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A Study Of Acoustic Wave Propagation In Porous Media: Analytical And Numerical Approaches

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Abstract: Wave propagation through porous media is a phenomenon of significant interest across diverse scientific and engineering disciplines, including soil mechanics, seismology, acoustics, earthquake engineering, ocean engineering, and geophysics. This review provides a comprehensive overview of the literature on this subject.We begin by examining Biot's seminal theory, which describes wave propagation in linear, elastic, fluid-saturated porous media. Biot's theory predicts the existence of two dilatational waves and one rotational wave within such media. Notably, the second dilatational wave exhibits strong attenuation and displays diffusion-like behavior. The influence of coupling between the solid and fluid phases has a diminishing effect on the first dilatational wave while amplifying the second.This review delves into various analytical and numerical solutions derived by researchers based on Biot's theory. We then explore the extension of Biot's theory to unsaturated soils, discussing common approaches that adapt equations for saturated media by modifying density and compressibility parameters to account for the presence of air and water.The review further investigates methods for determining the permeability of porous media through the attenuation of dilatational waves. We then focus on the extensive research in geophysics concerning the prediction of acoustic wave speeds and attenuations in marine sediments, alongside studies on the dissipation of water waves at ocean bottoms.The mixture theory, a prominent framework in continuum mechanics, is discussed within the context of wave propagation in porous media. Subsequently, we present an alternative approach to deriving governing equations for wave propagation in porous media based on macroscopic balance equations.Finally, we analyze wave propagation in fractured porous media saturated with two immiscible fluids.

***Index Terms* - Wave propagation, Porous media, Biot's theory, Dilatational waves, Rotational waves, Coupling effects, Permeability, Fractured porous media.**

I. INTRODUCTION

The dynamic response of porous media is crucial across various fields, including underground nuclear explosions, earthquake engineering, seismic studies, acoustic logging, biomechanics, and geotechnical engineering. These diverse applications encompass phenomena like shock wave propagation, liquefaction, wave attenuation, and soil-structure interaction. Despite the wide range of applications, the fundamental principles governing these behaviors remain consistent. The analysis typically treats the porous medium as a continuum comprising a solid phase (compressible or incompressible) and one or more fluids (gases or liquids) occupying the interconnected void space. This framework, incorporating conservation of mass and momentum principles, provides a unified approach to understanding wave propagation in porous media across different disciplines.

II. Literature Review

The design of low-power IoT sensors has emerged as a critical area of research, driven by the need for energy-efficient devices that ensure prolonged operation in resource-constrained environments. Studies on wave propagation, such as Biot's theory of elastic wave propagation in porous media (Biot, 1956a, 1956b), have provided a foundation for understanding wave dynamics that can be adapted for sensor development. This theory, which explains the interaction between solid and fluid phases in porous media, has influenced applications ranging from seismology to acoustic sensing. Biot's framework has been further elaborated in works like Geertsma and Smit (1961) and Berryman (1980), which introduced refinements to wave propagation models, considering compressibility and attenuation, aspects that are critical for designing sensors with minimal energy loss. Similarly, numerical modeling approaches such as those by Zienkiewicz et al. (1980, 1982) and Santos et al. (1986) demonstrated the use of finite element methods to simulate wave behavior, offering insights into how sensors can optimize energy consumption by targeting specific frequencies and minimizing dissipation.In more recent years, the integration of computational techniques has advanced sensor optimization. For example, Hassanzadeh (1991) applied finite difference methods for low-frequency transient wave modeling, emphasizing how environmental parameters influence wave propagation and attenuation—concepts that directly apply to IoT sensors for structural health monitoring. Similarly, Burridge and Keller (1981) provided theoretical justifications for Biot's equations by incorporating microstructural dynamics, highlighting the importance of material properties in energy-efficient designs. Advances in acoustic wave propagation modeling, as explored by Johnson (1982) and Attenborough and Chen (1990), demonstrated how understanding wave reflection and refraction can lead to improved sensor accuracy and reduced power usage in real-world applications.Furthermore, studies by Foda and Mei (1983) and Tajuddin (1991) on Rayleigh waves and surface waves in porous media emphasized the role of boundary interactions, which are critical for sensors placed in dynamic environments such as soil or water. These findings align with the increasing use of IoT sensors in agriculture and environmental monitoring, where wave dynamics must be accurately measured with minimal energy expenditure. The work of Stoll and Bryan (1970), which applied Biot's theory to marine sediments, further underscores the versatility of wave propagation principles in designing low-power sensors for underwater and geotechnical applications. Recent numerical simulations by Zhu and McMechan (1991) and Bougacha et al. (1993) highlighted the impact of porosity and permeability on wave dynamics, offering opportunities to customize IoT sensors for specific environmental conditions.The convergence of these foundational theories with modern computational and material science advancements provides a robust framework for the design of energy-efficient IoT sensors. However, challenges remain in integrating these principles into miniaturized devices capable of real-time data processing and wireless communication. Addressing these challenges requires continued exploration of wave mechanics and material interactions, alongside advances in machine learning and low-power electronics.

III. Mathematical Model

The study employs a **three-velocity, three-pressure mathematical model** to describe wave propagation in layered and fractured porous media. The governing equations include mass and momentum conservation for the skeleton and fluid phases (primary pores and fractures). The main components are as follows:

i. Mass Balance Equations:

Skeleton (s), fluid in primary pores (f), and fluid in fractures (fr)

Mass Balance Equations:

Skeleton (solid phase)

$$\partial \rho s/\partial t + \nabla\cdot(\rho s v s) = 0$$

Fluid in primary pores

$$\partial \rho f/\partial t + \nabla\cdot(\rho f v f) = -q$$

Fluid in fractures

$$\partial \rho fr/\partial t + \nabla\cdot(\rho fr v fr) = q$$

Where,

- ρs, ρf, ρfr are the densities of the skeleton, fluid in primary pores, and fluid in fractures, respectively.
- vs, vf, vfr are the velocities of the skeleton, fluid in primary pores, and fluid in fractures, respectively.
- q is the exchange rate of fluid between pores and fractures, defined as: $q = \zeta (\rho f0\ kf / \mu f) ((pf - pfr) / ab^2)$ where:
  - ζ is the permeability coefficient
  - ρf0 is the reference fluid density
  - kf is the permeability of the porous medium
  - μf is the dynamic viscosity of the fluid
  - pf, pfr are the pressures in primary pores and fractures, respectively
  - ab is the characteristic length scale for fluid exchange

ii. Momentum Equations:

- **Skeleton**
  $\rho s\ (dvs/dt) = -\alpha s \nabla pl + \nabla \cdot \sigma s + Ff + Ffr$

- **Fluid in pores**
  $\rho f\ (dvf/dt) = -\alpha f \nabla pf - Ff - vfq$

- **Fluid in fractures**
  $\rho fr\ (dvfr/dt) = -\alpha fr \nabla pfr - Ffr + vfrq$

Where,

- pl is the pore pressure
- σs is the stress tensor of the skeleton
- Ff, Ffr are the forces of interaction between the skeleton and the fluids in pores and fractures, respectively
- αs, αf, αfr are the effective stress coefficients

iii. Elasticity of the Skeleton:

$\sigma s = \alpha s\ [\lambda * \ tr(\varepsilon)\ I + 2\mu * \ \varepsilon + \nu * \ pl\ I]$

where:

- λ∗, μ∗ are the Lamé parameters of the skeleton
- ε is the strain tensor of the skeleton
- I is the identity tensor
- tr(ε) is the trace of the strain tensor (sum of diagonal elements)

iv. Boundary Conditions:

- At the interfaces between porous media layers:
  - Continuity of velocity
  - Continuity of fluid flow
  - Continuity of pressure
  - Continuity of stress components
- For open-pore conditions: Fluid exchange between layers is allowed.

  These equations provide a comprehensive mathematical framework for analyzing wave propagation in complex porous media, considering interactions between solid and fluid phases in both primary pores and fractures.

IV. Neumerical Integration Method

The numerical integration in this study employs the finite-difference MacCormack method, a predictor-corrector scheme for solving the system of partial differential equations.

i.Transition to Dimensionless Variables

The equations are transformed into dimensionless form:

- $\rho j = \rho j / \rho f0$
- $pj = pj / p0$
- $\sigma s^* = \sigma s^* / p0$
- $vj = vj / \sqrt{(p0 / \rho f0)}$

where $\rho f0$ is the reference fluid density, and p0 is the reference pressure.

ii. Discretization

The computational grid is defined as:

- $xi = x0 + i * hx$
- $zj = z0 + j * hz$
- $tn = t0 + n * \tau$

where hx and hz are spatial step sizes, and τ is the time step.

The system of equations is represented in matrix form:

$\partial U/\partial t + \partial W1/\partial x + V2\ \partial W2/\partial x + \partial W3/\partial z + V4\ \partial W4/\partial z + Q = 0$

where:

- U represents the vector of unknowns (density, velocity, stress, etc.)
- W1, W2, W3, W4 represent flux terms
- V2, V4 are coupling matrices
- Q represents source terms
-

iii.Steps in the MacCormack Scheme

- **Predictor Step (Forward Differencing)**
  $U^* = Un - \tau\ (\partial W1/\partial x + V2\ \partial W2/\partial x + \partial W3/\partial z + V4\ \partial W4/\partial z + Q)n$
- **Corrector Step (Backward Differencing)**
  $U^{**} = Un - \tau\ (\partial W1^*/\partial x + V2^*\ \partial W2^*/\partial x + \partial W3^*/\partial z + V4^*\ \partial W4^*/\partial z + Q^*)$
- **Averaging:**
  $Un+1 = 0.5 * (U^* + U^{**})$
- **Correction for Source Terms**
  $Un+1 = Un+1 - \tau * Qn+1$

iv. Boundary Conditions

- Ghost cells are introduced at the domain boundaries:
  - $U0,j\hat{}n = U1,j\hat{}n$
  - $UI+1,j\hat{}n = UI,j\hat{}n$
  - $Ui,0\hat{}n = Ui,1\hat{}n$
  - $Ui,J+1\hat{}n = Ui,J\hat{}n$

v. Stability Criterion

The time step τ is chosen to satisfy the Courant–Friedrichs–Lewy (CFL) stability condition:

$\tau < \min(hx, hz) / \max(|v| + c)$

where v is the velocity and c is the sound speed in the medium.

V. Critical Analysis of Mathematical and Numerical Methods

i.Mathematical Model: The mathematical model presented in the paper provides a robust framework for analyzing acoustic wave propagation in porous media. By incorporating a three-velocity and three-pressure approach, it effectively captures the complexities of wave behavior in both layered and fractured porous systems. The inclusion of mass and momentum conservation equations for the skeleton, fluid in primary pores, and fluid in fractures is a strength, as it allows the model to simulate fluid-structure interactions comprehensively.

However, some limitations should be acknowledged:

1. **Simplifying Assumptions**: The model assumes linear elasticity for the porous skeleton and small deformations, which may not hold for highly heterogeneous or non-linear materials. This restricts its applicability to environments where these assumptions are valid.
2. **Neglect of Non-Linear Effects**: Non-linear wave phenomena, such as shock wave propagation or high-frequency attenuation, are not addressed. These are critical in applications like seismic analysis and require additional modeling complexity.
3. **Homogeneous Layer Assumptions**: The assumption of homogeneity within each layer does not account for microstructural variations, such as anisotropy or gradient porosity, which could significantly influence wave dynamics.

ii.Numerical Integration Method: The use of the finite-difference MacCormack method is well-suited for solving the governing equations due to its simplicity, second-order accuracy, and efficiency. This method avoids the need for calculating effective parameters at half-integer grid points, which is a practical advantage over the Lax-Wendroff scheme.

Nonetheless, some challenges and limitations are evident:

1. **Stability Constraints**: The method requires strict adherence to the Courant–Friedrichs–Lewy (CFL) condition for stability. For highly heterogeneous media or fine spatial grids, this can result in very small time steps, increasing computational costs.
2. **Applicability to Complex Geometries**: While the MacCormack method performs well for structured grids, it may not be easily extendable to irregular geometries or three-dimensional problems without significant modifications.
3. **Numerical Dissipation and Dispersion**: Although the method is second-order accurate, it may still introduce numerical dissipation and dispersion, particularly in long-term simulations or for highly dynamic waveforms.

iii.Boundary Conditions: The implementation of open-pore boundary conditions to allow fluid exchange between layers is a notable strength, as it reflects realistic interactions in layered porous media. However

- The boundary conditions might oversimplify scenarios involving partial saturation, capillary effects, or dynamic pore closure.
- For fractured media, the model assumes ideal continuity, which may not fully capture the complexities of fracture propagation or fluid leakage.

iv .Overall Strengths:

1. **Comprehensive Framework**: The model bridges analytical theory and numerical methods, providing insights into wave behavior in complex porous systems.
2. **Scalability**: The approach can be extended to study different types of porous media (e.g., fluid-saturated, fractured, or multi-phase).
3. **Practical Implementation**: The MacCormack method is computationally efficient and relatively easy to implement, making it a practical choice for initial studies.

v.Recommendations for Future Enhancements

1. Extend the model to include non-linear effects, such as high-frequency wave attenuation or shock wave phenomena, to address more complex applications.
2. Investigate the impact of microstructural heterogeneities, such as anisotropy or gradient porosity, on wave propagation.
3. Explore alternative numerical methods, such as finite element or spectral methods, to improve accuracy and applicability to irregular geometries.
4. Integrate experimental validation to assess the accuracy of the mathematical and numerical methods in real-world scenarios.

VI. Understanding Wave Behavior in Porous Media: Key Findings

This study offers significant insights into the complex dynamics of wave propagation in porous media. By integrating analytical and numerical approaches, it highlights several critical aspects:

1. **Wave Propagation Dynamics**: The study elucidates how acoustic waves interact with both the solid skeleton and the fluid phases in porous media. It demonstrates the influence of material properties such as porosity, permeability, and elasticity on wave speed, attenuation, and dispersion.
2. **Behavior in Layered and Fractured Media**: The research emphasizes the effects of heterogeneities, such as layering and fractures, on wave propagation. These structural features introduce unique wave modes, affect energy transmission and reflection, and can significantly alter wave attenuation.
3. **Boundary Effects**: The study underscores the importance of boundary conditions, such as open-pore or partially open interfaces, in determining wave interactions at material boundaries. This understanding is vital for accurately modeling wave behavior in complex systems.
4. **Applications Across Fields**: The findings are applicable to various disciplines, including:
   - **Geophysics**: Analyzing seismic wave behavior in subsurface materials for resource exploration or earthquake studies.
   - **Environmental Engineering**: Monitoring fluid flow and stability in soil or porous rocks.
   - **Material Science**: Designing porous materials with specific acoustic properties for soundproofing or vibration control.
5. **Predictive Modeling**: The mathematical framework enables the prediction of wave behavior under varying conditions, offering a valuable tool for understanding and optimizing porous systems in practical applications.

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IX. Conclusion

This study successfully demonstrates a novel numerical approach for investigating the propagation of arbitrary waveform impulses within a two-dimensional layered medium comprising both porous and fractured-porous zones. The finite-difference MacCormack method is effectively employed to simulate wave propagation and transmission across the interface between these distinct media.Computer implementation and subsequent calculations validate the efficacy of this proposed methodology. While this study focuses on the simpler case of a planar boundary between media, the developed approach provides a robust foundation for future investigations.Further development of this methodology will enable the accurate calculation of wave propagation in porous media exhibiting more complex zonal inhomogeneities, significantly enhancing our understanding of wave behavior in such challenging geological environments.

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