*$–Closure** of A, denoted by $N\tau_{R_{1,2}}Cl(A)$ is defined as

$\cap \{ \mathbf{F} : \mathbf{A} \subseteq \mathbf{F} \text{ and } \mathbf{F} \text{ is } \mathbf{N}\boldsymbol{\tau}_{R_{1,2}}(X)\mathbf{Closed} \}$

2.5 Definition

Let A be subset of a Nano bitopological space $(U, \tau_{R_{1,2}}(X))$ and $A \subseteq U$.

The **Nano$(\mathbf{1},\mathbf{2})^*$ Semi open** if $A \subseteq N\tau_{R_{1,2}}cl[N\tau_{R_{1,2}}Int(A)]$

The **Nano $(\mathbf{1},\mathbf{2})^*$Semi closed** if $N\tau_{R_{1,2}}Int[N\tau_{R_{1,2}}cl(A)] \subseteq A$

2.6 Definition

If$(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X and if $A \subseteq X$, Then

(i)The **Nano$(\mathbf{1},\mathbf{2})^*$semi-closure** of A is defined as the intersection of all $\boldsymbol{N}(\mathbf{1},\mathbf{2})^*$ semi closed sets containing A and it is denoted by $N\tau_{R_{1,2}}scl(A)$. $N\tau_{R_{1,2}}scl(A)$ is the smallest $N(1,2)^*$semi closed set containing A.

(ii)The **Nano$(\mathbf{1},\mathbf{2})^*$semi-interior** of A is defined as the union of all $\boldsymbol{N}(\mathbf{1},\mathbf{2})^*$semi open subsets of A contained in A and it is denoted by $N\tau_{R_{1,2}}sInt(A)$. $N\tau_{R_{1,2}}sInt(A)$ is the largest $\boldsymbol{N}(\mathbf{1},\mathbf{2})^*$ semi open subset of A.

2.7 Definition

A subset A of $(U, \tau_{R_{1,2}}(X))$ is called **Nano $(\mathbf{1},\mathbf{2})^*$generalized-semi closed set** (briefly $\boldsymbol{N}(\mathbf{1},\mathbf{2})^*$gs closed) if $N\tau_{R_{1,2}}scl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N(1,2)^*$open in $(U, \tau_{R_{1,2}}(X))$.

2.7.1 Example

Let $U = \{a, b, c, d\}$ $U/R = \{\{c\}, \{d\}, \{a, b\}\}$ and

$X_1 = \{a, c\}$ and $\tau_{R_1}(X) = \{\emptyset, U, \{c\}\{a, b, c\}, \{a, b\}\}$.

$X_2 = \{a, b\}$ and $\tau_{R_2}(X) = \{\emptyset, U, \{d\}, \{a, b, d\}, \{a, b\}\}$.

Then $\tau_{R_{1,2}}(X) = \{\emptyset, U, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

The Nano$(1,2)^*$closed sets $= \{\emptyset, U, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

The Nano$(1,2)^*$semi closed sets $= \{\emptyset, U, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

The Nano$(1,2)^*$semi open sets $= \{\emptyset, U, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, b\}, \{d\}, \{c\}\}$

The Nano$(1,2)^*$generalized-semi open sets are

$\{\emptyset, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

The Nano$(1,2)^*$generalized-semi closed sets are

$\{\emptyset, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

2.8 Defenition

A subset A of $(X, \tau_{1,2})$ is called a **Nano$(\mathbf{1},\mathbf{2})^*$ generalized semi closed set** (briefly $\boldsymbol{\tau}_{1,2,}$ gsclosed set) if $\boldsymbol{\tau}_{1,2}scl(A) \subseteq U$, whenever $A \subseteq U$, U is $(1,2)^*$ open in X.

2.8.1 Example

Let $U = \{p, q, r, s\}$ with $U \backslash R_1 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_1 = \{p, r\}$. Then $\boldsymbol{\tau}_{R_1}(X) = \{U, \emptyset, \{r\}, \{p, q\}, \{p, q, r\}\}$.

$U \backslash R_2 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_2 = \{r, s\}$. Then

$\boldsymbol{\tau}_{R_2}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}\}$.

$\boldsymbol{\tau}_{R_{1,2}}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}, \{p, q\}, \{p, q, r\}\}$

are Nano$(1,2)^*$open sets.

$[\boldsymbol{\tau}_{R_{1,2}}]^c = \{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$ are Nano$(1,2)^*$ closed sets.

Nano$(1,2)^*$semi-open sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano$(1,2)^*$semi-closed sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano $(1,2)^*$ gs-closed sets are $\{U, \emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$.

2.9 Definition

A function $f : (U, \tau_{R_{1,2}}(\mathrm{X})) \rightarrow (V, \tau_{R'_{1,2}}(Y))$ from a Nano Bitopological space X with a Nano Bitopological space Y is called **Nano$(\mathbf{1},\mathbf{2})^*$ generalized semi continuous function** (briefly Nano$(1,2)^*gscf$) if $f^{-1}(V)$ is Nano$(1,2)^*$ gs-closed in X for every closed set V in Y.

2.9.1 Example

Let $U = \{p, q, r, s\}$ with $U \backslash R_1 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_1 = \{p, r\}$. Then $\boldsymbol{\tau}_{R_1}(X) = \{U, \emptyset, \{r\}, \{p, q\}, \{p, q, r\}\}$.

$U \backslash R_2 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_2 = \{r, s\}$. Then

$\boldsymbol{\tau}_{R_2}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}\}$.

$\boldsymbol{\tau}_{R_{1,2}}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}, \{p, q\}, \{p, q, r\}\}$ are Nano$(1,2)^*$ open sets.

$[\boldsymbol{\tau}_{R_{1,2}}]^c = \{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$ are Nano$(1,2)^*$ closed sets.

Nano$(1,2)^*$semi-open sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano$(1,2)^*$semi-closed sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano $(1,2)^*$ gs-closed sets are $\{U, \emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$.

Let $V = \{1,2,3,4\}$ with $V \backslash R_1 = \{\{1\}, \{3\}, \{2,4\}\}$ and $X_1 = \{2,3\}$,

Then $\tau_{R_1}(Y) = \{V, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$

$\tau_{R_{1,2}}(Y) = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$ are $N(1,2)^*$open sets.

$[\tau_R(Y)]^C = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}$ are $N(1,2)^*$closed sets.

Define a mapping $f: U \to V$ as $f^{-1}(1) = q, f^{-1}(2) = r, f^{-1}(3) = p$ and $f^{-1}(4) = s$. That is image of every Nano closed sets in $(V, \tau_R(Y))$ is $N(1,2)^*$gs-closed set in U. Therefore the function f is $N(1,2)^*$gscf.

2.10 Definition

Let $(U, \tau_{R_{1,2}}(X))$ and $(V, \tau_{R'_{1,2}}(Y))$ be two Nano$(1,2)^*$ topological space. Then a function $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is said to be Nano$(1,2)^*$generalized Semi Irresolute function (briefly N$(1,2)^*$gsif) if the inverse image of every N$(1,2)^*$gs open set in V is N$(1,2)^*$gs open in $\tau_{R_{1,2}}$U.

2.10.1 Example

Let $U = \{a, b, c, d\}$ with $U \backslash R = \{a, c\}, \{b\}, \{d\}\}$ and $X_1 = \{a, b\}, X_2 = \{c, d\}$

Then the Nano$(1,2)^*$topology is defined as

$\boldsymbol{\tau}_{R_1}(X) = \{U, \emptyset, \{a\}, \{c\}, \{a, b, d\}, \{b, c, d\}, \{b, d\}\}$.

Let $V = \{1,2,3,4\}$ with $V \backslash R = \{\{1,3\}, \{2\}, \{4\}\}$

Then $\tau_{R_{1,2}}(X) = \{V, \emptyset, \{1\}, \{2\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$.

Define $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ as $f(1) = b, f(2) = d, f(3) = a, f(4) = c$.

Then f is Nano$(1,2)^*$generalized Irresolute. Since the inverse image of every $N(1,2)^*$gs set in V is $N(1,2)^*$gs open set in U.

III. Theorems On Nano$(1,2)^*$ Generalized Semi Continuous Function

3.1 Theorem

Let $(U, \tau_{R_{1,2}}(X))$ and $(V, \tau_{R'_{1,2}}(Y))$ are any two Nano Bitopological spaces.Let $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ be $N(1,2)^*$continuous function. If f is $N(1,2)^*$cf, then f is $N(1,2)^*$gscf.

Proof:

Let S be any Nano $(1,2)^*$ closed set in $(V, \tau_{R'_{1,2}}(Y))$. Then $f^{-1}(S)$ is Nano$(1,2)^*$ gs closed in $(U, \tau_{R_{1,2}}(X))$. Since, every $N(1,2)^*$ closed set is Nano$(1,2)^*$ gs-closed. Hence, $f^{-1}(S)$ is $N(1,2)^*$ gs-closed in $(U, \tau_{R_{1,2}}(X))$. Therefore, f is $N(1,2)^*$ gscf.

3.1.2 Example

Let $U = \{p, q, r, s\}$ with $U \backslash R_1 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_1 = \{p, r\}$. Then $\boldsymbol{\tau}_{R_1}(X) = \{U, \emptyset, \{r\}, \{p, q\}, \{p, q, r\}\}$.

$U \backslash R_2 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_2 = \{r, s\}$. Then

$\boldsymbol{\tau}_{R_2}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}\}$.

$\boldsymbol{\tau}_{R_{1,2}}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}, \{p, q\}, \{p, q, r\}\}$

are Nano$(1,2)^*$ open sets.

$[\boldsymbol{\tau}_{R_{1,2}}]^c = \{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$ are Nano$(1,2)^*$ closed sets.

Nano $(1,2)^*$ gs-closed sets are $\{U, \emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$.

Let $V = \{1,2,3,4\}$ with $V \backslash R_1 = \{\{1\}, \{3\}, \{2,4\}\}$ and $X_1 = \{2,3\}$,

Then $\tau_{R_1}(Y) = \{V, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$

$\tau_{R_{1,2}}(Y) = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$ are $N(1,2)$*open sets.

$[\tau_R(Y)]^C = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}$ are $N(1,2)$*closed sets.

Define a mapping $f: U \to V$ as $f^{-1}(1) = q, f^{-1}(2) = r, f^{-1}(3) = p$ and $f^{-1}(4) = s$. That is image of every Nano closed sets in $(V, \tau_R(Y))$ is $N(1,2)^*$gs-closed set in U. Therefore the function f is $N(1,2)^*$gscf.

3.2 Theorem

Let $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ be $N(1,2)^*$ continuous function and $(U, \tau_{R_{1,2}}(X))$ and $(V, \tau_{R'_{1,2}}(Y))$ are any two Nano Bitopological spaces. If f is $N(1,2)^*$ s cf, then f is $N(1,2)^*$gs cf.

Proof:

Let S be any $N(1,2)^*$ closed set in $(V, \tau_{R'_{1,2}}(Y))$ then $f^{-1}(S)$ is $N(1,2)^*$ semi-closed in $(U, \tau_{R_{1,2}}(X))$ as f is $N(1,2)^*$ semi cf. Since, every $N(1,2)^*$semi-closed set is $N(1,2)^*$gs closed. Hence, $f^{-1}(S)$ is $N(1,2)^*$gs-closed in $(U, \tau_{R_{1,2}}(X))$. Therefore f is $N(1,2)^*$gs cf.

3.2.1 Remark

The converse of the Theorem need not be true as seen from the following example.

3.2.2 Example

Let $U = \{p, q, r, s\}$ with $U \backslash R_1 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_1 = \{p, r\}$. Then $\boldsymbol{\tau}_{R_1}(X) = \{U, \emptyset, \{r\}, \{p, q\}, \{p, q, r\}\}$.

$U \backslash R_2 = \{\{r\}, \{s\}, \{p, q\}\}$ and $X_2 = \{r, s\}$. Then

$\boldsymbol{\tau}_{R_2}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}\}$

$\boldsymbol{\tau}_{R_{1,2}}(X) = \{U, \emptyset, \{r\}, \{s\}, \{r, s\}, \{p, q\}, \{p, q, r\}\}$

are Nano$(1,2)^*$ open sets.

$[\boldsymbol{\tau}_{R_{1,2}}]^c = \{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$are Nano$(1,2)^*$closed sets.

Nano$(1,2)^*$semi-open sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano$(1,2)^*$semi-closed sets are $\{U, \emptyset, \{r\}, \{s\}, \{p, q\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}\}$.

Nano$(1,2)^*$ gs-closed sets are $\{U, \emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$.

Let $V = \{1,2,3,4\}$ with $V \backslash R_1 = \{\{1\}, \{3\}, \{2,4\}\}$ and $X_1 = \{2,3\}$,

Then $\tau_{R_1}(Y) = \{V, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$

$\tau_{R_{1,2}}(Y) = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$ are $N(1,2)^*$open sets.

$[\tau_R(Y)]^C = \{V, \emptyset, \{1\}, \{3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}$ are $N(1,2)^*$closed sets.

Define a mapping $f: U \to V$ as $f^{-1}(1) = r, f^{-1}(2) = s, f^{-1}(3) = p$ and $f^{-1}(4) = q$. That is image of every Nano closed sets in $(V, \tau_R(Y))$ is $N(1,2)^*$gs-closed set in U. Therefore the function f is $N(1,2)^*$gscf. But not f is pre closed as the inverse image of a closed set $\{1,2,4\}$ in $(V, \tau_R(Y))$ is $\{q, r, s\}$ is which is not closed in $(U, \tau_{R_{1,2}}(X))$.

3.3 Theorem

Let $(U, \tau_{R_{1,2}}(X))$ and $(V, \tau_{R'_{1,2}}(Y))$ be any two Nano Bitopologicalspaces.If a function $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is $N(1,2)^*$sif then it is $N(1,2)^*$gs cfs.

Proof:

Assume that $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is Nano$(1,2)^*$sif. Let S be any Nano$(1,2)^*$closed set in $(V, \tau_{R'_{1,2}}(Y))$. Since , every Nano$(1,2)^*$ closed set is Nano$(1,2)^*$ semi closed. Then S is Nano$(1,2)^*$ semi-closed in $(V, \tau_{R'_{1,2}}(Y))$. Since f is Nano$(1,2)^*$sif, $f^{-1}(S)$ is Nano$(1,2)^*$ semi-closed in $(U, \tau_{R_{1,2}}(X))$. We know that, every Nano$(1,2)^*$semi-closed set in $N(1,2)^*$gs-closed. Hence, $f^{-1}(S)$ is Nano$(1,2)^*$gs-closed in $(U, \tau_{R_{1,2}}(X))$.

IV. THEOREMS ON NANO$(1,2)^*$ GENERALIZED SEMI IRRESOLUTE FUNCTION

4.1 Theorem

A function $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is $N(1,2)^*$gs if, then f is $N(1,2)^*$gs cf.

Proof:

Every $N(1,2)^*$ open set in Nano generalized open. The inverse image of every $N(1,2)^*$gs open set in V is $N(1,2)^*$gs open set in U, Whenever the inverse image of every $N(1,2)^*$gs open set is $N(1,2)^*$gs open. Therefore, any $N(1,2)^*$gsif is $N(1,2)^*$gscf.

4.1.1 Remark

The converse of above theorem need not be true shown in the follwing example.

4.1.2 Example

Let $U = \{a, b, c, d\}$ with $U \backslash R = \{a, c\}, \{b\}, \{d\}\}$ and $X_1 = \{a, b\}, X_2 = \{c, d\}$

Then the $\text{Nano}(1,2)^*$topology is defined as

$\tau_{R_{1,2}}(X) = \{U, \emptyset, \{a\}, \{c\}, \{a, b, d\}, \{b, c, d\}, \{b, d\}\}$.

Let $V = \{1,2,3,4\}$ with $V \backslash R = \{\{1,3\}, \{2\}, \{4\}\}$

Then $\tau_{R_{1,2}}(X) = \{V, \emptyset, \{1\}, \{2\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$.

Define $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ as $f(1) = b, f(2) = d, f(3) = a, f(4) = c$.

Then f is $N(1,2)^*$gcf.

But f is not $N(1,2)^*$gif.

$f^{-1}(a) = \{3\}$ which is $\text{Nano}(1,2)^*$gs open in V where as {a} is $N(1,2)^*$gs open in V.

Thus a $N(1,2)^*$gscf is not $\text{Nano}(1,2)^*$gsif.

4.2 Theorem

Let $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ to be C and $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ to be irresolute but are unrelated.

Proof:

Every $\text{Nano}(1,2)^*$ Nano open set in Nano generalized open.

The inverse image of every $N(1,2)^*$gs open set in V is $\text{Nano}(1,2)^*$gs open set in U, whenever the inverse image of every $\text{Nano}(1,2)^*$gs open set is $N(1,2)^*$gs open.

Therefore, any $N(1,2)^*$gs-I function is $\text{Nano}(1,2)^*$gscf.

$f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ to be continuous.

$f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ to be Irresolute.

But they are unrelated.

4.3 Theorem

$f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is $N(1,2)^*$ g-I iff the inverse image every Nano $(1,2)^*$ semi closed in V is $N(1,2)^*$ gs closed in U.

Proof:

Let V be any $\text{Nano}(1,2)^*$ open set in $(V, \tau_{R'_{1,2}}(Y))$

Since f is $N(1,2)^*$ cf

$f^{-1}(Y)$ is $N(1,2)^*$ open in U.

Since every $N(1,2)^*$ open set in $N(1,2)^*$ gs open set.

So $f^{-1}(Y)$ is $N(1,2)^*$ gs open in U.

Thus, the inverse image of every $\text{Nano}(1,2)^*$ gs open set.

Thus f is $N(1,2)^*$ gs closed set in U.

4.4 Theorem

If X is $\text{Nano}(1,2)^*$ generalized closed set in $(U, \tau_{R_{1,2}}(X))$ and if the function $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ is $N(1,2)^*$ closed, then f(X) is $N(1,2)^*$ gs closed in $(V, \tau_{R'_{1,2}}(Y))$.

Proof:

Let $f: (U, \tau_{R_{1,2}}(X)) \to (V, \tau_{R'_{1,2}}(Y))$ be $N(1,2)^*$ g scf and Y be $\text{Nano}(1,2)^*$ open in $(V, \tau_{R'_{1,2}}(Y))$. Then F(Y) is closed in $(V, \tau_{R'_{1,2}}(Y))$. Since f is $N(1,2)^*$ gsclosed in $(U, \tau_{R_{1,2}}(X))$

$f^{-1}(Y)$ is $N(1,2)^*$ gs open in $(U, \tau_{R_{1,2}}(X))$

thus the inverse image of every Nano $(1,2)^*$ open set in $(V, \tau_{R'_{1,2}}(Y))$ is $N(1,2)^*$ gsopen in $(U, \tau_{R_{1,2}}(X))$.

Conversely, assume that $f^{-1}(Y)$ is Nano gs open in $(U, \tau_{R_{1,2}}(X))$ for each $\text{Nano}(1,2)^*$ open set in F in $(V, \tau_{R'_{1,2}}(Y))$. Let Y be a $\text{Nano}(1,2)^*$ closed set in $(V, \tau_{R'_{1,2}}(Y))$. Then generalized closed set in $\text{Nano}(1,2)^*$ in $(V, \tau_{R'_{1,2}}(Y))$ by assumption. $f^{-1}(X)$ is $N(1,2)^*$ gsopen in $(U, \tau_{R_{1,2}}(X))$

F is $N(1,2)^*$ gscf

F is $N(1,2)^*$ gs closed set.

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