



ESTIMATION OF SHAPE (α) AND SCALE (λ) FOR TWO-PARAMETER NEW WEIGHTED EXPONENTIAL DISTRIBUTION BY MEDIAN RANKS REGRESSION METHOD

1Mudavath Nandini, 2Veera Aruna Chittibomma, 3G. V. S. R. Anjaneyulu

1Research Scholar, 2Associate professor, 3Professor (Retd.),

1Acharya Nagarjuna University,

2Ramachandra College of Engineering,

3Acharya Nagarjuna University

ABSTRACT

The objective of this paper is to estimate the parameters of New Weighted Exponential distribution with shape parameter(α) and scale parameter(λ). In this heuristic algorithm, the least squares method is used to Estimates the parameters of New Weighted Exponential Distribution. We also computed Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Error (RE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators. Keywords: New Weighted Exponential Parameters, Grouped sample, Median Rank regression method, simulation.

1. Median Ranks Regression Method to Estimate the New Weighted Exponential Distribution parameters:

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from NWED(α, λ), its Probability Density Function (pdf) and cumulative distribution function(CDF) are given by

$$f(x, \alpha, \lambda) = (1+\lambda)\alpha \cdot e^{-\lambda x} - (1+\lambda)\alpha x; x > 0, \alpha > 0, \lambda > 0 \quad (1.1)$$

α =shape parameter

λ =scale parameter

$$\text{CDF } F(x; \alpha, \lambda) = 1 - e^{-\lambda x} - (1+\lambda)\alpha x; x > 0, \alpha > 0, \lambda > 0 \quad (1.2)$$

$$1 - F(X) = e^{-\lambda x} - (1+\lambda)\alpha x \quad (1.3)$$

$$1 + F(X) = 2 - e^{-\lambda x} - (1+\lambda)\alpha x \quad (1.4)$$

$$\frac{1-F(X)}{1+F(X)} = \frac{e^{-(1+\lambda)\alpha x}}{2 - e^{-(1+\lambda)\alpha x}} \quad (1.5)$$

Applying log on both sides to (1.5)

$$\text{Log} \frac{1-F(X)}{1+F(X)} = \log_{2 - e^{-(1+\lambda)\alpha x}} e^{-(1+\lambda)\alpha x}$$

$$\text{Log} \frac{1-F(X)}{1+F(X)} = -(1+\lambda)\alpha x - \log(2 - e^{-(1+\lambda)\alpha x}) \quad (1.6)$$

Let us consider $U=A+BV$

$$U = \text{Log} \frac{1-F(X)}{1+F(X)}, V = -X, A = -\log(2 - e^{-(1+\lambda)\alpha x}), B = (1+\lambda)\alpha$$

$F(x)$ is obtained by using the median rank method (also known as Benard's approximation), which is a good estimate of the median rank estimate. Benard's median rank was used because it showed the best performance and was the most used rank for prediction ($F(x)$). The procedure for ranking the complete data is as follows:

1. List the time to failure data from small to large.
2. Use Benard's formula to assign median ranks to each failure.
3. Estimate the α and λ
4. $F(x) = \frac{i-0.3}{N+0.4}$, $i=1, 2, \dots, N$.

Where $i \rightarrow$ the ranked position of data point and

$N \rightarrow$ The total number of units in the sample

$$\tilde{A} = \frac{\left(\sum_{i=1}^n v_i^2\right)\left(\sum_{i=1}^n u_i\right) - \left(\sum_{i=1}^n v_i\right)\left(\sum_{i=1}^n v_i u_i\right)}{\left(n \sum_{i=1}^n v_i^2\right) - \left(\sum_{i=1}^n v_i\right)^2}$$

$$\tilde{B} = \frac{n\left(\sum_{i=1}^n v_i u_i\right) - \left(\sum_{i=1}^n v_i\right)\left(\sum_{i=1}^n u_i\right)}{\left(n \sum_{i=1}^n v_i^2\right) - \left(\sum_{i=1}^n v_i\right)^2}$$

$F(X)$ is estimated from the median ranks. Once A and B obtained the values of

$\hat{\alpha}$ and $\hat{\lambda}$ can easily be obtained.

2. Simulation Study:

In order to obtain the median ranks method estimators of shape (α) and scale (λ) is used to obtain estimators and to study their predictive properties by Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB). If $\hat{\psi}_{lm}$ is Median Ranks Method estimate of $\hat{\psi}_m$, $m=1, 2$ where ψ_m is a general notation that can be replaced by $\psi_{m1} = \lambda, \psi_{m2} = \alpha$ based on sample l , ($l=1, 2, \dots, r$), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

$$\text{Average Estimate } (\hat{\psi}_m) = \frac{\sum_{i=1}^r \hat{\psi}_{lm}}{r}$$

$$\text{Variance } (\hat{\psi}_m) = \frac{\sum_{i=1}^r (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$$

$$\text{Standard Deviation } (\hat{\psi}_m) = \sqrt{\frac{\sum_{i=1}^r (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}}$$

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^r |\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}|}{r}$$

$$\text{Mean Square Error } (\hat{\psi}_m) = \frac{\sum_{i=1}^r (\hat{\psi}_{lm} - \psi_m)^2}{r}$$

$$\text{Relative Absolute Bias } (\hat{\psi}_m) = \frac{\sum_{i=1}^r |\hat{\psi}_{lm} - \psi_m|}{r \psi_m}$$

Simulated Data Sets:

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the NWED (α, λ) and Newton-Raphson simulation for two parameter combinations, and the process is repeated 10,000 times for different sample sizes $n = 2$ (50) 200 considered.

MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of the median rank method to estimators of scale and shape parameters. Population parameters shape=1 and Scale =1.

TABLE-1

Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.005147	0.000238	0.000000	0.009264	0.020125	0.037880
	λ	0.089706	0.072425	0.005245	0.161471	0.901060	0.089706
3	α	0.002824	0.000072	0.000000	0.005083	0.008324	0.032084
	λ	0.094352	0.080121	0.006419	0.169834	0.900319	0.094352
5	α	0.000654	0.000004	0.000000	0.001177	0.063424	0.002604
	λ	0.098692	0.087660	0.007684	0.177645	0.900017	0.098692
10	α	0.001361	0.000017	0.000000	0.002449	3.843354	0.000694
	λ	0.097278	0.085168	0.007254	0.175101	0.900074	0.097278

15	α	0.001260	0.000014	0.000000	0.002268	4.459219	0.000596
	λ	0.097480	0.085521	0.007314	0.175464	0.900064	0.097480
20	α	0.000454	0.000002	0.000000	0.000816	0.287899	0.000845
	λ	0.099093	0.088375	0.007810	0.178367	0.900008	0.099093
25	α	0.000738	0.000005	0.000000	0.001329	13.282641	0.000202
	λ	0.098524	0.087363	0.007632	0.177343	0.900022	0.098524
50	α	0.162812	0.000005	0.000001	0.001752	0.0391	0.022202
	λ	0.754326	0.00547	0.000003	0.05703	0.065825	0.000604
100	α	0.168612	0.000002	0.000000	0.001121	0.023341	9.644795
	λ	1.101465	0.000189	0.000000	0.010745	0.010484	0.101466
200	α	0.168919	0.000008	0.000002	0.002439	0.020424	5.488071
	λ	1.180636	0.002108	0.000004	0.033769	0.034738	0.180636

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the NWED with Shape(α) and Scale(λ) parameters, Newton- Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Median Ranks method for the estimator so shape and scale parameters of the NWED are evaluated. The simulated results were presented in Table-2 for the values of the parameters Shape=3 and Scale=1.5.

TABLE-2
Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.005147	0.000238	0.000000	0.009264	0.020125	0.037880
	λ	0.096569	0.083930	0.007044	0.173824	0.900118	0.096569
3	α	0.000850	0.000006	0.000000	0.001529	0.034647	0.004586
	λ	0.099434	0.088983	0.007918	0.178980	0.900003	0.099434
5	α	0.000279	0.000001	0.000000	0.000502	0.000624	0.011059
	λ	0.099814	0.089665	0.008040	0.179665	0.900000	0.099814
10	α	0.000564	0.000003	0.000000	0.001015	1.612752	0.000444
	λ	0.099624	0.089324	0.007979	0.179323	0.900001	0.099624
15	α	0.002514	0.000057	0.000000	0.004526	258.604065	0.000156
	λ	0.098324	0.087008	0.007570	0.176983	0.900028	0.098324
20	α	0.001668	0.000025	0.000000	0.003002	53.017784	0.000229
	λ	0.098888	0.088010	0.007746	0.177999	0.900012	0.098888
25	α	0.000497	0.000002	0.000000	0.000894	12.764274	0.000139
	λ	0.099669	0.089405	0.007993	0.179404	0.900001	0.099669
50	α	0.283081	0.000226	0	0.011969	0.060711	6.621125
	λ	1.196363	0.004942	0.000024	0.053498	0.0435	0.196363
100	α	0.217824	0.000016	0	0.002471	0.033421	5.21368
	λ	1.260835	0.011148	0.000124	0.078972	0.079183	0.260835
200	α	0.189768	0.000009	0	0.002336	0.026201	5.794604
	λ	1.1696	0.001041	0.000001	0.024226	0.029806	0.1696

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the NWED

with Shape (α) and Scale(λ) parameters, Newton- Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation(MAD), Mean Square Error(MSE) and Relative Absolute Bias(RAB) of Median Ranks method for the estimators of location and scale parameters of the NWED are evaluated. The simulated results were presented in Table-3 for the values of the parameters Shape=1.5 and Scale=3

TABLE-3
Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.000462	0.000002	0.000000	0.000831	0.010066	0.004622
	λ	0.098154	0.086707	0.007518	0.176676	8.507420	0.032718
3	α	0.005043	0.000229	0.000000	0.009078	19.164406	0.001151
	λ	0.079826	0.057350	0.003289	0.143687	8.584765	0.026609
5	α	0.000195	0.000000	0.000000	0.000351	0.594174	0.000253
	λ	0.099221	0.088603	0.007850	0.178597	8.503123	0.033074
10	α	0.000217	0.000000	0.000000	0.000391	3.190330	0.000122
	λ	0.099132	0.088444	0.007822	0.178437	8.503481	0.033044
15	α	-0.000254	0.000001	0.000000	0.000458	11.848330	0.000074
	λ	0.101018	0.091841	0.008435	0.181831	8.495940	0.033673
20	α	-0.000268	0.000001	0.000000	0.000482	35.077366	0.000045
	λ	0.101072	0.091940	0.008453	0.181930	8.495723	0.033691
25	α	-0.000191	0.000000	0.000000	0.000343	11.192948	0.000057
	λ	0.100762	0.100762	0.008350	0.181372	8.496957	0.033587
50	α	0.162541	0.000005	0.000002	0.00172	0.039265	0.021473
	λ	0.752674	0.005356	0.000029	0.056378	5.05583	0.000119
100	α	0.168257	0.000002	0.000001	0.001115	0.023638	10.59145
	λ	1.093493	0.000169	0.000001	0.010014	3.634937	0.000284
200	α	0.168529	0.000008	0.000002	0.002404	0.020755	5.880935
	λ	1.170447	0.001976	0.000004	0.032543	3.349241	0.00158

We evaluate the performance of the Median Ranks Regression(MRR) method for estimating the NWED with Shape (α) and Scale(λ) parameters, Newton- Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation(MAD), Mean Square Error(MSE) and Relative Absolute Bias(RAB) of Median Ranks method for the estimators of location and scale parameters of the NWED are evaluate. The simulated results were presented in Table-4 for the values of the parameters Shape= 2 and Scale=1.5.

TABLE-4

Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.005147	0.000238	0.000000	0.009264	0.020125	0.037880
	λ	0.094853	0.080974	0.006557	0.170736	0.900265	0.094853
3	α	0.005147	0.000238	0.000000	0.009264	0.020125	0.037880
	λ	0.094853	0.080974	0.006557	0.170736	0.900265	0.094853
5	α	0.000482	0.000002	0.000000	0.000868	0.059660	0.001978
	λ	0.099518	0.089134	0.007945	0.179132	0.900002	0.099518
10	α	-0.000496	0.000002	0.000000	0.000892	20.122671	0.000110
	λ	0.100496	0.090894	0.008262	0.180892	0.900002	0.099504
15	α	-0.000035	0.000000	0.000000	0.000063	5.377807	0.000015
	λ	0.100035	0.090063	0.008111	0.180063	0.900000	0.099965
20	α	-0.000550	0.000003	0.000000	0.000991	35.374756	0.000093
	λ	0.100550	0.090994	0.008280	0.180991	0.900003	0.099450
25	α	-0.000014	0.000000	0.000000	0.000025	3.982592	0.000007
	λ	0.100014	0.090025	0.008104	0.180025	0.900000	0.099986
50	α	0.279479	0.000081	0	0.007628	0.056414	5.63328
	λ	1.174558	0.000562	0	0.017744	0.031032	0.174558
100	α	0.255352	0.000014	0	0.002991	0.040799	3.78194
	λ	1.242392	0.00267	0.000007	0.031157	0.061424	0.242392
200	α	0.209085	0.000019	0	0.002659	0.027575	3.852895
	λ	1.21962	0.000062	0	0.006235	0.048295	0.21962

We evaluate the performance of the Median Ranks Regression(MRR) method for estimating the NWED with Shape(α) and Scale(λ) parameter. Newton- Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation(SD), Mean Absolute Deviation(MAD), Mean Square Error(MSE) and Relative Absolute Bias(RAB) of Median Ranks method for the estimators of shape and scale parameters of the NWED are evaluated. The simulated results were presented in Table-5 for the values of the parameters Shape=1.5 and Scale=2.

TABLE-5

Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.005338	0.000256	0.000000	0.009609	0.020125	0.039364
	λ	0.083985	0.063481	0.004030	0.151173	3.734596	0.041992
3	α	0.000536	0.000003	0.000000	0.000964	0.017163	0.004107
	λ	0.098393	0.087130	0.007592	0.177107	3.703240	0.049196
5	α	0.000823	0.000006	0.000000	0.001481	0.150561	0.002116
	λ	0.097532	0.085612	0.007329	0.175557	3.704997	0.048766
10	α	0.000437	0.000002	0.000000	0.000786	43.483498	0.000066
	λ	0.098690	0.087658	0.007684	0.177642	0.177642	0.049345
15	α	-0.000042	0.000000	0.000000	0.000076	8.429236	0.000015

	λ	0.100127	0.090228	0.008141	0.180228	3.699747	0.050063
20	α	-0.000013	0.000000	0.000000	0.000023	6.130487	0.000005
	λ	0.100038	0.090069	0.008112	0.180069	3.699924	0.050019
25	α	-0.000268	0.000001	0.000000	0.000482	25.087473	0.000053
	λ	0.100803	0.091452	0.008363	0.181446	3.698400	0.050402
50	α	0.162674	0.000005	0.000002	0.001735	0.039192	0.021811
	λ	0.753445	0.005409	0.000029	0.056686	1.559309	0.000237
100	α	0.16843	0.000002	0.0000001	0.001117	0.023501	10.12731
	λ	1.097241	0.000177	0.0000001	0.010346	0.815151	0.000475
200	α	0.168718	0.000008	0.0000002	0.002421	0.020605	5.694363
	λ	1.175219	0.002043	0.000004	0.033155	0.682307	0.002367

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the NWED with Shape (α) and Scale (λ) parameter. Newton-Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Median Ranks method for the estimators of shape and scale parameters of the NWED are evaluated. The simulated results were presented in Table-6 for the values of the parameters Shape= 4 and Scale= 4.5.

TABLE-6

Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.002059	0.000038	0.000000	0.003706	0.019063	0.015152
	λ	0.097427	0.085427	0.007298	0.175368	15.315506	0.024357
3	α	-0.000001	0.000000	0.000000	0.000002	3.536517	0.000000
	λ	0.100001	0.090002	0.008100	0.180002	15.299994	0.025000
5	α	0.000253	0.000001	0.000000	0.000455	0.071618	0.000945
	λ	0.099684	0.089432	0.007998	0.179431	15.301897	0.024921
10	α	-0.000065	0.000000	0.000000	0.000117	7.465598	0.000024
	λ	0.100081	0.090147	0.008126	0.180146	15.299512	0.025020
15	α	0.000003	0.000000	0.000000	0.000006	2.470282	0.000002
	λ	0.099996	0.089993	0.008099	0.179993	15.300024	0.024999
20	α	-0.000140	0.000000	0.000000	0.000252	17.791813	0.000033
	λ	0.100175	0.090315	0.008157	0.180315	15.298950	0.025044
25	α	-0.000007	0.000000	0.000000	0.000013	7.936385	0.000003
	λ	0.100009	0.090017	0.008103	0.180017	15.299944	0.025002
50	α	0.162412	0.000005	0	0.001706	0.039321	0.021177
	λ	0.752003	0.005308	0.000028	0.056099	10.55479	0.000063
100	α	0.168091	0.000002	0	0.001112	0.023759	11.04423
	λ	1.090108	0.000163	0	0.009726	8.467635	0.000195
200	α	0.16835	0.000008	0	0.002389	0.020885	6.055254
	λ	1.166155	0.001911	0.000004	0.031959	8.032592	0.00118

We evaluate the performance of the Median Ranks Regression (MRR) method for estimating the NWED With Shape (α) and Scale (λ) parameter. Newton- Raphson simulation for two parameter combinations was done and the process is repeated 10,000 times for different sample sizes $n=2(50)200$. The MRR and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Median Ranks method for the estimators of shape and scale parameters of the NWED

are evaluated. The simulated results were presented in Table-7 for the values of the parameters Shape= 4 and Scale= 1.

TABLE-7

Median Ranks Regression- NWED

Sample Size	Parameters	AE	VAR	SD	MAD	MSE	RAB
2	α	0.005147	0.000238	0.000000	0.009264	0.020125	0.037880
	λ	0.097427	0.085427	0.007298	0.175368	0.900066	0.097427
3	α	0.000658	0.000004	0.000000	0.001185	0.041557	0.003241
	λ	0.099671	0.089408	0.007994	0.179407	0.900001	0.099671
5	α	0.000545	0.000003	0.000000	0.000981	0.054308	0.002345
	λ	0.099727	0.089510	0.008012	0.179509	0.900001	0.099727
10	α	0.001654	0.000025	0.000000	0.002977	0.597261	0.002135
	λ	0.099173	0.088518	0.007835	0.178512	0.900007	0.099173
15	α	0.001094	0.000011	0.000000	0.001969	0.420837	0.001683
	λ	0.099453	0.089018	0.007924	0.179016	0.900003	0.099453
20	α	-0.004363	0.000171	0.000000	0.007853	230.809082	0.000287
	λ	0.102181	0.093969	0.008830	0.183927	0.900048	0.097819
25	α	-0.000226	0.000000	0.000000	0.000406	7.673898	0.000081
	λ	0.100113	0.090203	0.008137	0.180203	0.900000	0.099887
50	α	0.218515	0.000029	0	0.0044	0.033068	4.945992
	λ	1.223872	0.000811	0.000001	0.022901	0.05093	0.223872
100	α	0.185533	0.000028	0	0.003659	0.022107	4.022534
	λ	1.192547	0.000416	0	0.018457	0.03749	0.192547
200	α	0.192303	0.000002	0	0.001171	0.023078	3.760593
	λ	1.264356	0.00312	0.00001	0.03813	0.073004	0.264356

3. OBSERVATIONS AND CONCLUSIONS FROM SIMULATION

RESULTS OBSERVATIONS:

1. The Average Estimate (AE), Variance (VAR), Standard deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are independent of true values of the parameters of Shape (α) and Scale (λ).
2. Average Estimate (AE) of Shape parameter (α) and Scale parameter (λ) by Median Ranks Method are increasing when sample size (n) is decreasing in most of the situations.
3. Variance (VAR) of Shape parameter (α) and Scale parameter (λ) by Median Ranks Method are decreasing in some cases and increasing in some cases when sample size (n) is increasing.
4. Standard Deviation (SD) of Shape parameter (α) is zero and Scale parameter (λ) by Median Ranks Method are decreasing when sample size (n) is increasing.

5. Mean Absolute Deviation (MAD) of Shape parameter (α) is decreasing in some cases and increasing in some cases and Scale parameter (λ) by Median Ranks Method are increasing in most of the cases when sample size (n) is increasing.
6. Mean Square Error(MSE) of Shape parameter (α) is suddenly increasing in some cases when sample size is increasing and Scale parameter (λ) by Median Ranks Method are decreasing in most of the cases when sample size(n) is increasing.
7. Relative Absolute Bias(RAB) of Shape parameter(α) is decreasing when sample size is increasing but at some large sample size it's value suddenly increasing and Scale parameter(λ) by Median Ranks Method are almost similar when sample size (n)is increasing but at large sample size it's value suddenly decreasing.

REMARKS:

In this Median Ranks Regression Estimation method, the parameters in New Weighted Exponential Distribution it is observed that

1. MSE of the Shape parameter (α) is suddenly increasing when Sample size (n) increases.
2. When compared with small sample sizes and large sample sizes the Shape parameter (α)and Scale Parameter (λ)estimators are suddenly increasing in large samples.

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