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In Generalized Classes Of Estimator b_{hT} In Linear Regression Model The Sampling Distribution, Concentration Probability Of An Estimators.

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ABSTRACT

For small sigma asymptotic approach due to Kadane and Shukla studies he properties of two families of estimators considered by Srivastava and Chandra from the idea of concentration probabilities of estimators around the two unknown parameters stochastically restricted a linear regression model. To analyses the properties of more generalized classes of estimators by Sing et.al (1995) from concentration probabilities theme choice of an estimator.

Key Words : Quadratic Risk Criterion, Stochastic, Regression, Exact linear constraints, Coefficient vector.

Let the classical linear regression modal is

$$Y = X \quad \beta \ + \quad u \qquad \ldots(1)$$

Where Y is Tx1 vector of observations on the variable to be explained, X is a non- stochastic Txp full column matrix of observations on p explanatory variable β is px1 vector of regression coefficients and u is Tx1 vector of disturbances following a multivariate normal distribution with mean vector is zero and dispersion matrix $\sigma^2 I_T$

$$E(u) = 0$$

And dispersion matrix

$$E(u\,u') = \sigma^2 I_T$$

σ^2 being unknown variance of disturbances.

Concentration probability of estimator b_{hT} which is generalized estimator and we derive the small sigma asymptotically approcle, expression for the sampling distribution of the estimator b_{hT}.

The estimation error of b_{hT} may be written as

$$(b_{hT} - \beta) = \sigma\gamma_1^* + \sigma_2^2\gamma_2^* + \sigma^3\gamma_3^* + \sigma^4\gamma_4^* + \sigma^5\gamma_5^* + 0(\sigma^6) \qquad ...(2)$$

$$\gamma_1^* = (X'X)^{-1}X'w$$

$$\gamma_2^* = \frac{1}{n}(w'\bar{P}_X w)(X'X)^{-1}Q'\,\Psi^{-1}v$$

$$\gamma_3^* = -\frac{1}{n}(w'\bar{P}_X w)(X'X)^{-\frac{1}{2}}A\,Z$$

$$\gamma_4^* = -\frac{1}{n^2}(w'\bar{P}_X w)^2(X'X)^{-\frac{1}{2}}A(X'X)^{-\frac{1}{2}}Q'\Psi^{-1}v + \frac{\left(g'(0)\right)^2}{n\theta^2}(w'P_X w)^2(\beta'\,X'X\beta)(X'X)^{-1}Q'\Psi^{-1}v$$

$$\gamma_5^* = \frac{1}{n^2}(w'\bar{P}_X w)^2(X'X)^{-\frac{1}{2}}\;A\,Z - \frac{\left(g'(0)\right)^2}{n\theta^2}(w'\bar{P}_X w)^2(\beta'\,X'X\beta)(X'X)^{-\frac{1}{2}}\;A\,Z$$
$$- \frac{4\left(g'(0)\right)^2}{n\theta^2}(w'\bar{P}_X w)^2(\beta'\,X'X\beta)\beta'C(X'X)^{-\frac{1}{2}}\;Z\,.(X'X)^{-1}Q'\Psi^{-1}v$$
$$+ \frac{2\left(g'(0)\right)^2}{n\theta^2}(w'\bar{P}_X w)^2\beta'^{(X'X)^{-\frac{1}{2}}}\,Z.(X'X)^{-1}Q'\Psi^{-1}v$$

Again defining the vector

$$r_h = \frac{1}{\sigma}(X'X)^{\frac{1}{2}}(b_{hT} - \beta) \qquad ...(3)$$
$$= A_0^* + \sigma A_1^* + \sigma^2 A_2^* + \sigma^3 A_3^* + \sigma^4 A_4^* + 0(\sigma^5)$$

Where

$$\left.\begin{aligned}
A_0^* &= Z \\
A_1^* &= \frac{1}{n}(w'\bar{P}_X w)(X'X)^{-\frac{1}{2}}\, Q'\Psi^{-1}v \\
A_2^* &= -\frac{1}{2}(w'\bar{P}_X w)\ \ A\ \ Z \\
A_3^* &= -\ \frac{1}{n}(w'\bar{P}_X w)^2 H(X'X)^{-\frac{1}{2}}\, Q'\Psi^{-1}v \\
A_4^* &= \frac{1}{n}(w'\bar{P}_X w)^2\ H\ A\ Z + \frac{2\big(g'(0)\big)}{n\theta^2}(w'\bar{P}_X w)^2(\alpha_1' z).(X'X)^{-\frac{1}{2}}Q'\Psi^{-1}v \\
&\qquad - \frac{4\big(g'(0)\big)^2}{n\theta^3}(w'P_X w)^2.(\alpha_1'\alpha_1)(\alpha_2' Z)(X'X)^{-\frac{1}{2}}Q'\Psi^{-1}v
\end{aligned}\right\}\ldots(4)$$

For fixed vector h, the characteristic function of the vector r_h to order $0(\sigma^4)$,can be written as

$$\eta_{r_h}(h) = E\left(e^{ih'r_h}\right) \qquad \ldots(5)$$

$$= E\left[\left(e^{ih'r_h}\right).\, e^{\left(\sigma ih'A_1^* + \sigma^2 ih'A_2^* + \sigma^3 ih'A_3^* + \sigma^4 ih'A_4^* + 0(\sigma^5)\right)}\right]$$

$$= E\left[e^{ih'A_*^3}\left\{1 + \sigma(ih'A_1^*) + \sigma^2\left(ih'A_2^* + \frac{1}{2}(ih'A_1^*)^2\right) + \sigma^3\left((ih'A_3^*) + (ih'A_1^*)(ih'A_2^*) + \frac{1}{6}(ih'A_1^*)^3\right) + \sigma^4\left((ih'A_4^*) + (ih'A_1^*)(ih'A_3^*) + \frac{1}{2}(ih'A_2^*)^2 + \frac{1}{2}(ih'A_1^*)^2(ih'A_2^*) + \frac{1}{24}(ih'A_1^*)^4\right)\right\}\right]$$

We get the characteristic function of vector r_h, up to order $0(\sigma^4)$, as follows.

$$\eta_{r_h}(h) = (1 + \sigma^2\eta_2^* + \sigma^4\eta_4^*)e^{-\frac{1}{2}h'h} \qquad \ldots(6)$$

$$\eta_2^* = \frac{(n-2\,)}{2n}(h'Ah) \qquad \ldots(7)$$

$$\eta_4^* = \left[\frac{(n-2\,)}{2n}\left\{\frac{(n-2)(n-4\,)+16}{4n^2}\,(h'Ah)^2 - \frac{(n-8\,)}{n}(h'A^2h) - \frac{8\big(h'(0)\big)^2}{\theta^2}.(\alpha_1'\alpha_1)(h'Ah)\right\}\right].0$$

Noting the concentration probability of the estimator b_{hT} around β for the region bounded by the constant $\bar{m}_1, \bar{m}_2, \ldots\ldots\ldots\ldots..\bar{m}_p$ in the p – dimensional ecludian space to be

$$CP(b_{hT}) = P(|b_{hT} - \beta| < \bar{m}) \qquad \ldots(8)$$

Now with the help of results given by

$$g(r_g) = \frac{1}{(2\pi)^p}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\ldots\ldots\ldots\int_{-\infty}^{\infty} e^{ihr_g}\,\phi_g(h)\,dh \qquad \ldots(9)$$

$$\frac{1}{(2\pi)^p}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\ldots\ldots\ldots\int_{-\infty}^{\infty} e^{-ih'r_h - \frac{1}{2}r_h' r_h}\ \ dh$$

$$= [(tr\ A)^2 + 2tr\ A^2 - 4(r_h'\ A^2 r_h) + (r_h' A r_h)^2 - 2(r_h' A r_h)\ tr\ A\ ]\ \xi(r_h)$$

And the inversion theorem (9), we arrive at results

$$f(r_h) = (1 + \zeta_2\sigma^2 + \sigma^4\zeta^4)\xi(r_h)$$

$$\zeta_2 = \frac{(n-2)}{2n}(tr\ A - r_h'\ A\ r_h)$$

$$\zeta_4 = \left(\frac{n+2}{4n}\right)\left\{\frac{(n-2)(n-4)+16}{2n^2}\right\}(tr\ A - r_h'\ A\ r_h)^2 - \left\{\frac{(n+2)(n-12)}{n^2}\right\}(tr\ A^2 - r_h'A^2 r_h) - \left\{\frac{(n-2)(n-4)+16}{n^2}\right\}r_h'\ A^2\ r_h - \left\{\frac{16\left(h'(0)\right)}{\theta^2}\right\}(\alpha_1'\alpha_1)(tr\ A - r_h'Ar_h)$$

$$CP(b_{hT}) = P\left[\left|r_{hg}^*\right| < m_j; j = 1,2,3,,,\dots\dots\dots p\right]$$

$$= \int_{-m_p}^{m_p} \dots\dots\dots.\int_{-m_1}^{m_1} f(r_h)\quad dr_{h_1}.dr_{h_2}\dots\dots\dots.dr_{h_p}$$

$$= 1 + \frac{\sigma^2(n-2)}{2n}tr\ A\ E + \frac{\sigma^4(n+2)}{4n}\left\{\frac{(n-2)(n-4)+16}{2n^2}\left(3tr\ (A\ \#\ A) - (tr\ A)^2 - 2tr\ A^2 + 4tr\ A^2E - 3tr(A\#\ A)E - tr\ (A\#\ A)\#Emm' + \Sigma_{i\neq j=1}^{p}\left(a_{ii}a_{jj} + 2a_{ij}\right).(1-e_i)\left(1-e_j\right)\right) - \frac{8\left(h'(0)\right)^2}{\theta^2}(\beta'X'X\beta)tr\ AE - \frac{(n-8)}{n}tr\ A^2E\right\}\emptyset(m)$$

We have the concentration probability of the mixed estimator

$$CP(b_T) = \left[1 + \sigma^2\left\{\frac{n-2}{2n}\ tr\ AE\right\}\right]\phi(m)$$

And the Concentration probability of the generalized estimator b_{hT}

$$CP(b_{hT}) = \left[1 + \sigma^2\frac{(n-2)}{2n}\ tr\ AE + \sigma^4\frac{(n+2)}{4n}\left\{\frac{(n-2)(n-4)+16}{2n^2} - \left(3tr(A\#A) - (trA)^2 - 2trA^2 + 4trA^2E - 3tr\ (A\#A)E - tr(A\#A)\#Em'm + \Sigma_{i\neq j=1}^{p}\left(a_{ii}a_{jj} + 2a_{ij}\right).(1-e_i)\left(1-e_j\right)\right) - \frac{8\left(h'(0)\right)^2}{\theta^2}(\beta'xx'\beta)trAE - \frac{(n-8)}{n}tr\ A^2E\right\}\right]\phi(m)$$

\# denotes the Hadamard product of matrices so that Which is the generalized classes of estimator b_{hT} in linear regression model the concentration probability of an estimators.

For small sigma asymptotic approach due to Kadane and Shukla studies he properties of two families of estimators considered by Srivastava and Chandra from the idea of concentration probabilities

of estimators stochastically restricted a linear regression model. To analyses the properties of more generalized classes of estimators by Sing et.al (1995) from concentration probabilities theme choice of an estimator.We see that generalized estimator b_{hT} will have concentration probability up to order $o(\sigma^4)$,around p then the mixed regression estimator the generalized class b_{hT} of mixed type estimator has the value for comparing both we see that the range of b_{hT_1} *over* b_T is wider than that is better of b_{TS} *over* b_T in the extended range of b_{hT_1} *over* b_T , the estimator b_{hT_1} *over* b_T, the estimator b_T *and* b_{TS}. We can say that the concentration probability an estimator around the true parameter for generalized classes of estimator in linear regression model. Which is the generalized classes of estimator b_{hT} in linear regression model the sampling distribution, concentration probability of an estimators.

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