IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

ON CORDIAL LABELING : PS_n, $A(PS_n)$, I_n

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Abstract: Let f be a function from the vertices of G to {0,1} and for each edge xy assign the label $|f(x) - f(y)|$. The function f is called cordial labeling of G if the number of vertices labelled 0 and the number of vertices labelled 1 differ by at most 1, and the number of edges labelled 0 and number of edges labelled differ at most by 1. In this paper, we have proved that Pentagonal snake graph, Ice – cream graph, Alternative pentagonal snake graph admit cordial labeling.

Key words: Cordial labeling, Ice – cream graph, Pentagonal snake graph, Alternative pentagonal snake graph.

1. INTRODUCTION

Graph Theory plays a vital role in various fields such as Communication networks, Sociology,Chemistry, Microbiology and in Sports also. In graph theory we have many problems such as Existence problems, Enumeration problems and Optimization problems. Graph labelling comes under enumeration problems. In this paper we concentrate only on graph labeling. Graph labeling is an assignment of integers (non negative) to the vertices, edges or both of a graph, subject to certain conditions. The concept graph labeling initiated by Rosa A [5] in 1967.

ThePentagonal snake graphPS_n isobtained from the path $P_n$by replacing every edge of a path by a cycle C_5. A. Uma Maheswari, S. Azhagarasi et. al. [9] proved that Pentagonal snake $PS_n$is AUM block sum labeling. S. Dhanalakshmi, S.Thirunavukkarasu et. al. [4] proved that Penta0gonal snake PS_k is mean square cordial labelling,

TheAlternative pentagonal snake graph$A(PS_n)$ is obtained from the path $P_n$by replacing every alternate edge of a path by a cycleC_5.S.Dhanalakshmi, S.Thirunavukkarasu et. al. [4] proved that Pentagonal snake PS_k is mean square cordial labelling, A. Uma Maheswari, S. Azhagarasi et. al. [9] proved that Pentagonal snake $PS_n$is AUM block sum labeling.

A shell graph [3] is defined as a cycle C_n with (n-3) chords sharing a common end point called apex. Shell graphs are denoted as C (n, n-3). Meena, Renugha et. al. [6] proved the followingresults: Shell graphs are cordial, two copies of shell graphs joined by a path of arbitrary length is cordial, multiple shell graph is cordial, the cycle of shell graph is cordial. An Ice - cream graph $I_n$is obtained by joining a shell graph and a path P_3 graph keeping v_1 and v_{n-1} common where $n > 3$ sharing common end point called the apex vertex v_0. It is denoted by I_n.Swedha V.P, Vanithashree R [8] proved that ice – cream graphs $I_n$are 7 – cordial.

2. MAIN RESULTS

***Theorem* 2.1:** Pentagonal snake graph PS_n admits cordial, when n is even.

proof

Let G be a pentagonal snake graph, PS_n. We describe G as follows:

The graph, G is obtained from path P_n having n vertices by replacing every edge of that by a cycle C_5. Let v_1, v_2, v_3, v_4, v_5 be the vertices in the first cycle C_5. We denote the vertices of the next cycle C_5as v_6, v_7, v_8, v_9 and so on. The vertices of last cycle $4n - 7$,$4n - 6$, $4n - 5$, $4n – 4$, $4n – 3$. G has total number of vertices $p = 4n – 3$ and total number of edges $q = 5n – 5$. We prove the theorem in three cases.

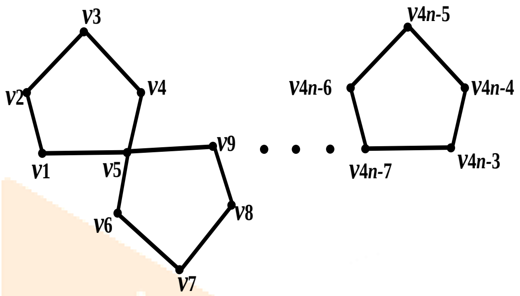

Figure 1 Pentagonal snakegraphPS_n

***Case*1:** When $n = 4$

We define the vertex labeling as follows.

$f(v_1) = f(v_2) = f(v_5) = f(v_6) = f(v_7) = f(v_9) = f(v_{12}) = 1$ and

$f(v_3) = f(v_4) = f(v_5) = f(v_{10}) = f(v_{11}) = f(v_{13}) = 0$

Here, $|v_f(0)| = 6$; $|v_f(1)| = 7 \Rightarrow |v_f(0) - v_f(1)| = 1$.

and $|e_{f^*}(0)| = 7$; $|e_{f^*}(1)| = 8. \Rightarrow |e_{f^*}(0) - e_{f^*}(1)| = 1$.

This proves that Pentagonal snake graph PS_4 is Cordial.

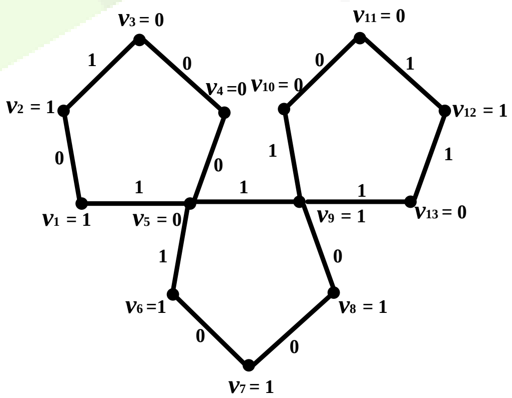

Figure 2 ACordial Pentagonal snakegraph,PS_4

***Case*2:** when $n = 4m - 2, m \geq 2$

Define $f : V(G) \rightarrow \{0, 1\}$ as follows

$f(v_{8i+1}) = 1 \quad 0 \leq i \leq \frac{n}{2} - 1$

$f(v_{8i+5}) = 0 \quad 0 \leq i \leq \frac{n}{2} - 1$

$f(v_{16i+2}) = 1 \quad 0 \leq i \leq \frac{n-2}{4}$

$f(v_{16i+3}) = 0 \quad 0 \leq i \leq \frac{n-2}{4}$

$f(v_{16i+4}) = 0 \quad 0 \leq i \leq \frac{n-2}{4}$

$f(v_{16i+6}) = 1 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+7}) = 1 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+8}) = 1 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+10}) = 0 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+11}) = 0 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+12}) = 1 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+14}) = 0 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+15}) = 0 \quad 0 \leq i \leq \frac{n-2}{4} - 1$

$f(v_{16i+16}) = 1 \quad 0 \leq i \leq \frac{n-2}{4} - 1 \quad (1)$

From the above definition given in equations (1) it is clear that all the vertices have been covered. Alsothe vertices satisfy the cordial labeling conditions.

Here, $|v_f(0)| = 2n - 1$; $|v_f(1)| = 2n - 2 \Rightarrow |v_f(0) - v_f(1)| = 1.$

We compute the edge labels as follows. If $e = uv$ then $f^*(e) = |f^*(u) - f^*(v)|$. Alsothe edges satisfy the cordial labeling condition. Here, $|e_{f^*}(0)| = \frac{5n-6}{2}$; $|e_{f^*}(1)| = \frac{5n-6}{2} + 1 \Rightarrow |e_{f^*}(0) - e_{f^*}(1)| = 1.$

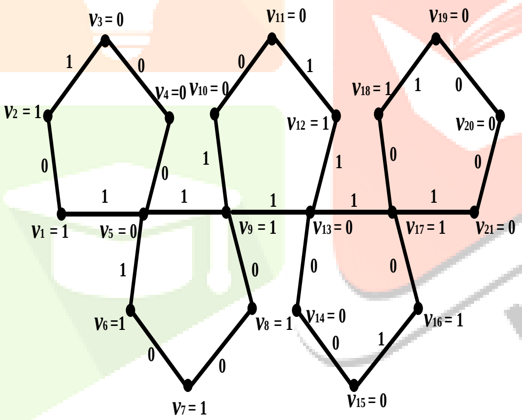

Figure 3 ACordial Pentagonal snakegraph,PS_6

Case 3: when $n = 4m, m \geq 2$

Define $f : V(G) \rightarrow \{0, 1\}$ as follows

$f(v_{8i+1}) = 1 \quad 0 \leq i \leq \frac{n}{2} - 1$

$f(v_{8i+5}) = 0 \quad 0 \leq i \leq \frac{n}{2} - 1$

$f(v_{16i+2}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+3}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+4}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+6}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+7}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+8}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+10}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+11}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+12}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 1$

$f(v_{16i+14}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 2$

$f(v_{16i+15}) = 0 \quad 0 \leq i \leq \frac{n}{4} - 2$

$f(v_{16i+16}) = 1 \quad 0 \leq i \leq \frac{n}{4} - 2 \quad (2)$

From the above definition given in equations (2) it is clear that all the vertices have been covered. Alsothe vertices satisfy the cordial labeling conditions.

Here, $|v_f(0)| = 2n-2$; $|v_f(1)| = 2n-1 \Rightarrow |v_f(0) - v_f(1)| = 1$.

We compute the edge labels as follows. If $e = uv$ then $f^*(e) = |f^*(u) - f^*(v)|$. Alsothe edges satisfy the cordial labeling condition. Here, $|e_{f^*}(0)| = \frac{5n-6}{2}$; $|e_{f^*}(1)| = \frac{5n-6}{2} + 1 \Rightarrow |e_{f^*}(0) - e_{f^*}(1)| = 1$.

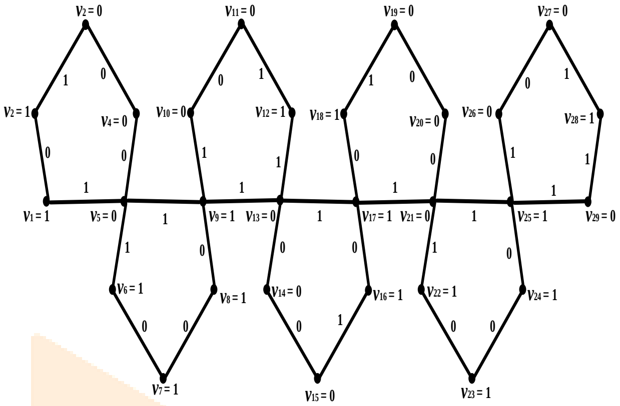

Figure 4 ACordial Pentagonal snakegraph,PS_8

In all the cases G satisfies the cordial labeling conditions. Hence Pentagonal snake graph $P(S_n)$ admits cordial, when n is even.

***Theorem* 2.2:** AlternatePentagonal snake graph $A(PS_n)$ admits cordial, when n is even.

Proof

Let G be aAlternate pentagonal snake graph, $A(PS_n)$. We describe G as follows:

The graph$G$ is obtained from path P_n having n vertices by replacing every edge of that by a cycle C_5. Let v_1, v_2, v_3, v_4, v_5 be the vertices in the first cycle C_5. We denote the vertices of the next cycle C_5 as v_6, v_7, v_8, v_9 ,v_{10} and so on. The vertices of last cycle $\frac{5n-8}{2}$, $\frac{5n-6}{2}$, $\frac{5n-4}{2}$,$\frac{5n-2}{2}$, $\frac{5n}{2}$. G has total number of vertices $p = \frac{5n}{2}$ and total number of edges $q = 3n - 1$.

We prove the theorem in two cases: viz., when $n = 4m + 2$, $m \geq 1$ and when $n = 4m$, $m \geq 1$.

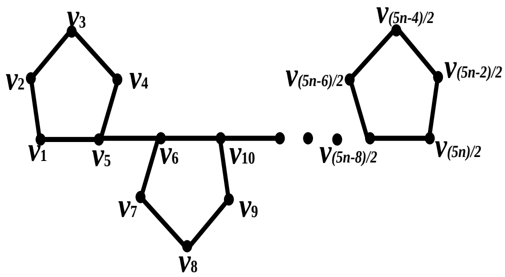

Figure 5 ACordial AlternatePentagonal snake graph, $A(PS_n)$

***Case* 1:**$n = 4m$, $m \geq 1$

$f(v_{5i+1})$ =	1	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{10i+3})$ =	1	$0 \leq i \leq \frac{n}{4} - 1$
$f(v_{5i+5})$ =	0	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{10i+8})$ =	0	$0 \leq i \leq \frac{n}{4} - 1$
$f(v_{5i+2})$ =	1	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{5i+4})$ =	0	$0 \leq i \leq \frac{n-2}{2}$ (1)

From the above definition given in equations (1) it is clear that all the vertices have been covered. Alsothe vertices satisfy the cordial labeling conditions.

Here, $|v_f(0)| = |v_f(1)| = \frac{5n}{4} \Rightarrow |v_f(0) - v_f(1)| = 1$.

We compute the edge labels as follows. If $e = uv$ then $f^*(e) = |f^*(u) - f^*(v)|$. Alsothe edges satisfy the cordial labeling condition. Here, $|e_{f^*}(0)| = \frac{3n}{2}$; $|e_{f^*}(1)| = \frac{3n}{2} - 1 \Rightarrow |e_{f^*}(0) - e_{f^*}(1)| = 1$.

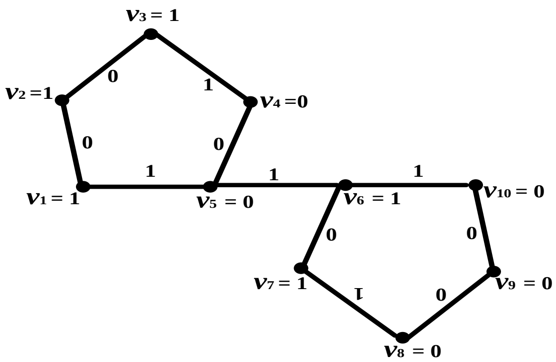

Figure 6 ACordial AlternatePentagonal snake graph, $A(PS_4)$

***Case* 2:** $n = 4m + 2, \ m \geq 1$

$f(v_{5i+1})$ =	1	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{10i+3})$ =	1	$0 \leq i \leq \frac{n-2}{4}$
$f(v_{5i+5})$ =	0	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{10i+8})$ =	0	$0 \leq i \leq \frac{n-6}{4}$
$f(v_{5i+2})$ =	1	$0 \leq i \leq \frac{n}{2} - 1$	$f(v_{5i+4})$ =	0	$0 \leq i \leq \frac{n-2}{2}$ (2)

From the above definition given in equations (2) it is clear that all the vertices have been covered. Alsothe vertices satisfy the cordial labeling conditions.

Here, $|v_f(0)| = \frac{5n-2}{4}$; $|v_f(1)| = \frac{5n-2}{4} + 1 \Rightarrow |v_f(0) - v_f(1)| = 1$.

We compute the edge labels as follows. If $e = uv$ then $f^*(e) = |f^*(u) - f^*(v)|$. Alsothe edges satisfy the cordial labeling condition.Here, $|e_{f^*}(0)| = \frac{3n}{2}$; $|e_{f^*}(1)| = \frac{3n}{2} - 1 \Rightarrow |e_{f^*}(0) - e_{f^*}(1)| = 1$.

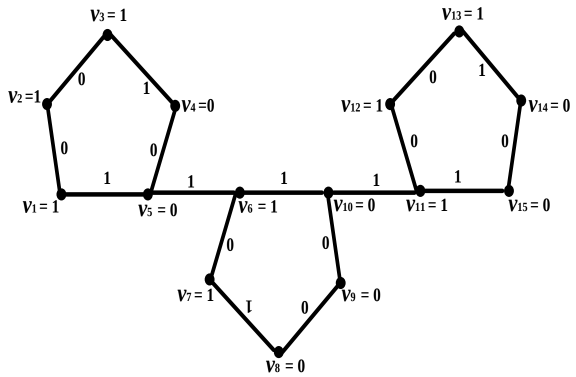

Figure 7 ACordial AlternatePentagonal snake graph, $A(PS_6)$

In all the cases G satisfies the cordial labeling conditions.Hence Alternate Pentagonal snake graph $A(PS_n)$ admits cordial, when n is even.

***Theorem* 2.3:** All ice – cream graphs I_n are cordial.

Let G be the ice – cream graph. Consider $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$ be the path vertices of $I_n$and v_0 be the apex vertex. G has total number of vertices $p = n + 1$ and total number of edges $q = 2n - 1$. In all cases we define $v_0 = 0$.We prove the theorem in two cases: when n is even and when n is odd

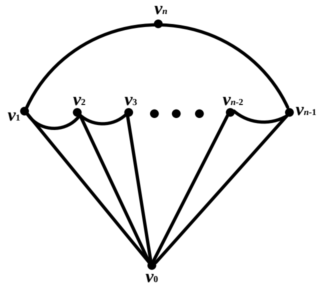

Figure 8 AIce cream graph I_n

***Case* 1:**when n is even

***Subcase* 1.1:**when$n \equiv 0 \ (mod\ 4)$

$$f(v_{4i+1}) = 1 \qquad 0 \le i \le \frac{n-4}{4}$$
$$f(v_{4i+2}) = 1 \qquad 0 \le i \le \frac{n-4}{4}$$
$$f(v_{4i+3}) = 0 \qquad 0 \le i \le \frac{n-4}{4}$$
$$f(v_{4i+4}) = 0 \qquad 0 \le i \le \frac{n-4}{4}$$

***Subcase* 1.2:** when$n \equiv 2 \ (mod\ 4)$

$$f(v_{4i+1}) = 1 \qquad 0 \le i \le \frac{n-2}{4}$$
$$f(v_{4i+2}) = 1 \qquad 0 \le i \le \frac{n-2}{4}$$
$$f(v_{4i+3}) = 0 \qquad 0 \le i \le \frac{n-6}{4}$$
$$f(v_{4i+4}) = 0 \qquad 0 \le i \le \frac{n-6}{4}$$

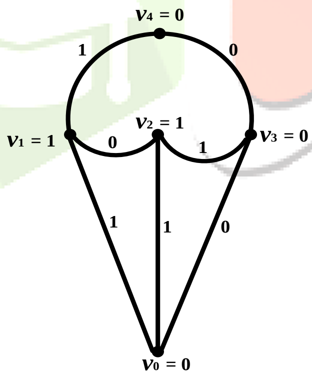

Figure 9ACordial Ice cream graph, I_4

***Case* 2:**when n is odd

***Subcase* 2.1:**when$n \equiv 1 \ (mod\ 4)$

$$f(v_{4i+1}) = 1 \qquad 0 \le i \le \frac{n-1}{4}$$
$$f(v_{4i+2}) = 1 \qquad 0 \le i \le \frac{n-5}{4}$$
$$f(v_{4i+3}) = 0 \qquad 0 \le i \le \frac{n-5}{4}$$
$$f(v_{4i+4}) = 0 \qquad 0 \le i \le \frac{n-5}{4}$$

***Subcase* 2.2:**when $n \equiv 3 \ (mod\ 4)$

we define vertex labeling$v_1 = 0$

$f(v_{4i+2}) =$	1	$0 \le i \le \frac{n-3}{4}$	$f(v_{4i+4}) =$	0	$0 \le i \le \frac{n-7}{4}$
$f(v_{4i+3}) =$	1	$0 \le i \le \frac{n-3}{4}$	$f(v_{4i+5}) =$	0	$0 \le i \le \frac{n-7}{4}$

In each case the graph under consideration satisfies the vertex conditions and edge condition for cordial labeling as shown in Table 1. Hence ice – cream graphs $I_n$is cordial.

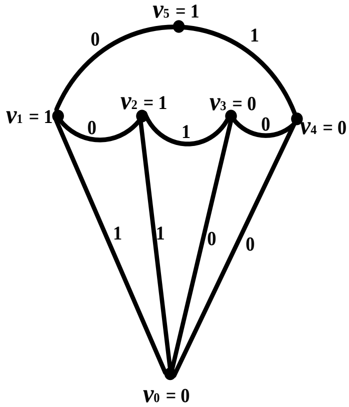

Figure 10 ACordial Ice cream graph,I_5

Sub case	Vertex condition	Edge condition
1.1	$v_f(0) = \frac{n+2}{2}, v_f(1) = \frac{n+2}{2} - 1$	$e_{f^*}(0) = n-1,\ e_{f^*}(1) = n$
1.2	$v_f(0) = \frac{n}{2}, v_f(1) = \frac{n}{2} + 1$	$e_{f^*}(0) = n,\ e_{f^*}(1) = n-1$
2.1	$v_f(0) = v_f(1) = \frac{n+1}{2}$	$e_{f^*}(0) = n,\ e_{f^*}(1) = n-1$
2.2	$v_f(0) = v_f(1) = \frac{n+1}{2}$	$e_{f^*}(0) = n-1,\ e_{f^*}(1) = n$

Table 1 Vertex and Edge labels

3. CONCLUSION

In this paper, we have proved that the Pentagonal snakegraph PS_n, when n is even, Alternative Pentagonal snakegraph $A(PS_n)$, when is n is even, Ice cream I_n admit cordial.

4. REFERENCE

[1] Bondy J. A., and Murty U.S.R.,"Graph theory with applications", The Macmillan Press Ltd, (1976).

[2] Cahit I., Cordial graphs: A weaker version of graceful and harmonious Graphs, Ars Combinatoria, 23(1987), 201-207.

[3] Deb P., and Limaye N. B., "On Harmonious Labeling of some cycle related graphs", Ars Combinatorics. Vol. 65, (2002), 177 – 197.

[4] Dhanalakshmi S., Thirunavukkarasu S., "Mean Square Cordial Labelling ofSome Pentagonal Snake Graphs", *Proceedings of the First International Conference onComputing, Communication and Control System*, I3CAC 2021, 7-8 June 2021, BharathUniversity, Chennai, India, EAI DOI 10.4108/eai.7-6-2021.2308859.

[5] Gallian J. A., "A Dynamic Survey ofGraphLabeling", *Electronic Journal of Combinatorics* 16(2009), DS6.

[6] Meena S., Renughu M., and Sivasakthi M., "Cordial labeling for different types of shell graph", International Journal of Scientific and Engineering Research, Vol. 6 (9), (2015), 1282 – 1288.

[7] Rosa A., "On certain valuations of the vertices of a graph", Theory of graphs(Internat. Symposium Rome, July1966) Gordon and Breach, N. Y – Dunod Paris (1967), 349 – 355.

[8] Swedha V.P., and VanithashreeR.," A study on combination of shell graph and path P_2 Graph", *jetir*. Vol. 6 Issue 5, (2019), 578 – 580, 2349 – 5162.

[9] Uma Maheswari A., Azhagarasi S., " AUM Block sum labeling for some special graphs ", *International Journal of Mechanical Engineering* Vol. 7 (Special Issue 5, April 2022), 102 - 110.