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A Note On Replenishment Policy For An Inventory Model Of Deteriorating Items With Additive Exponential Life Time Under Linear Trended Stochastic Demand

Dr. Biswaranjan Mandal
Associate Professor of Mathematics
Acharya Jagadish Chandra Bose College, Kolkata, West Bengal, India

Abstract:

A collaborative replenishment policy in an inventory system between the venders and the customers is most important. In the present paper, we deal with an replenishment policy of an inventory model for deteriorating with stochastic demand in which shortages are allowed with fully backlogged. Here deteriorating items with lifetime is followed an additive exponential distribution. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is minimized to the total average cost by finding optimal values. The developed model is illustrated by a numerical example and lastly the sensitivity analysis of the optimal solutions towards the changes in the values of key parameters has been presented.

Key Words: Replenishment, deteriorating items, additive exponential, linear trended, stochastic demand and shortages.
Subject classification: AMS Classification No. 90B05

1. Introduction:

The traditional inventory model of Haris F[1] considers the idea in which depletion of inventory is caused by a constant demand rate alone. In real-life, it was noticed that the depletion of inventory may take place due to deterioration also. Many researchers like Shah [2], Datta and Pal [3] etc names only a few. Moreover several researchers like Goyal et al [4], Martin [5], Rao [6], R. Li, H. Lan et al [7], Biswaranjan Mandal [8, 9] have developed the inventory model of deteriorating items under additive exponential lifetime in nature. In this paper my development based on the assumption where the lifetime of the commodity is random which is sum of two variables namely natural life and extended life. This extended life of commodity occurs mainly due to cold storage facilities, humidity, chemical treatment etc. So the lifetime of the commodity is to be approximated with an additive exponential distribution having the probability density function of the form

$$f(t) = \frac{e^{-\frac{t}{\theta_1}} - e^{-\frac{t}{\theta_2}}}{\theta_1 - \theta_2}, \theta_1 > \theta_2 \text{ and } t \geq 0.$$

The assumption of constant demand rate may not be always appropriate for many inventory management systems. For examples, milk items, vegetables, fruits, cosmetics etc have a negative impact on demand due to loss of confidence of consumers on the quality of such products for their age of inventory. Also we noticed that the demand of seasonal foods and garments is highly dependent on time. So it can be concluded that demand for items varies with respect to time. However, in order to match with real-life criteria, many authors have developed new types of inventory models with a variable demand rate. Also the acceptance of some constant demand rate is not reasonable for many inventory items such as electronic goods, fashionable garments, tasty foods, volatile liquids etc , as they fluctuate in the demand rate. As a result, linear trended dependent rate on to time with a stochastic error has a prominent role in inventory control system. Researchers like U Dave et al [10], T K Datta et al [11], J T Teng et al [12], Biswaranjan[13], M Mallick et al [14], A Kundu [15] are mentioned a few.

The word shortage means a state or situation in which the needed items cannot be obtained in sufficient amounts or totally absent. It has a great importance for many models, especially when delay in payment is considered. When a shortage occurs but the company offers delay in payment, it can gain more orders from the customers. So shortages have an important role on optimization in inventory theory.

On the above facts and features, efforts have been made to develop a replenishment policy for an inventory model assuming time-varying deteriorating items having linear time dependent demand rate with a stochastic error assuming life time of the commodity as random and following exponential distribution. Shortages are allowed and fully backlogged. The model is minimized to the total average cost by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of key parameters has been presented.

2. Assumptions and Notations:

2.1 Assumptions:

The present inventory model is developed on the basis of the following assumptions

i. Lead time is zero.
ii. Replenishment rate is infinite but size is finite.
iii. The time horizon is finite.
iv. There is no repair of deteriorated items occurring during the cycle.
v. Deteriorating items with lifetime is followed an additive exponential distribution..
vi. The demand rate is a linear function of time with stochastic error.
vii. Shortages are allowed and completely backlogged.

2.2 Notations:

The following notations are used in the proposed model:

i. Q : The total amount of inventory.
ii. S : The size of initial inventory.
iii. The instantaneous rate of deterioration of the on-hand inventory is

$$\theta(t) = \frac{e^{-\frac{t}{\theta_1}} - e^{-\frac{t}{\theta_2}}}{\theta_1 e^{-\frac{t}{\theta_1}} - \theta_2 e^{-\frac{t}{\theta_2}}}, \theta_1 > \theta_2; t \geq 0$$

. iv. t_1 : The time length in which the stock is completely diminished.
v. T : The fixed length of each production cycle.
vi. C_3 : Ordering cost of inventory.
vii. c : The unite cost per item.

viii. c_h : The inventory holding cost per unit item.

ix. c_s : The shortage cost per unit item.

x. D(t) : Demand rate D(t) = $a+bt+\varepsilon$, where a, b>0 so that the demand is positive throughout the demand, ε (stochastic error). Here the shape of the demand curve is deterministic while the scaling parameter representing the size of the market is random. From practical stand point if "a" is large relative to the variance of ε, unbounded probability distribution such as the normal distribution provides adequate approximation. We assume that F(.) and f(.) represent the cumulative distribution and probability density function of ε, respectively having mean μ and standard deviation δ.

xi. q(t) : The level of inventory $q(t)=\left|\begin{matrix} q_1(t), 0\le t\le t_1 \\ q_2(t), t_1\le t\le T\end{matrix}\right\rangle$

xii. ATC : Average total cost per unit time.

xiii. <ATC> : Expected average total coat per unit time.

3. Formulation and Solution:

In this model, we consider the variation of the inventory level during the period $[0,T]$. The inventory level is depleted only due to demand and deterioration and ultimately falls to zero at t $=t_1$. The shortages occur during time period [t_1, T] which are completely backlogged. The differential equations pertaining to the above situations are given by

$$\frac{dq_1(t)}{dt}+\theta(t)q_1(t)=-D(t), 0\le t\le t_1 \tag{3.1}$$

And $$\frac{dq_2(t)}{dt}=-D(t), t_1\le t\le T \tag{3.2}$$

The initial conditions are $q_1(0)=S$, $q_1(t_1)=0$ and $q_2(t_1)=0$ (3.3)

Putting the values of $\theta(t)=\dfrac{e^{-\frac{t}{\theta_1}}-e^{-\frac{t}{\theta_2}}}{\theta_1 e^{-\frac{t}{\theta_1}}-\theta_2 e^{-\frac{t}{\theta_2}}}$ and $D(t)=a+bt+\varepsilon, a,b>0$, and then solving the equations (3.1) and (3.2), using the initial condition (3.3), and neglecting the higher powers of $\frac{1}{\theta_1}$ *and* $\frac{1}{\theta_2}$, we get the following

$$q_1(t)=S(1-\frac{t^2}{2\theta_1\theta_2})-\{(a+\varepsilon)t+\frac{b}{2}t^2-\frac{a+\varepsilon}{3\theta_1\theta_2}t^3-\frac{b}{8\theta_1\theta_2}t^4)\},\ 0\le t\le t_1 \tag{3.4}$$

And $$q_2(t)=(a+\varepsilon)(t_1-t)+\frac{b}{2}(t_1^2-t^2),\quad t_1\le t\le T \tag{3.5}$$

Since $q_1(t_1)=0$, we get the following expression of initial inventory from the equation (3.4), neglecting second and higher order powers of $\frac{1}{\theta_1}$ *and* $\frac{1}{\theta_2}$,

$$S=(a+\varepsilon)t_1+\frac{b}{2}t_1^{\,2}+\frac{a+\varepsilon}{6\theta_1\theta_2}t_1^{\,3}+\frac{b}{8\theta_1\theta_2}t_1^{\,4} \tag{3.6}$$

The number of items backlogged at the beginning of the period is

$$Q-S=\int_{t_1}^{T}(a+bt)dt$$

or, $$Q = (a+\varepsilon)T + \frac{b}{2}T^2 + \frac{a+\varepsilon}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4 \quad (3.7)$$

4. Cost Components:

The total cost over the period [0, T] consists of the following cost components:

- The ordering cost for raw materials is fixed at C_3.
- The cost of deterioration of items (C_D) over entire cycle (0,T) is given by

$$C_D = cQ = c\,[(a+\varepsilon)T + \frac{b}{2}T^2 + \frac{a+\varepsilon}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4] \quad (4.1)$$

- Holding cost for carrying inventory **(**C_H**)** over the period [0,T]

$$C_H = cc_h\int_0^{t_1} q_1(t)dt$$

Putting the values of $q_1(t)$ from (3.4), and integrating and then substituting the value of S from (3.6), we get the following expression after neglecting second and higher order powers of $\frac{1}{\theta_1}$ *and* $\frac{1}{\theta_2}$,

$$C_H = cc_h\{\frac{a+\varepsilon}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{a+\varepsilon}{12\theta_1\theta_2}t_1^4 + \frac{b}{15\theta_1\theta_2}t_1^5\} \quad (4.2)$$

- Cost due to shortage **(**C_S**)** over the period [0,T]

$$C_S = c_s\int_{t_1}^{T}(a+bt+\varepsilon)(T-t)dt$$

Or, $$C_S = \frac{c_s}{6}(T-t_1)^2\{3(a+\varepsilon)+b(T+2t_1)\} \quad (4.3)$$

Thus the average total cost per unit time of the system during the cycle [0, T] will be

$$ATC(t_1) = \frac{1}{T}\,[C_3 + C_D + C_H + C_S]$$

$$= C_3 + c\,\{(a+\varepsilon)T + \frac{b}{2}T^2 + \frac{a+\varepsilon}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4\} + cc_h\{\frac{a+\varepsilon}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{a+\varepsilon}{12\theta_1\theta_2}t_1^4 + \frac{b}{15\theta_1\theta_2}t_1^5\}$$
$$+ \frac{c_s}{6}(T-t_1)^2\{3(a+\varepsilon)+b(T+2t_1)\} \quad (4.4)$$

Let us suppose that

$$f(\varepsilon) = \frac{1}{k_1-k_0},\ k_1 \le \varepsilon \le k_0$$

$$= 0, \text{ elsewhere}$$

Where (μ,σ) are mean and standard deviation.

Therefore the expected average total cost (EATC) is given by

$$EATC = \langle ATC(t_1)\rangle = \frac{1}{T}\,[C_3 + c\,\{(a+\mu)T + \frac{b}{2}T^2 + \frac{a+\mu}{6\theta_1\theta_2}t_1^3 + \frac{b}{8\theta_1\theta_2}t_1^4\} +$$
$$cc_h\{\frac{a+\mu}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{a+\mu}{12\theta_1\theta_2}t_1^4 + \frac{b}{15\theta_1\theta_2}t_1^5\} + \frac{c_s}{6}(T-t_1)^2\{3(a+\mu)+b(T+2t_1)\}\,] \quad (4.5)$$

To minimize the cost, the necessary condition is $\frac{d\langle ATC(t_1)\rangle}{dt_1} = 0$

This gives

$$\alpha_1 t_1^4 + \beta_1 t_1^3 + \gamma_1 t_1^2 + \delta_1 t_1 + \nu_1 = 0 \quad (4.6)$$

where $\alpha_1 = \frac{bcc_h}{3\theta_1\theta_2}$, $\beta_1 = \frac{c}{\theta_1\theta_2}(\frac{b}{2} + \frac{(a+\mu)c_h}{3})$, $\gamma_1 = \frac{(a+\mu)c}{2\theta_1\theta_2} + bcc_h - \frac{bc_s}{3}$,

$\delta_1 = (a+\mu)cc_h + \frac{c_s}{3}(bT + 3a + 3\mu)$, and $\nu_1 = -c_s(a+\mu)T$

Since $\alpha_1 > 0$, $\beta_1 > 0$, $\delta_1 > 0$ and $\nu_1 < 0$,, we must have atleast one positive value of t_1.

For minimum, the sufficient condition $\frac{d^2 <ATC(t_1)>}{dt_1^2}$ >0 would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1.

The optimal expected values $<S^*>$ of S, $<Q^*>$ of Q and $<ATC^*>$ of <ATC> are obtained from the expressions (3.6), (3.7) and (4.5) by putting the value $t_1 = t_1^*$.

5. A Special Case:

Absence of deterioration:

If the deterioration of items is switched off, then the expressions (3.6), (3.7) and (4.5) of initial inventory(S), the total amount of inventory (Q) and the expected average total cost per unit time (<ATC (t_1)>) during the period [0, T] become

$$S = (a+\varepsilon)t_1 + \frac{b}{2}t_1^2 \tag{5.1}$$

$$Q = (a+\varepsilon)T + \frac{b}{2}T^2 \tag{5.2}$$

And

$$< ATC(t_1) > = \frac{1}{T}\left[C_3 + c\{(a+\mu)T + \frac{b}{2}T^2\} + cc_h\{\frac{a+\mu}{2}t_1^2 + \frac{b}{3}t_1^3\} + \frac{c_s}{6}(T-t_1)^2\{3(a+\mu) + b(T+2t_1)\}\right] \tag{5.3}$$

The equation (4.6) becomes

$$\gamma_1 t_1^2 + \delta_1 t_1 + \nu_1 = 0 \tag{5.4}$$

where, $\gamma_1 = bcc_h - \frac{bc_s}{3}$, $\delta_1 = (a+\mu)cc_h + \frac{c_s}{3}(bT + 3a + 3\mu)$, and $\nu_1 = -c_s(a+\mu)T$

This gives the optimum value of t_1.

6. Numerical Analysis:

To exemplify the above model numerically, let the values of parameters be as follows:

C_3 = \$200/order; c_h = \$0.12/year ; c_s = \$10 /unit/year; θ_1 = 5; θ_2 = 3; a = 5; b = 2; c = 20, T = 1 year.

Also we assume that a uniformly-distributed random demand component exhibited an error span of u= 20 with $[k_0, k_1]$=[10,20], and a mean μ =20.

Solving the equation (4.6) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.763 year; $<S^*>$ = 19.78, $<Q^*>$ = 26.13 units and $<ATC^*>$ = Rs. 731.00

It is checked that this solution satisfies the sufficient condition for optimality.

For Special Cases:

Nature of deterioration	t_1^*	$<S^*>$	$<Q^*>$	$<ATC^*>$
Absence of deterioration	0.793	20.45	26.00	726.73

7. Sensitivity Analysis and Discussion:

We now study the effects of changes in the system $c_h = 0.12$; $c_s = 10$; $\theta_1 = 5$; $\theta_2 = 3$; a = 5; b = 2; c = 20 and μ =20 on the optimum expected amount of initial inventory.$<S^*>$, the optimum expected amount of inventory ($<Q^*>$) and expected average total cost per unit time (<ATC(t_1)>) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	The optimum time length (t^*)	% change in $<S^*>$	% change in $<Q^*>$	% change in $<ATC^*>$
c_h	-50	0.844	11.13	0.18	- 0.50
	-20	0.793	4.16	0.06	- 0.22
	+20	0.735	- 3.83	- 0.06	0.23
	+50	0.696	-9.04	- 0.12	0.59
c_s	-50	0.624	-18.74	- 0.22	0.05
	-20	0.722	-5.55	- 0.08	0.03
	+20	0.793	4.12	0.06	- 0.03
	+50	0.826	8.63	0.13	- 0.07
θ_1	-50	0.736	- 3.07	0.39	0.52
	-20	0.756	- 0.79	0.11	0.13
	+20	0.767	0.54	- 0.07	- 0.09
	+50	0.772	1.09	- 0.16	-0.19
θ_2	-50	0.736	- 3.07	0.39	0.52
	-20	0.756	- 0.79	0.11	0.13
	+20	0.767	0.54	- 0.07	- 0.09
	+50	0.772	1.09	- 0.16	-0.19
a	-50	0.761	-9.89	- 9.62	- 6.97
	-20	0.762	-3.96	- 3.85	- 2.79
	+20	0.763	3.96	3.85	2.79
	+50	0.764	9.89	9.62	6.97
b	-50	0.769	- 0.56	- 1.91	- 1.46
	-20	0.765	- 0.22	- 0.76	- 0.58
	+20	0.759	0.22	0.76	0.58
	+50	0.756	0.53	1.91	1.46
c	-50	0.862	13.59	0.22	- 36.30
	-20	0.799	4.99	0.08	- 14.53
	+20	0.729	- 4.99	- 0.06	14.53
	+50	0.686	- 11.45	- 0.14	36.34
μ	-50	0.754	-39.58	- 38.47	- 27.89
	-20	0.760	-15.82	- 15.39	- 11.16
	+20	0.764	15.82	15.39	11.16
	+50	0.767	39.55	38.47	27.89

Analyzing the results of table A, the following observations may be made:

(i) The optimum expected amount of initial inventory $<S^*>$ increase or decrease with the increase or decrease in the values of the system parameters c_s, θ_1, θ_2, a , b and μ. On the other hand $<S^*>$ increase or decrease with the decrease or increase in the values of the system parameters c_h and c. The results obtained show that $<S^*>$ is very highly sensitive to changes in the value of parameter μ;

moderate sensitive to the changes of parameters c_h, c_s a and c ; and less sensitive to the changes of parameters θ_1 , θ_2 and b.

(ii) The optimum expected amount of inventory $< Q^* >$ increase or decrease with the increase or decrease in the values of the system parameters c_s, a , b and μ. On the other hand $< Q^* >$ increase or decrease with the decrease or increase in the values of the system parameters c_h, θ_1, θ_2 and c. The results obtained show that $< Q^* >$ is very highly sensitive to changes in the value of parameter μ; moderate sensitive to the changes of parameters a and b ; and less sensitive to the changes of parameters c_h, c_s, θ_1 , θ_2 and c.

(iii) The optimum expected average total cost $< ATC^* >$ increase or decrease with the increase or decrease in the values of the system parameters c_h , a , b , μ and c. On the other hand $< ATC^* >$ increase or decrease with the decrease or increase in the values of the system parameter c_s, θ_1 and θ_2. The results obtained show that $< ATC^* >$ is very highly sensitive to changes in the value of parameters μ and c ; moderate sensitive for a and b ; and less sensitive to the changes of parameters c_h , c_s, θ_1 and θ_2 .

From the above analysis, it is seen that μ is highly sensitive parameter in the sense that any error in the estimation of this parameter result in significant errors in the optimal cost solution. Hence estimation of such parameter needs adequate attention.

Scope of future work:

The present paper deals with an replenishment policy of an inventory model for deteriorating with stochastic demand in which shortages are allowed with fully backlogged. The deteriorating items with lifetime is followed an additive exponential distribution. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is minimized to the total average cost by finding optimal values. Eventually, a researcher can extend this model considering cubic demand under stochastic behaviour along with salvage value and permissible delay in payments.

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