



# Application Of Goal Programming Approach For Optimization Of Nutrient Management Of Fertilizers Used For Rubber Plantation In Tripura: A Comparative Study

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## **Abstract:**

Natural rubber is the dominant cash crop of Tripura. Proper application of fertilizer namely nitrogen, phosphorus and potassium are necessary for the soil to increase the production of latex. So, nutrient management of fertilizers used for rubber plantation is vital part for the production. In the paper of Sen & Nandi(2012), a mathematical model was formulated. In this paper I try to consider four cases changing basic structure of the model and a comparative study is made for nutrient management of fertilizer.

**Index Terms:** Fertilizer combination, model, goal programming, priority factors, optimization.

## **1. Introduction:**

Most of the soil in the North Eastern region is highly degenerated due to shifting cultivation and heavy nutrient removed by thatch grass. In general, available phosphorus and potassium levels are low but the available magnesium is high. Based on this, a separate fertilizer recommendation with slightly higher levels of N (nitrogen), P (phosphorus) and K (potassium) was formulated for the main field in the North Eastern region where the state Tripura belongs. Healthy rubber tree growth requires continuous supply of all essential nutrients from soil. Under the agro-climatic conditions, rubber plants after planting takes about seven years to attain tap ability in most cases. The fertilizer requirements of rubber during the period of immaturity depend to a great extent on the cultivation practices, such as mulching the plant bases during the initial years and the establishment and maintenance of leguminous ground covers in the field. These cultivation practices will result in the improvement of the soil conditions and availability of plant nutrients, particularly nitrogen. The general

fertilizer recommendation for rubber is based on the results of the fertilizer experiments conducted by the Rubber Research Institute of India on rubber of different age groups. Application of N-P-K higher than the required level is likely to cause diverse effects on plants and result in a substantial decrease in their yields. Hence, for sustainable production of latex, it is necessary to produce optimum fertilizer combination for nutrient application to the soil. By doing so, damage to the soil's health is minimized. This results in increased production of latex and as a result, improves the economic position of the planters. There may be several objectives in nutrient management and thereby optimum fertilizer combination problems. Basic objective is achievement of nutrient requirements and minimization of cost.

## **2. Application of Mathematical Approaches:**

In the paper of Sen and Nandi (2012), they reviewed many research works of scholars in the field of nutrient management of fertilizers. They reviewed the works of Romero, 1986, Ghosh (1993, 1995), Ghosh, Sharma and Mattison (2005), (Rehman and Romero, 1984, 1987), Minguez, 1988 and many more. They studied many types of mathematical approaches.

## **3. Problem Statement:**

On the basis of the data obtained during the survey and from the secondary sources like Rubber Board of India, TRPC, TFDPC etc., a comparative study has been done on nutrient management of fertilizer combination for rubber plantation. For the proposed model, different goals are prioritized with change of basic structure involving change of deviational variables and one extra case of the original model which is itself a new model have been studied and validated with data. Four models including original one have been solved and analyzed. In the second model, change of priority factor between all constraints of second goal with all constraints of third goal have been considered and analyzed. In the third model, change of priority factor between set of constraints of first goal with first set of constraints of third goal have been considered and analyzed. In the fourth model, change of priority factor between set of constraints of first goal with third set of constraints of second goal have been considered and analyzed. In all the models, main aim is to minimize the total cost goal in fertilizer combination and to minimize the under utilization of the lower limit of nutrients and to minimize the over utilization of the upper limit of nutrients. A priority based linear goal programming technique has been used to obtain the nutrient requirement for rubber plantation by determining the optimum fertilizer combination.

## **4. Mathematical Models:**

The model developed by Sen & Nandi (2012) is presented here in numerical form. For detail discussion of the model, one can go through paper.

In this paper, a priority based linear goal programming technique has been used for nutrient management of fertilizers system which will cover by and large all relevant factors. To formulate a GP model, the symbols used and model components i.e. goal constraints and achievement function are explained.

## 5. Priorities of goals:

As per the decision-making environment, priority level of the problem can be defined as follows:

Goal	Priority
Lower limit of nutrients	$P_1$
Cost	$P_2$
Upper limit of nutrients	$P_3$

Therefore achievement function is

$$\text{Minimize } Z = P_1 (d_{i2}^- + d_{i3}^- + d_{i4}^-) + P_2 d_{i1}^+ + P_3 (d_{i5}^+ + d_{i6}^+ + d_{i7}^+)$$

## 6. Application of proposed model with Numerical Illustration- Model-1:

For numerical illustration, here 7 years is considered for use of fertilizer. Our fertilizer combination plan for rubber plant is for the North-Eastern state Tripura, India. Basically, four types of fertilizers are applied in soil. The fertilizers are Urea, Ammonium Phosphate (AP), Rock Phosphate (RP) and Muriate of Potash (MOP). Nutrient management of fertilizer is expressed as

$$\text{Minimize } Z = P_1 (d_{i2}^- + d_{i3}^- + d_{i4}^-) + P_2 d_{i1}^+ + P_3 (d_{i5}^+ + d_{i6}^+ + d_{i7}^+)$$

Subject to

### Goal constraints

Total cost:

$$P_2: \sum_{n=1}^4 C_{in} x_{in} + d_{i1}^- - d_{i1}^+ = T_i, i = 1, 2, \dots, 7$$

Minimum limit of nutrients:

$$P_1: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i2}^- - d_{i2}^+ = L_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_1: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i3}^- - d_{i3}^+ = L_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_1: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i4}^- - d_{i4}^+ = L_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

Maximum limit of nutrients:

$$P_3: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i5}^- - d_{i5}^+ = U_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i6}^- - d_{i6}^+ = U_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i7}^- - d_{i7}^+ = U_i^{(pot)}, i = 1, 2, \dots, 7. (\text{Potassium})$$

## 7. Explanation of symbols:

### Decision variables:

$x_{in}$  = Amount of fertilizers ( $n=1,2,\dots,N$ ) in the mixture and  $i=1,2,\dots,7$ (year)

### Coefficients and constants:

$C_{in}$  = unit cost for fertilizers  $x_{in}(n=1,2,\dots,N)$  in the mixture and  $i=1,2,\dots,7$ (year)

$A_{in}^{(q)}$  = unit amount of nutrient,  $q(q=1,2,\dots,Q)$  in fertilizer  $x_{in}$

$L_i^{(q)}$  = Minimum limit of nutrient,  $q(q=1,2,\dots,Q)$

$U_i^{(q)}$  = Maximum limit of nutrient,  $q(q=1,2,\dots,Q)$

$T_i$  = Total cost of fertilizer

Table 1: Variables corresponding to respective fertilizers

Variable	Urea	Ammonium Phosphate(AP)	Rock Phosphate(RP)	Muriate of Potash(MOP)
Year(i)				
1	$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$
2	$x_{21}$	$x_{22}$	$x_{23}$	$x_{24}$
3	$x_{31}$	$x_{32}$	$x_{33}$	$x_{34}$
4	$x_{41}$	$x_{42}$	$x_{43}$	$x_{44}$
5	$x_{51}$	$x_{52}$	$x_{53}$	$x_{54}$
6	$x_{61}$	$x_{62}$	$x_{63}$	$x_{64}$
7	$x_{71}$	$x_{72}$	$x_{73}$	$x_{74}$

The above model was developed from the information based on the survey regarding the utilization of different fertilizers containing essential nutrients in Tripura. Collected data are presented in tabular form in table 2 and table 3.

Table 2: Summary of relevant data of parameters

Year	1	2	3	4	5	6	7
Price of Urea in Rs/kg	C <sub>11</sub>	C <sub>21</sub>	C <sub>31</sub>	C <sub>41</sub>	C <sub>51</sub>	C <sub>61</sub>	C <sub>71</sub>
	8.00	8.50	9.00	9.60	10.10	10.80	11.40
Price of AP in Rs/kg	C <sub>12</sub>	C <sub>22</sub>	C <sub>32</sub>	C <sub>42</sub>	C <sub>52</sub>	C <sub>62</sub>	C <sub>72</sub>
	30	31.5	---	-----	----	----	----
Price of RP in Rs/kg	C <sub>13</sub>	C <sub>23</sub>	C <sub>33</sub>	C <sub>43</sub>	C <sub>53</sub>	C <sub>63</sub>	C <sub>73</sub>
	10.50	11.10	12.00	12.80	13.40	14.20	15
Price of MOP in Rs/kg	C <sub>14</sub>	C <sub>24</sub>	C <sub>34</sub>	C <sub>44</sub>	C <sub>54</sub>	C <sub>64</sub>	C <sub>74</sub>
	20.00	21.50	23.00	24.00	25.50	27.00	28.00
$A_{i1}^{(nit)}$ (in kg in Urea)	.46	.461	.47	.48	.482	.481	.483
$A_{i2}^{(nit)}$ (in kg in AP)	.13	.132	---	---	---	---	----
$A_{i3}^{(nit)}$ (in kg in RP)	---	---	---	---	----	---	----
$A_{i4}^{(nit)}$ (in kg in MOP)	---	---	---	---	----	---	----
$A_{i1}^{(pho)}$ (in kg in Urea)	---	---	---	---	----	---	----
$A_{i2}^{(pho)}$ (in kg in AP)	.17	.13	---	---	----	---	----
$A_{i3}^{(pho)}$ (in kg in RP)	.6	.18	.18	.181	.19	.182	.183
$A_{i4}^{(pho)}$ (in kg in MOP)	---	---	---	---	----	---	----
$A_{i1}^{(pot)}$ (in kg in Urea)	---	---	---	---	----	---	----
$A_{i2}^{(pot)}$ (in kg in AP)	---	---	---	---	----	---	----
$A_{i3}^{(pot)}$ (in kg in RP)	---	---	---	---	----	---	----
$A_{i4}^{(pot)}$ (in kg in MOP)	.6	.63	.64	.63	.61	.6	.62

Table 3: Summary of data related to bounds of target goal

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$
$T_i$ (in Rs)	6898.00	9457.00	6619.00	5908.80	5514.00	5122.20	5391.80
$L_i^{(nit)}$	$\geq 43$	$\geq 54$	$\geq 64$	$\geq 53.50$	$\geq 47$	$\geq 41.50$	$\geq 42$
$U_i^{(nit)}$	$\leq 49$	$\leq 58$	$\leq 69$	$\leq 55.50$	$\leq 50$	$\leq 42.50$	$\leq 44$
$L_i^{(pho)}$	$\geq 41.50$	$\geq 55$	$\geq 64.50$	$\geq 54$	$\geq 49$	$\geq 41$	$\geq 42$
$U_i^{(pho)}$	$\leq 45$	$\leq 56$	$\leq 68$	$\leq 57.50$	$\leq 50$	$\leq 43.50$	$\leq 44.50$
$L_i^{(pot)}$	$\geq 20.50$	$\geq 27$	$\geq 31$	$\geq 27$	$\geq 24$	$\geq 20$	$\geq 20$
$U_i^{(pot)}$	$\leq 25.50$	$\leq 28$	$\leq 34$	$\leq 30.50$	$\leq 26$	$\leq 22$	$\leq 22.50$

## 8. Different cases of the above model:

The present study will give the idea about the effects on the model when priorities are rearranged. Accordingly, we have considered three different cases which are also treated as different models.

### 8.1. Model -2: Changing priority factor between all constraints of second goal with all constraints of third goal.

$$\text{Minimize } Z = P_1 (d_{i2}^- + d_{i3}^- + d_{i4}^-) + P_2 d_{i1}^+ + P_3 (d_{i5}^+ + d_{i6}^+ + d_{i7}^+)$$

Subject to

Goal Constraints

Total cost:

$$P_2: \sum_{n=1}^4 C_{in} x_{in} + d_{i1}^- - d_{i1}^+ = T_i, i = 1, 2, \dots, 7$$

Minimum limit of nutrients:

$$P_3: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i5}^- - d_{i5}^+ = L_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i6}^- - d_{i6}^+ = L_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i7}^- - d_{i7}^+ = L_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

Maximum limit of nutrients:

$$P_1: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i2}^- - d_{i2}^+ = U_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_1: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i3}^- - d_{i3}^+ = U_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_1: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i4}^- - d_{i4}^+ = U_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

### 8.2. Model -3: Changing priority factor between set of constraints of first goal with first set of constraints of third goal.

$$\text{Minimize } Z = P_1 (d_{i2}^- + d_{i3}^- + d_{i4}^-) + P_2 d_{i1}^+ + P_3 (d_{i5}^+ + d_{i6}^+ + d_{i7}^+)$$

Subject to

#### Goal constraints

Total cost:

$$P_3 : \sum_{n=1}^4 C_{in} x_{in} + d_{i5}^- - d_{i5}^+ = T_i, i = 1, 2, \dots, 7$$

Minimum limit of nutrients:

$$P_1 : \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i2}^- - d_{i2}^+ = L_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_1 : \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i3}^- - d_{i3}^+ = L_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_1 : \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i4}^- - d_{i4}^+ = L_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

Maximum limit of nutrients:

$$P_2 : \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i1}^- - d_{i1}^+ = U_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_3 : \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i6}^- - d_{i6}^+ = U_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_3 : \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i7}^- - d_{i7}^+ = U_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

### 8.3. Model -4: Changing priority factor between set of constraints of first goal with third set of constraints of second goal.

$$\text{Minimize } Z = P_1 (d_{i2}^- + d_{i3}^- + d_{i4}^-) + P_2 d_{i1}^+ + P_3 (d_{i5}^+ + d_{i6}^+ + d_{i7}^+)$$

Subject to

#### Goal constraints

Total cost:

$$P_1 : \sum_{n=1}^4 C_{in} x_{in} + d_{i4}^- - d_{i4}^+ = T_i, i = 1, 2, \dots, 7$$

Minimum limit of nutrients:

$$P_1: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i2}^- - d_{i2}^+ = L_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_1: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i3}^- - d_{i3}^+ = L_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_2: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i1}^- - d_{i1}^+ = L_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

Maximum limit of nutrients:

$$P_3: \sum_{n=1}^4 A_{in}^{(nit)} x_{in} + d_{i5}^- - d_{i5}^+ = U_i^{(nit)}, i = 1, 2, \dots, 7. \text{ (Nitrogen)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pho)} x_{in} + d_{i6}^- - d_{i6}^+ = U_i^{(pho)}, i = 1, 2, \dots, 7. \text{ (Phosphorus)}$$

$$P_3: \sum_{n=1}^4 A_{in}^{(pot)} x_{in} + d_{i7}^- - d_{i7}^+ = U_i^{(pot)}, i = 1, 2, \dots, 7. \text{ (Potassium)}$$

## 9. Solution:

The models outlined above have been solved by LING 13.0 using the table 2 and table 3. The results obtained in initial model and its three different cases are shown in table 4. More over table 5 shows the extent to which target goals in different models mentioned above are achieved.

## 10. Analysis of the Results & discussion:

### 10.1. Analysis of the Results of First Model:

Analysis of 1<sup>st</sup> model indicates that minimization of under utilization of the limit of all the nutrients including nitrogen, phosphorous and potash are fully achieved except for the 3<sup>rd</sup> and 4<sup>th</sup> year. Underachievement values 0.3089162 and 0.1209866 in phosphorous nutrient for 3<sup>rd</sup> and 4<sup>th</sup> years show that there is a scope to use more phosphorous that is 0.3089162 kg and 0.1209866 kg phosphorous as nutrient in fertilizer combination for 3<sup>rd</sup> and 4<sup>th</sup> year respectively. Results of 1<sup>st</sup> model also reveal that minimization of over utilization of the upper limit of all the nutrients for all seven years are fully achieved. Optimal solutions show that 618.46108 kg Urea, 388.5006 kg Ammonium Phosphate, 1558.93432 kg Rock Phosphate and 274.06056 kg Muriate of Potash are required as fertilizer combination up to seven years. Here the fertilizer combination to be applied for seven years is 2839.95656 kg in one hectare which includes 338.89 kg/ha of nitrogen, 467.088 kg/ha of phosphorous and 164.436 kg/ha of potash.

### 10.2. Analysis of the Results of Second Model:

When priority factors are changed between goals, result changes and second model reveals that values of quantity of all the essential nutrients are zero which implies that no nutrient is necessary but practically which is not possible.

### 10.3. Analysis of the Results of Third Model:

Results of third model reveal that it is exactly same as 1<sup>st</sup> model in all respect including optimal solutions. Fourth model indicates that all goals are fully achieved except the goal constraints for minimization of over utilization of the lower limit of potash nutrient are under achieved for all seven years and optimal solutions are different from 1<sup>st</sup> and 3<sup>rd</sup> model. Optimal solutions show that 598.12877 kg Urea, 459.8687 kg Ammonium Phosphate, 1579.5174 kg Rock Phosphate and 165.30021 kg Muriate of Potash are required as fertilizer combination up to seven years. Here the fertilizer combination to be applied for seven years is 2802.81508 kg in one hectare which includes 339.5206 kg/ha of nitrogen, 505.0491 kg/ha of phosphorous and 99.180 kg/ha of potash.

### 10.4. Analysis of the Results of Fourth Model:

But due to little change in the requirement of Ammonium Phosphate and Rock Phosphate, slight increase in phosphate nutrient is noticeable in fourth model compare to 1<sup>st</sup> and 3<sup>rd</sup> models. Requirement of phosphate nutrient in 4<sup>th</sup> model is 505.0491 kg/ha for all the seven years.

Table 4: Summary of the values of variables in above stated models

Variables		Models			
		Model -1	Model -2	Model -3	Model -4
Urea	$x_{11}$	39.37856	0.0000	39.37856	31.58034
	$x_{21}$	60.70854	0.0000	60.70854	48.17445
	$x_{31}$	136.1702	0.0000	136.1702	136.1702
	$x_{41}$	111.4583	0.0000	111.4583	111.4583
	$x_{51}$	97.51037	0.0000	97.51037	97.51037
	$x_{61}$	86.27859	0.0000	86.27859	86.27859
	$x_{71}$	86.95652	0.0000	86.95652	86.95652
AP	$x_{12}$	191.4297	0.0000	191.4297	219.0234
	$x_{22}$	197.0709	0.0000	197.0709	240.8453
RP	$x_{13}$	14.92825	0.0000	14.92825	7.110038
	$x_{23}$	163.2266	0.0000	163.2266	131.6117
	$x_{33}$	356.6171	0.0000	356.6171	377.7778
	$x_{43}$	297.6741	0.0000	297.6741	317.6796
	$x_{53}$	263.1244	0.0000	263.1244	263.1579
	$x_{63}$	231.7177	0.0000	231.7177	239.0110
MOP	$x_{73}$	231.6459	0.0000	231.6459	243.1694
	$x_{14}$	34.16667	0.0000	34.16667	0.00000
	$x_{24}$	42.85714	0.0000	42.85714	0.00000
	$x_{34}$	48.43750	0.0000	48.43750	37.39716

$x_{44}$	42.85714	0.0000	42.85714	32.18757
$x_{54}$	39.34426	0.0000	39.34426	39.32664
$x_{64}$	33.33333	0.0000	33.33333	29.49760
$x_{74}$	33.06452	0.0000	33.06452	26.89124

Table 5: Summary of analysis of different goals of models

Priority	Description and Achievement according of different objectives (goals)			
	Model 1	Model 2	Model 3	Model 4
$P_1$	Minimize under utilization of the lower limit of nitrogen nutrient.	Minimize under utilization of the upper limit of nitrogen nutrient.	Minimize under utilization of the lower limit of nitrogen nutrient.	Minimize the under achievement of the total cost goal in fertilizer combination.
	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years
	Minimize under utilization of the lower limit of phosphorous nutrient.	Minimize under utilization of the upper limit of phosphorous nutrient.	Minimize under utilization of the lower limit of phosphorous nutrient.	Minimize under utilization of the lower limit of phosphorous nutrient.
	Achieved except for 3 <sup>rd</sup> and 4 <sup>th</sup> year. For 3 <sup>rd</sup> year, under achieved by 0.3089162 and for 4 <sup>th</sup> year, under achieved by 0.1209866.	Achieved for all 7 years	Achieved except for 3 <sup>rd</sup> and 4 <sup>th</sup> . For 3 <sup>rd</sup> year, under achieved by 0.3089162, for 4 <sup>th</sup> year, under achieved by 0.1209866.	Achieved for all 7 years
	Minimize under utilization of the lower limit of potash nutrient.	Minimize under utilization of the upper limit of potash nutrient.	Minimize under utilization of the lower limit of potash nutrient.	Minimize under achievement of the total cost goal in fertilizer combination.
	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years
$P_2$	Minimize over achievement of the total cost goal in fertilizer combination.	Minimize over achievement of the total cost goal in fertilizer combination.	Minimize over utilization of the upper limit of nitrogen nutrient.	Minimize over utilization of the lower limit of potash nutrient.
	Achieved for all 7 years	Under achieved for all 7 years	Achieved for all 7 years	Under achieved for all 7 years
$P_3$	Minimize over utilization of the upper limit of nitrogen nutrient.	Minimize over utilization of the lower limit of nitrogen nutrient.	Minimize the over achievement of the total cost goal in fertilizer combination.	Minimize over utilization of the upper limit of nitrogen nutrient.
	Achieved for all 7 years	Under achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years
	Minimize over utilization of the upper limit of phosphorous nutrient.	Minimize over utilization of the upper limit of phosphorous nutrient.	Minimize over utilization of the upper limit of phosphorous nutrient.	Minimize over utilization of the upper limit of phosphorous nutrient.
	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years
	Minimize over utilization of the upper limit of potash nutrient.	Minimize over utilization of the upper limit of potash nutrient.	Minimize over utilization of the upper limit of potash nutrient.	Minimize over utilization of the upper limit of potash nutrient.
	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years	Achieved for all 7 years

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**References:**

- [1] Ghosh, D.; Sharma, D.K.; Mattison, D.M. (2005). Goal programming formulation in nutrient management for rice production in West Bengal. *International Journal of Production Economics*. 95: 1-7.
- [2] Ghosh, D.; Pal, B.B.; Basu, M. (1993). Determination of optimal land allocation in agricultural planning through goal programming with penalty functions. *OPSEARCH*. 30(1): 15-34.
- [3] Ghosh, D.; Pal, B.B.; Basu, M. (1995). Determination of optimal solution for a MCDM model in agriculture planning: A strategy. *International journal of Management and System*. 11(2): 267-283.
- [4] Minguez, M.I.; Romero, C.; Domingo, J. (1988). Determining optimum fertilizer combinations through goal programming with penalty functions: An application to sugar beet production in Spain. *The Journal of Operational Research Society*. 39(1): 61-70.
- [5] Rehman, T.; Romero, C. (1984). Multi-criteria decision making techniques and their role in livestock ration formulation. *Agricultural System*. 15: 23-49.
- [6] Rehman, T.; Romero, C. (1987). Goal programming with penalty functions and livestock ration formulation. *Agricultural System*. 23: 117-132.
- [7] Romero, C. (1986). A survey of generalized goal programming (1970-1982). *European Journal of Operational Research*. 25: 183-191.
- [8] Sen, N.; Nandi, M.(2012). A goal programming formulation in nutrient management of fertilizers used for Rubber plantation in Tripura. *International Journal Of Research In Commerce, It & Management*. 2(9) : 142-144.

