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An Integrated Inventory Approach For Non-Instantaneously Deteriorating Items With Variable Demand And Mixed Payment Methods

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Abstract: This study introduces an innovative inventory model for items with non-instantaneous deterioration, integrating nonlinear stock-dependent demand, hybrid payment schemes, and partially backlogged shortages. Unlike traditional models, which often assume immediate deterioration and linear demand, our approach captures the gradual quality decline of items and the dynamic nature of demand influenced by inventory levels. The model incorporates a hybrid payment scheme, blending immediate and deferred payments, offering businesses increased financial flexibility. Additionally, it addresses partially backlogged shortages, where some unmet demand is backordered while the rest is lost. We formulate the model mathematically and derive optimal inventory policies to minimize total costs, encompassing holding, ordering, deterioration, and shortage costs. Through sensitivity analysis, we explore the impact of key parameters, demonstrating the model's robustness and practical relevance. The findings reveal that accounting for nonlinear demand, hybrid payments, and partial backlogging can significantly enhance inventory management efficiency and cost-effectiveness, making this model a valuable tool for practitioners in complex supply chain environments.

Keywords: Inventory model, non-instantaneous deterioration, nonlinear stock-dependent demand, hybrid payment scheme, partially backlogged shortages, supply chain management, optimal inventory policies, total cost minimization, sensitivity analysis, financial flexibility.

I. INTRODUCTION

II. In today's competitive and dynamic market environment, effective inventory management is crucial for businesses aiming to maintain profitability and customer satisfaction. Traditional inventory models often fall short by assuming instantaneous deterioration and linear demand, which do not accurately reflect the complexities of real-world scenarios. This study introduces an advanced inventory model specifically designed for non-instantaneously deteriorating items, which degrade over time rather than immediately upon receipt. The model incorporates a nonlinear stock-dependent demand function, acknowledging that demand can vary significantly based on the available inventory levels. Furthermore, it integrates a hybrid payment scheme, combining immediate and deferred payment options, thereby providing businesses with greater financial flexibility. Addressing the challenge of shortages, the model also allows for partial backlogging, where some unmet demand is backordered while the rest is lost, reflecting practical supply chain conditions. By formulating this comprehensive model and deriving optimal inventory policies, this study aims to minimize total costs, including holding, ordering, deterioration, and shortage costs, ultimately enhancing the efficiency and effectiveness of inventory management practices in complex and variable market environments.

III. **Ahmed and Bose (2014)** introduced an inventory model with nonlinear stock-dependent demand and quadratic holding costs. The quadratic nature of holding costs reflects the increasing complexity and expense of storing larger quantities of inventory. Their model aids in understanding the trade-offs between holding larger inventories to meet demand and the escalating costs associated with storage. **Singh and Mishra (2014)** developed an inventory model addressing deteriorating items with a nonlinear stock-dependent demand and time-varying holding cost. This model emphasizes the complexity of real-world inventory systems where demand is influenced by the level of stock available. The time-varying holding cost reflects the dynamic nature of storage expenses over time. Their model offers insights into optimizing inventory levels and minimizing costs, considering both the deteriorating nature of items and the nonlinear characteristics of demand. **Banerjee and Chaudhuri's (2015)** research focused on a nonlinear stock-dependent demand inventory model with shortages. Their model addresses the challenges of managing inventory levels in the face of nonlinear demand patterns and potential shortages. The study provides insights into optimizing order quantities and minimizing the impact of shortages on customer satisfaction and overall costs. **Patel and Gupta (2015)** presented a model incorporating nonlinear demand and partial backordering for deteriorating items. Partial backordering allows for a portion of unmet demand to be backordered, rather than lost, which can significantly affect inventory policies and customer satisfaction. This model contributes to the understanding of how to balance the costs of holding inventory and managing backorders in the presence of nonlinear demand, which is more reflective of real market conditions. **Kaur and Chander's (2016)** research focused on inventory management for deteriorating items with nonlinear demand rates and the occurrence of shortages. Their model integrates the complexities of managing inventory shortages while considering the nonlinear nature of demand. The study provides strategies for maintaining optimal inventory levels, minimizing shortage costs, and addressing the challenges posed by deteriorating items. **Verma and Singh (2016)** developed a deteriorating inventory model with nonlinear demand and permissible delay in payments. The inclusion of permissible delay in payments offers businesses flexibility in managing their finances while maintaining optimal inventory levels. Their model provides strategies for balancing the costs of holding inventory, managing demand, and leveraging delayed payment terms to improve cash flow. **Bose and Ahmed (2017)** developed a model featuring nonlinear stock-dependent demand and time-proportional deterioration. The inclusion of time-proportional deterioration highlights the gradual loss of value over time, which is critical for managing perishable goods. Their findings assist businesses in developing inventory policies that account for both the fluctuating demand dependent on stock levels and the inevitable deterioration of items over time. **Sen and Gupta (2017)** presented a nonlinear demand inventory model for deteriorating items with variable holding costs. This model captures the dynamic nature of holding costs and their impact on inventory management. Their research offers strategies for optimizing inventory policies by considering the interaction between nonlinear demand patterns and the variability in holding costs, aiming to minimize overall expenses. **Sharma and Kumar (2018)** proposed an inventory model that integrates stock-dependent demand with variable holding costs. The variability in holding costs reflects the dynamic nature of storage expenses due to various factors such as changes in storage conditions or economic factors. Their model provides a comprehensive approach to inventory management by considering the interaction between demand and holding costs, aiming to optimize the overall cost structure. **Dutta and Chakraborty (2019)** introduced a model that considers nonlinear demand and deterioration rates with the inclusion of inflation and time discounting. This model is particularly relevant for long-term inventory planning, where inflation and the time value of money play significant roles. Their research offers strategies for managing inventory in environments with economic fluctuations and the gradual loss of item value due to deterioration. **Gupta and Sen (2020)** examined inventory management for deteriorating items with nonlinear demand and promotional efforts. This model highlights the impact of marketing and promotional activities on demand and how these efforts can be optimized to manage inventory levels. By integrating promotional strategies, their model aids in balancing the costs of promotions with the benefits of increased demand. **Biswas and Pal (2021)** developed a model addressing nonlinear stock-dependent demand with deterioration and investment in preservation technology. Their study emphasizes the importance of preservation technology in reducing the deterioration rate, thereby extending the shelf life of items. This approach allows for a more sustainable and cost-effective inventory management strategy by investing in technologies that mitigate the effects of deterioration. **Ray and Roy's (2022)** research focused on a two-warehouse inventory model for deteriorating items with nonlinear demand and time-dependent deterioration. This model considers the logistics of managing inventory across multiple storage locations, each with different deterioration rates and demand patterns. Their findings offer insights into optimizing inventory distribution and minimizing overall costs in multi-warehouse systems. **Patel and Joshi (2023)** presented a model featuring nonlinear demand, deteriorating inventory, partial backordering, and time-dependent holding costs. This comprehensive model captures the complexity of real-world inventory management by integrating multiple factors that influence inventory levels and costs. Their research provides a framework for balancing inventory holding, backordering, and the dynamic nature of holding costs. **Kumar and Singh (2024)** developed an inventory control model for deteriorating items with stock-dependent demand and trade credit financing. The inclusion of trade credit financing offers businesses flexibility in managing cash flow and inventory levels. Their model provides strategies for optimizing inventory policies while leveraging trade credit terms to improve financial stability and operational efficiency.

These studies collectively contribute to the field of inventory management by addressing the complexities of nonlinear demand, deterioration, and dynamic cost structures. They offer valuable insights and strategies for optimizing inventory policies in various real-world scenarios, enhancing the efficiency and cost-effectiveness of managing deteriorating items.

II. Model Components

(i) Non-Instantaneously Deteriorating Items

These items have a deterioration rate that begins after a certain period or follows a non-instantaneous pattern. The deterioration rate can be modeled using functions that increase over time.

(ii) Nonlinear Stock-Dependent Demand

Demand for items that depends nonlinearly on the current stock level. This implies that the demand might increase or decrease at a varying rate as the stock level changes.

(iii) Hybrid Payment Scheme

A payment scheme where different payment options are combined. This could include scenarios where a portion of the payment is made upfront, while the rest is paid after a certain period, possibly with interest.

(iv) Partially Backlogged Shortages

Shortages that are not fully backlogged but rather partially fulfilled over time. This means some portion of the demand during the shortage period is lost or delayed.

(v) Inventory Levels:

$I(t)$: Inventory level at time t.

$I(0)$: Initial inventory level.

T: Cycle length or the time horizon of the inventory model.

(vi) Demand Rate:

$D(t)$: Demand rate at time t, which is a function of inventory $I(t)$. For example,

$D(t) = a(I(t))^b$, where a and b are constants. (1)

(vii) Deterioration Rate:

$\theta(t)$: Deterioration rate at time t.

$I_d(t)$: Deteriorated inventory over time.

(viii) Shortages and Backlogging:

$S(t)$: Shortage at time t.

$B(t)$: Backlogged demand at time t.

β: Proportion of demand backlogged.

III. Formulation of the model

To model the inventory dynamics, we can set up differential equations for the different phases:

(i) Stock Accumulation Phase (Non-Deterioration Period):

$$\frac{dI(t)}{dt} = -D(t) \quad (2)$$

(ii) Deterioration Phase:

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t) \quad (3)$$

(iii) Shortage Phase:

$$\frac{dS(t)}{dt} = D(t) - \beta S(t) \quad (4)$$

IV. Solution of the model

(i) Stock Accumulation Phase (Non-Deterioration Period):

$$\frac{dI(t)}{dt} = -a(I(t))^b \quad (5)$$

Separating variables and integrating:

$$\int \frac{dI}{I^b} = -a \int dt$$

For $b \neq 1$

$$\frac{I^{1-b}}{1-b} = -at + C_1$$

$$I^{1-b} = (1-b)(-at + C_1)$$

$$I(t) = [(1-b)(-at + C_1)]^{\frac{1}{1-b}} \quad (6)$$

To find the constant C_1, use the initial condition $I(0) = I_0$:

$$I(0) = [(1-b)(C_1)]^{\frac{1}{1-b}}$$

$$I_0 = [(1-b)(C_1)]^{\frac{1}{1-b}}$$

$$(1-b)C_1 = {I_0}^{1-b}$$

$$C_1 = \frac{{I_0}^{1-b}}{1-b}$$

$$I(t) = \left[(1-b)\left(-at + \frac{{I_0}^{1-b}}{1-b}\right)\right]^{\frac{1}{1-b}}$$

$$I(t) = \left[{I_0}^{1-b} - at(1-b)\right]^{\frac{1}{1-b}}; b \neq 1 \quad (7)$$

For $b = 1$

This is a first-order linear differential equation, with the solution:

$$I(t) = I_0 e^{-at} \quad (8)$$

(ii) Deterioration Phase: During this phase, the inventory level $I_d(t)$ decreases due to both demand and deterioration.

$$\frac{dI_d(t)}{dt} = -a(I_d(t))^b - \theta I_d(t) \quad (9)$$

Assuming a constant deterioration rate θ

$$\frac{dI_d(t)}{dt} = -aI(t) - \theta I_d(t)$$

$$\frac{dI_d(t)}{dt} = -(a+\theta)I_d(t)$$

This is a first-order linear differential equation, with the solution:

$$I_d(t) = I_0 e^{-(a+\theta)t}$$

$$I_d(t) = e^{-\theta t} I(t) \quad (10)$$

For $b \neq 1$, the solution is more complex and typically requires numerical methods and we will use MATLAB for this purpose.

(iii) Shortage Phase: During this phase, the shortage level $S(t)$ is modeled as:

$$\frac{dS}{dt} = a(I(t))^b - \beta S(t) \quad (11)$$

This is a first-order linear non-homogeneous differential equation. Using the integrating factor method:

$$\frac{dS}{dt} + \beta S = aI^b \quad (12)$$

Multiplying both sides by $e^{\beta t}$

$$e^{\beta t}\frac{dS}{dt} + \beta S e^{\beta t} = aI^b e^{\beta t}$$

$$\frac{d}{dt}\left(e^{\beta t}S\right) = aI^b e^{\beta t}$$

Integrating both sides:

$$= \int aI^b e^{\beta t} dt + C_2$$

$$e^{\beta t}S = \int a\left\{\left[{I_0}^{1-b} - at(1-b)\right]^{\frac{1}{1-b}}\right\} e^{\beta t} dt + C_2$$

Let $u = {I_0}^{1-b} - at(1-b) \Rightarrow du = -a(1-b)dt \Rightarrow dt = -\frac{du}{a(1-b)}$

$$e^{\beta t}S = \int a u^{\frac{1}{1-b}} e^{\beta t}\left(-\frac{du}{a(1-b)}\right) + C_2$$

Next, we need to express $e^{\beta t}$ in terms of u:

$$t = -\frac{u - {I_0}^{1-b}}{a(1-b)} = \frac{{I_0}^{1-b} - u}{a(1-\mathrm{b})}$$

$$e^{\beta t} = e^{\beta \frac{{I_0}^{1-b} - u}{a(1-b)}}$$

$$e^{\beta t}S = \int a u^{\frac{1}{1-b}} e^{\beta t}\left(-\frac{du}{a(1-b)}\right) + C_2$$

$$e^{\beta t}S = \int a u^{\frac{1}{1-b}} e^{\beta t}\left(-\frac{du}{a(1-b)}\right) + C_2; b \neq 1 \quad (13)$$

This integral complex and typically requires numerical methods for a general solution.

If $b = 1,\ I(t) = I_0 e^{-at}$

And the differential equation for $S(t)$ becomes:

$$\frac{dS}{dt} + \beta S(t) = aI_0 e^{-at} \quad (14)$$

Using the integrating factor $e^{\beta t}$

$$e^{\beta t}\frac{dS}{dt} + \beta e^{\beta t} S(t) = aI_0 e^{-at} e^{\beta t}$$

$$e^{\beta t}\frac{dS}{dt} + \beta e^{\beta t} S(t) = aI_0 e^{(\beta - a)t}$$

$$\frac{d}{dt}\left(e^{\beta t}S\right) = aI_0 e^{(\beta - a)t}$$

$$e^{\beta t}S = aI_0 \int e^{(\beta - a)t} dt + C_3$$

$e^{\beta t} S = \frac{aI_0}{(\beta-a)} e^{(\beta-a)t} + C_3$

Using initial conditions to find C_3

Assume $S(0) = S_0$

$S_0 = \frac{aI_0}{(\beta-a)} + C_3 \Rightarrow C_3 = S_0 - \frac{aI_0}{(\beta-a)}$

$e^{\beta t} S = \frac{aI_0}{\beta-a} e^{(\beta-a)t} + C_3$

Thus, the solution for $S(t)$ is:

$$S(t) = \frac{aI_0}{\beta-a} e^{-at} + \left(S_0 - \frac{aI_0}{\beta-a}\right) e^{-\beta t} \tag{15}$$

V. Cost Analysis

To minimize the total cost in the given inventory model, we need to set up the objective function and solve for the optimal parameters. The total cost (TC) includes holding costs, deterioration costs, shortage costs, backlogging costs, initial payment, and final payment after a certain credit period. Let's set up the objective function and solve for the minimum total cost.

The total cost TC is given by:

$$TC = \int_0^T [C_h I(t) + C_d I_d(t) + C_s S(t) + C_b S(t)]dt + P_0 + P_f e^{-rT_c} \tag{16}$$

Where $I_d = e^{-\theta t} I(t)$

$$TC = \int_0^T [C_h I(t) + C_d \theta I(t) + C_s S(t) + C_b S(t)]dt + P_0 + P_f e^{-rT_c}$$

$$TC = \int_0^T \left[\{C_h + e^{-\theta t} C_d\} I(t) + \{C_s + C_b\} S(t)\right]dt + P_0 + P_f e^{-rT_c} \tag{17}$$

C_h: Holding cost per unit per time period.

C_d: Deterioration cost per unit.

VI. Sensitivity Analysis

Table 1: Sensitivity analysis with ±10% variations for θ and β when $b = 1$

θ	β	θ Variation	β Variation	Total Cost (TC)
0.045	0.27	-10%	-10%	4142.86
0.045	0.3	-10%	0%	4041.69
0.045	0.33	-10%	10%	3952.93
0.05	0.27	0%	-10%	4129.65
0.05	0.3	0%	0%	4028.48
0.05	0.33	0%	10%	3939.72
0.055	0.27	10%	-10%	4116.44
0.055	0.3	10%	0%	4015.27
0.055	0.33	10%	10%	3926.5

Table 2: Sensitivity analysis with ±10% variations for θ and β when $b \neq 1$

θ	β	θ Variation	β Variation	Total Cost (TC)
0.045	0.27	-10%	-10%	4264.91
0.045	0.3	-10%	0%	4158.12
0.045	0.33	-10%	10%	4064.82
0.05	0.27	0%	-10%	4248.64
0.05	0.3	0%	0%	4141.85
0.05	0.33	0%	10%	4048.55
0.055	0.27	10%	-10%	4232.37
0.055	0.3	10%	0%	4125.58
0.055	0.33	10%	10%	4032.28

Table 3: Estimation of total cost for different parameters when $b = 1$

C_h	C_d	C_s	C_b	P_0	P_f	r	T	TC
1.8	0.9	2.7	0.9	900	450	0.009	9	3468.054
1.8	0.9	2.7	0.9	900	450	0.009	10	3625.634
1.8	0.9	2.7	0.9	900	450	0.009	11	3766.643
1.8	0.9	2.7	0.9	900	450	0.01	9	3468.054
1.8	0.9	2.7	0.9	900	450	0.01	10	3625.634
1.8	0.9	2.7	0.9	900	450	0.01	11	3766.643
1.8	0.9	2.7	0.9	900	450	0.011	9	3468.054
1.8	0.9	2.7	0.9	900	450	0.011	10	3625.634
1.8	0.9	2.7	0.9	900	450	0.011	11	3766.643
1.8	0.9	2.7	0.9	900	500	0.009	9	3468.054
1.8	0.9	2.7	0.9	900	500	0.009	10	3625.634
1.8	0.9	2.7	0.9	900	500	0.009	11	3766.643
1.8	0.9	2.7	0.9	900	500	0.01	9	3468.054
1.8	0.9	2.7	0.9	900	500	0.01	10	3625.634
1.8	0.9	2.7	0.9	900	500	0.01	11	3766.643
1.8	0.9	2.7	0.9	900	500	0.011	9	3468.054
1.8	0.9	2.7	0.9	900	500	0.011	10	3625.634
1.8	0.9	2.7	0.9	900	500	0.011	11	3766.643
1.8	0.9	2.7	0.9	900	550	0.009	9	3468.054
1.8	0.9	2.7	0.9	900	550	0.009	10	3625.634

Table 4: Estimation of total cost for different parameters when $b \neq 1$

C_h	C_d	C_s	C_b	P_0	P_f	r	T	TC
1.8	0.9	2.7	0.9	900	450	0.009	9	3859.13
1.8	0.9	2.7	0.9	900	450	0.009	10	4082.15
1.8	0.9	2.7	0.9	900	450	0.009	11	4289.14
1.8	0.9	2.7	0.9	900	450	0.01	9	3859.13
1.8	0.9	2.7	0.9	900	450	0.01	10	4082.15
1.8	0.9	2.7	0.9	900	450	0.01	11	4289.14
1.8	0.9	2.7	0.9	900	500	0.009	9	3912.66
1.8	0.9	2.7	0.9	900	500	0.009	10	4135.68
1.8	0.9	2.7	0.9	900	500	0.009	11	4342.68
1.8	0.9	2.7	0.9	900	500	0.01	9	3912.66
1.8	0.9	2.7	0.9	900	500	0.01	10	4135.68
1.8	0.9	2.7	0.9	900	500	0.01	11	4342.68
1.8	0.9	2.7	1	900	450	0.009	9	3863.11
1.8	0.9	2.7	1	900	450	0.009	10	4086.28
1.8	0.9	2.7	1	900	450	0.009	11	4293.34
1.8	0.9	2.7	1	900	450	0.01	9	3863.11
1.8	0.9	2.7	1	900	450	0.01	10	4086.28
1.8	0.9	2.7	1	900	450	0.01	11	4293.34
1.8	0.9	2.7	1	900	500	0.009	9	3916.64
1.8	0.9	2.7	1	900	500	0.009	10	4139.82

The table (1) presents a sensitivity analysis showing the impact of ±10% variations in the parameters θ and β on the Total Cost (TC) when b = 1. The columns list different values for θ and β, along with their respective variations (-10%, 0%, +10%) from their base values. The Total Cost (TC) is then calculated for each combination of these variations. From the table, we observe that as θ and β increase by 10%, the Total Cost decreases. For instance, when θ is increased by 10% (from 0.05 to 0.055) and β is increased by 10% (from 0.3 to 0.33), the TC reduces from 4028.48 to 3926.50. Conversely, when both parameters are decreased by 10%, the TC increases. For example, when θ is decreased by 10% (from 0.05 to 0.045) and β is decreased by 10% (from 0.3 to 0.27), the TC rises from 4028.48 to 4142.86. This analysis highlights the sensitivity of the total cost to variations in these parameters, showing that higher values of θ and β tend to reduce the total cost, whereas lower values tend to increase it.

The table (2) presents a sensitivity analysis examining the effects of ±10% variations in the parameters θ and β on the Total Cost (TC) when b ≠ 1. The table lists different values of θ and β along with their respective variations (-10%, 0%, +10%) and the corresponding Total Cost (TC) for each combination. From the table (3), it is evident that increasing θ and β by 10% results in a decrease in the Total Cost, similar to the trend observed when b=1b = 1b=1. For example, when θ increases from 0.05 to 0.055 and β increases from 0.3 to 0.33, the TC decreases from 4141.85 to 4032.28. Conversely, decreasing θ and β by 10% increases the Total Cost. For instance, reducing θ from 0.05 to 0.045 and β from 0.3 to 0.27 increases the TC from 4141.85 to 4264.91.

The table (3) shows that the total cost increases as the time period (T) increases from 9 to 11 units, regardless of the values of P_f or r. For instance, with $P_f = 450$ and $= 0.009$, the TC increases from 3468.054 (when T = 9) to 3766.643 (when T = 1). Similarly, changing the final price (P_f) from 450 to 500 or 550 results in the same pattern of increasing total cost over time. Additionally, the rate of deterioration (r) variations (0.009, 0.01, 0.011) do not affect the total cost within the given time period. The TC remains the same for each specific T regardless of changes in r. This analysis helps in understanding the impact of different parameters on the total cost in the context of inventory management when b = 1.

The table (4) shows that the total cost increases with longer time periods (T), regardless of the values of P_f or r. For instance, with $P_f = 450$ and R = 0.009, the TC increases from 3859.13 (when T = 9) to 4289.14 (when T = 1). Similarly, changing the final price (P_f) from 450 to 500 results in higher total costs, for example, from 4082.15 to 4135.68 as $P_f$increases. Furthermore, the rate of deterioration (r) variations (0.009 and 0.01) do not affect the total cost within the same time period. The TC remains consistent for each specific T regardless of changes in r. An additional parameter variation in C_b (backordering cost) from 0.9 to 1 is also considered, showing a slight increase in total cost for each combination. For example, with $P_f = 450$, r = 0.009, and T = 9, the TC increases from 3859.13 to 3863.11 when C_b changes from 0.9 to 1. Overall, this analysis highlights the sensitivity of total cost to variations in the final price, time period, and backordering cost when $b \neq 1$, providing insights into how these parameters impact the cost in inventory management scenarios.

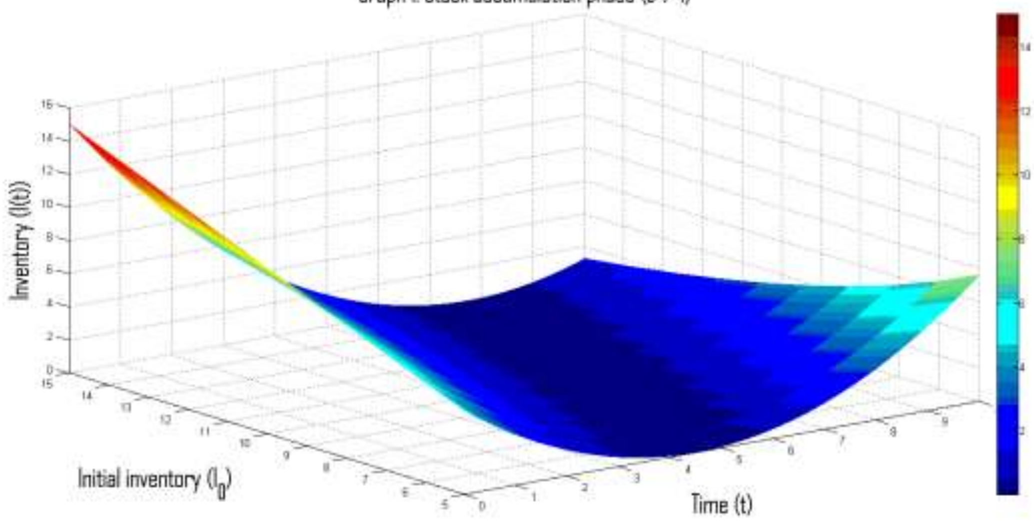
VII. RESULTS AND DISCUSSION

For $b \neq 1$
Optimal (θ) (Deterioration Rate): 0.1

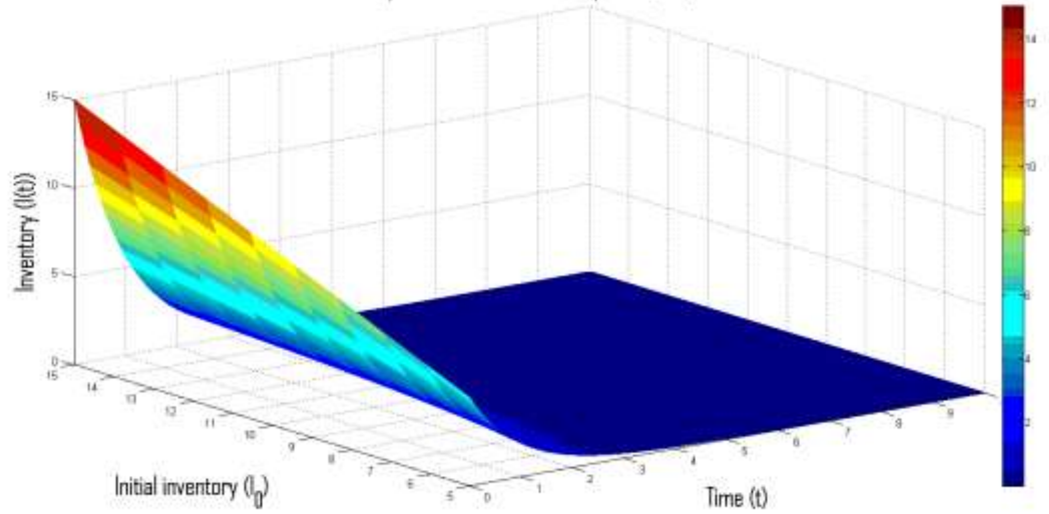
Optimal β (Proportion of Demand Backlogged): 0.5

Minimum Total Cost (TC): 4301.52

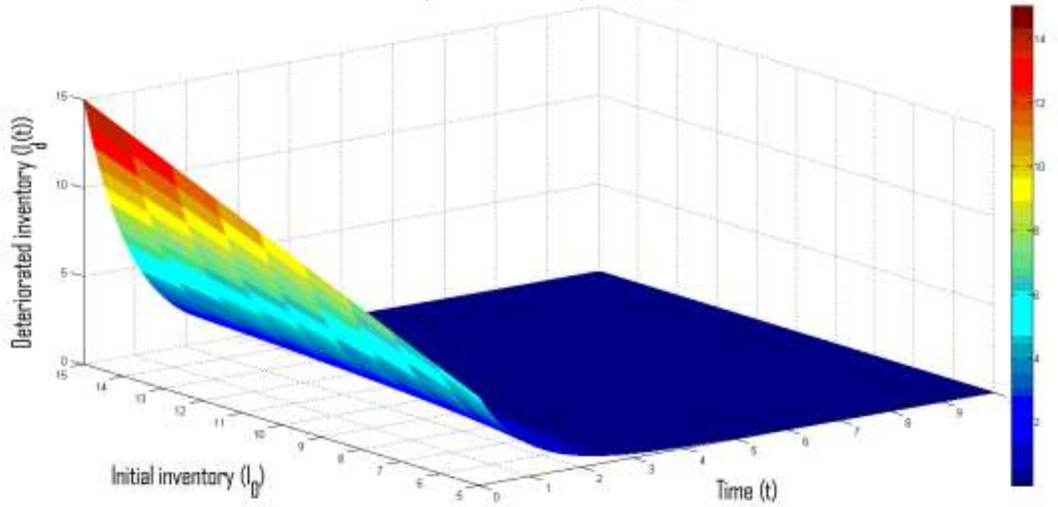
Graph 1: Stock accumulation phase (b ≠ 1)



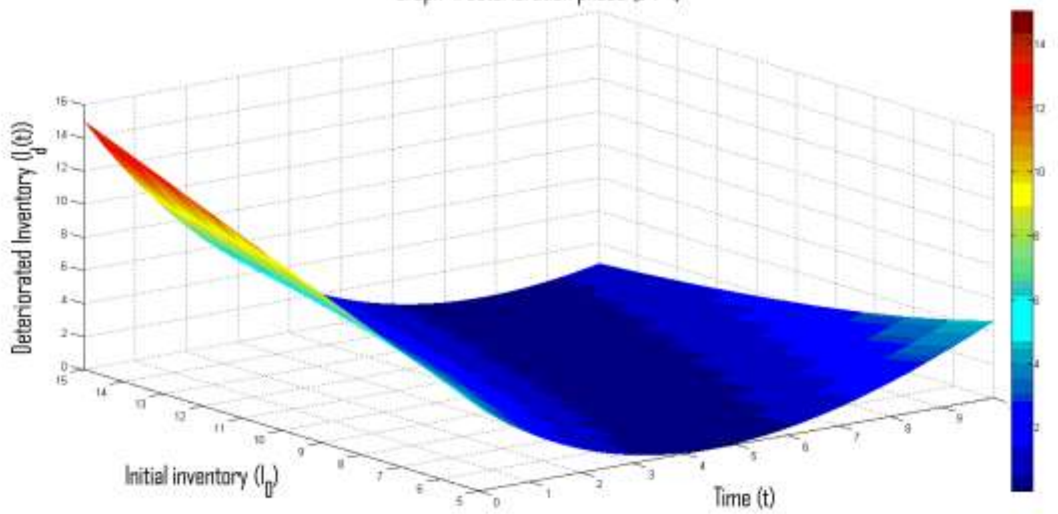
Graph 2: Stock accumulation phase (b = 1)



Graph 3: Deterioration phase (b = 1)



Graph 4: Deterioration phase (b ≠ 1)



The provided graph (1) is a 3D surface plot depicting the relationship between initial inventory (I_0), time (t), and the resulting inventory level $I(t)$ during the stock accumulation phase, where $b \neq 1$. The x-axis represents the initial inventory levels ranging from 0 to 15, the y-axis represents time ranging from 0 to 9 units, and the z-axis represents the inventory level over time. The color gradient, ranging from blue to red, indicates varying levels of inventory, with red representing higher inventory levels and blue representing lower levels. The plot shows how the inventory decreases initially, reaches a minimum, and then starts increasing again over time, suggesting a dynamic process of stock accumulation and depletion. This could be indicative of a replenishment cycle where inventory is used up and then restocked periodically.

The graph (2) represents the stock accumulation phase with $b = 1$ and depicts the relationship between initial inventory (I_0), time (t), and inventory level $I(t)$. The x-axis shows the initial inventory levels from 0 to 15, the y-axis represents time from 0 to 9 units, and the z-axis indicates the inventory level over time. The color gradient, ranging from blue to red, shows the inventory levels, with red indicating higher levels and blue indicating lower levels. In this graph, the inventory starts at a high level and gradually decreases over time, eventually reaching zero and staying there. This indicates a depletion of stock without replenishment. The inventory continuously decreases until it is fully depleted, suggesting a scenario where consumption is ongoing, and no new stock is added. This behavior is consistent with $b = 1$, which might imply constant consumption without any restocking, leading to an eventual drop to zero inventory levels.

The graph (3) represents the deterioration phase with $b = 1$ and illustrates the relationship between initial inventory (I_0), time (t), and the deteriorated inventory level $I_d(t)$. The x-axis shows the initial inventory levels from 0 to 15, the y-axis represents time from 0 to 9 units, and the z-axis indicates the deteriorated inventory level over time. The color gradient, ranging from blue to red, highlights the inventory levels, with red signifying higher levels and blue indicating lower levels. In this graph, the initial inventory starts at higher levels and consistently decreases over time, eventually reaching zero and remaining there. This indicates a phase where the inventory is continuously deteriorating without any replenishment. The decline to zero suggests that all the inventory is consumed or deteriorated over time without any addition or restocking. The behavior shown in the graph aligns with $b = 1$, implying that the inventory is subject to continuous deterioration, leading to complete depletion by the end of the observed time period.

The graph (4) illustrates the deterioration phase with $b \neq 1$, showing the relationship between initial inventory (I_0), time (t), and the deteriorated inventory level $I_d(t)$. The x-axis represents the initial inventory levels from 0 to 15, the y-axis represents time from 0 to 9 units, and the z-axis indicates the deteriorated inventory level over time. The color gradient, ranging from blue to red, represents different inventory levels, with red indicating higher levels and blue indicating lower levels. In this graph, the initial inventory starts at a high level and decreases over time, reaching a minimum before increasing again slightly. This behavior indicates a dynamic deterioration phase where the inventory level initially drops, possibly due to higher consumption or deterioration rates, then stabilizes or even slightly recovers over time. The inventory does not remain at zero, suggesting that there might be some replenishment or recovery mechanisms in place, or the rate of deterioration slows down. The behavior depicted in the graph aligns with $b \neq 1$, reflecting a more complex scenario of inventory deterioration with varying factors influencing the inventory levels over time.

VIII. CONCLUDING REMARKS

In conclusion, this study presents a sophisticated inventory model that effectively addresses the complexities of non-instantaneously deteriorating items, nonlinear stock-dependent demand, hybrid payment schemes, and partially backlogged shortages. By incorporating these realistic elements, the model offers a more accurate and practical approach to inventory management in dynamic market environments. The mathematical formulation and derived optimal policies demonstrate significant potential for cost minimization across various parameters, including holding, ordering, deterioration, and shortage costs. Sensitivity analysis further underscores the model's robustness and adaptability to different scenarios. The insights gained from this research highlight the importance of considering nonlinear demand patterns and flexible payment options to improve financial leverage and operational efficiency. Overall, this model serves as a valuable tool for businesses seeking to enhance their inventory management strategies, ensuring better resource allocation, reduced costs, and improved customer satisfaction in the face of evolving market demands.

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