



# A DETERIORATIVE INVENTORY MODEL FOR ITEMS

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**Abstract:** This study presents a different approach to get the best features out of Mak's depreciating production-inventory model. Here, the production lot-size model with constant deterioration's exact average total cost expression has been found. Next, the ideal production cycle time and the ideal total inventory cycle time are found using a traditional computerised search procedure. The findings of this study are demonstrated numerically, and they are then contrasted with Mak's methods for figuring out the "optimal" values of the approximation equations.

**Index Terms - Demand, deterioration, inventory, storage**

## I. INTRODUCTION

The idea that products have an endless shelf life in storage has long been a fundamental implicit assumption of most inventory systems. One can overlook the effects of degradation or decay if their rate is minimal. Nonetheless, specific consideration of degradation is necessary in many instances, such as the deterioration of fruits and vegetables, the evaporation of volatile liquids (alcohol or petrol), or the decay of radioactive substances.

Mak took into account an exponentially decaying item manufacturing lot-size inventory model with backlog. He arrived at approximations for the production cycle, total inventory cycle time, production cycle average, and ideal production lot size. A set of formulas for a production inventory model with a constant rate of deterioration for scenarios without shortages and backlogs has also been produced by Shah and Jaiswal. They computed the average carrying inventory as half of the greatest amount of inventory and approximated the inventory depletion curve as linear in order to determine the average total cost function. The "optimum" inventory cycle time was then determined by applying a Newton-Raphson iterative process. The work of Misra, who initially introduced a deteriorating inventory model for production lot-size inventory systems, is expanded upon in the aforementioned models.

To get the precise average total inventory cost function for the aforementioned inventory model, a thorough study is conducted in this paper. It is also decided on exact expressions for the production cycle, inventory cycle time, and production lot size. The ideal value of the inventory cycle time and the production cycle are then computed using an automated search method on the precise cost equation. Additionally, the additional features of the inventory model can be obtained by directly substituting these results into the corresponding equations. A numerical example from Mak's work is utilised to demonstrate how the outcomes of the two approaches are compared.

## II. Model Assumptions And Notations :

The work of Misra, Shah and Jaiswal, and Mak is extended in this paper's mathematical model of the inventory system, which is based on the following presumptions:

1. The production rate is higher than the demand rate, and both the demand and production rates are known and steady.
2. After manufacture, items are readily available to meet demand.
3. No item that deteriorates during a certain inventory cycle is repaired or replaced.
4. There is an expected zero lead time.
5. Items do not begin to deteriorate until they are placed in inventory; this does not happen beforehand. It is believed that the rate of degradation will never change.
6. The quantity of items is considered a continuous variable and the production rate is independent of the size of the production lot.
7. Only inventory goods that have not degraded are subject to carrying (keeping) costs.
8. The size of the production batch is set and won't change from cycle to cycle.

In this paper, the following notations are used:

$I(t)$  = is the inventory amount at  $t$ .

$Q$  = size of production lot / order amount;

$I_m$  = stands for maximum inventory level.

$I_b$  = stands for minimum inventory level (maximum backlog of unfilled orders);

$C$  = carrying cost of inventory (S/item/unit time);

$C_2$  = inventory shortage cost (S/item/unit time)

$C_3$  = production/set-up cost (S/set-up)

$C_4$  = deterioration cost (S/unit) [manufacturing cost plus disposal cost minus any salvage value];

$K()$  = average total cost

$T$  = inventory cycle time

$T_1$  = the time when the inventory level is at maximum

$T_2$  = the time at which the inventory level reaches zero

$T_3$  = the time at which the backlog reaches its peak

$D$  = expected total number of units deteriorating during a given inventory cycle

$P$  = constant production rate (units/unit time)

$d$  = constant demand rate (units/unit time)

$I_1$  = total carrying inventory

$I_2$  = total backlog

$h$  = Constant rate of deterioration, where  $(1/h)$  is the anticipated mean life of an inventory item.

### III. Model Development :

The behaviour of the production lot size inventory model with backlog and constant rate of deterioration. Only the more general situation is given because the inventory model without shortages is a special case of the inventory model with backlog.

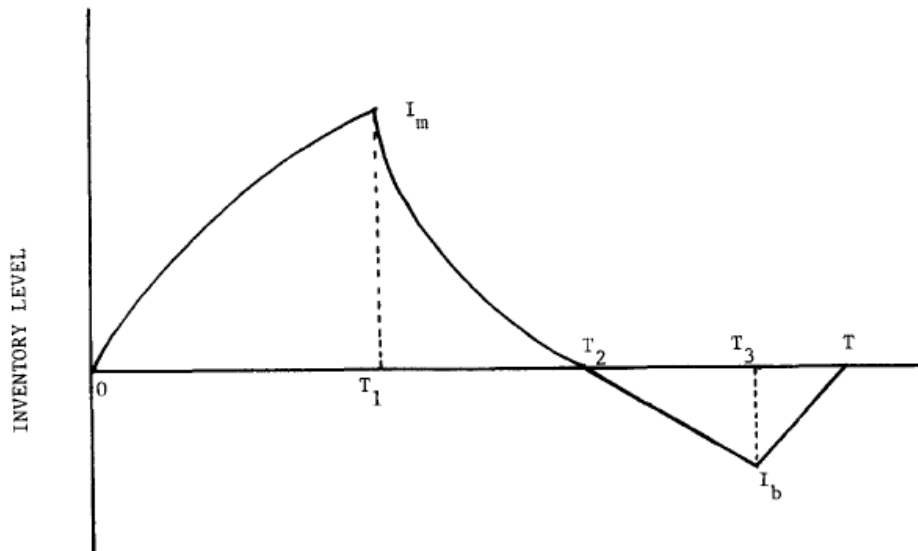


Fig. 1 : A manufacturing lot-size inventory system's inventory cycle with backlog that is continuously degrading

The inventory cycle has four phases if  $T$  is the inventory cycle time. Production happens at a rate of  $p$  and demand happens at a rate of  $d$  units per unit time in the first segment  $(0, T_1)$ . There is no production in the second segment  $(T_1, T_2)$ , and inventory is used to meet demand at a steady pace of  $d$  units per unit time. Deterioration is also occurring in segments one and two as a function of inventory level at a constant rate,  $h$ . Demand is backlogged in the third segment  $(T_2, T_3)$ , and it is finally lowered at a rate of  $(p - d)$  units per unit time in the fourth segment  $(T_3, T)$ . However, take note that the third and fourth components of this inventory model are stable.

Given that Mak's equation, the deterioration rate,  $h$ , is assumed to be constant.

$$I(t) = \frac{\int_0^t (p - d) \exp\left(\int_0^t h \, dt\right) dt}{\exp\left(\int_0^t h \, dt\right) dt}$$

which expresses the inventory level can be rephrased as follows

$$I(t) = \frac{\int_0^t (p - d) \exp(ht) \, dt}{\exp(ht)} = \left(\frac{p - d}{h}\right) [\exp(-ht)][\exp(ht) - 1] = \left(\frac{p - d}{h}\right) [1 - \exp(-ht)].$$

$0 \leq t \leq T_1 \quad (1)$

In the same way, Mak's formula ,

$$I(t) = \frac{I_m - \int_0^t d \exp\left(\int_0^t h \, dt\right) dt}{\exp\left(\int_0^t h \, dt\right) dt}$$

can be expressed as follows for a continuous decline

$$I(t) = [\exp(-ht)] \{I_m - (d/h) [\exp(ht) - 1]\}$$

Given that  $I(t) = 0$  at  $T_2$ ,  $I_m$ 's value equals:

$$I_m = (d/h) \exp(hT_2 - 1)$$

This, when substituted in the preceding equation, produces the inventory level expression shown below

$$I(t) = (-d/h) \{-\exp[h(T_2 - t)] + 1\}, \quad T \leq t \leq T_2 \quad (2)$$

Moreover, the inventory level can be expressed easily as follows after correcting for the beginning conditions, as there is no decline in the  $T_2$  to  $T$  range:

$$I(t) = -d(t - T_2), \quad T_2 \leq t \leq T_3 \quad (3)$$

$$I(t) = (p - d)(t - T), \quad T_3 \leq t \leq T. \quad (4)$$

The following equations linking  $T_1$  and  $T_2$  can be written using this identity since equations (1) and (2) are equivalent at  $t = T_1$ .

$$T_1 = (1/h) \ln \{1 + (d/p)[\exp(hT_2) - 1]\} \quad (5)$$

$$T_2 = (1/h) \ln \{1 + (p/d)[\exp(hT_1) - 1]\} \quad (6)$$

Additionally, equations (3) and (4) are equivalent at  $t = T_3$ , allowing the relationship below to be constructed.

$$T_3 = (d/p) T_2 + (P - d/p) T \quad (7)$$

Equations (5) and (7) can be used to determine the total number of goods generated during a specific inventory cycle,  $Q$ , as a function of  $T_2$  and  $T$ .

$$Q = pT_1 + p(T - T_3) = (p/h) \ln \{1 + (d/p)[\exp(hT_2) - 1]\} + d(T - T_2) \quad (8)$$

Since the entire number of units that are demanded during an inventory cycle equals  $dT$ , the total number of units that deteriorate can be calculated as follows:

$$D = Q - dT = (p/h) \ln \{1 + (d/p)[\exp(hT_2) - 1]\} - dT_2 \quad (9)$$

The total carrying inventory and the total backlog for a specific inventory cycle period are the only remaining amounts to be computed. Rather than approximating the total carrying inventory by assuming that  $Q(t)$  is linear in  $t$  over  $(0, T_2)$ , as suggested in Refs [2], [3] and [6] the carrying inventory is precisely determined by integrating equations (1) and (2) throughout this period. Even while the linearity assumption yields "good" approximations, it is incorrect. Next, the carrying inventory total can be expressed as follows:

$$I_1 = \int_0^{T_1} I(t) dt + \int_{T_1}^{T_2} I(t) dt = [(pT_1 - dT_2)/h] + [\{-p + (p - d) \exp(-hT_2 + d(\exp(hT_2 - hT_1)))\}/h^2] \quad (10)$$

Although equation (10) is expressed as a function of  $T_1$  and  $T_2$ , it can also be expressed as a  $I_1$  function of a single variable because equations (5) and (6) connect the values of  $T_1$  and  $T_2$ .

$I_1$  can be rewritten as, by adding equation (6) to the above expression and making additional simplifications.

$$I_1 = [(pT_1 - (d/h) \ln \{1 + (p/d) [\exp(hT_1) - 1]\})/h] \quad (11)$$

and total backlog as

$$I_2 = d/2 (T - T_2)^2 (p - d/p) \quad (12)$$

Equation (5) establishes a relationship between the value of  $T_1$  and  $T_2$ , so equation (11) only functions as a  $I_1$  function of  $T_2$ . Furthermore,  $T$  and  $T_2$  determine the precise average total cost equation, which is,

$$K(T_2, T) = [C_1 I_1 + C_2 I_2 + C_3 + C_4 D]/T \quad (13)$$



Differential calculus can theoretically be used to obtain the optimal values of  $T_2$  and  $T$  in order to find the minimum of the above cost function; however, approximations to the exponential and logarithmic terms would be required. In addition, the optimal  $T_2$  and  $T$  values from the required circumstances require more approximations in addition to numerical methods. Consequently, by employing an automated search method, these stages can be removed, and the "optimal" solutions can be found straight out of the precise cost equation. For instance, a pattern search like Hooke and Jeeves, which is easy to use and does not require differentiability of the cost function (see Keuster and Mize for an explanation and FORTRAN implementation of this technique), can be employed in this process. In the scenario where  $T = T_2$  and there is no backlog, equation (13) functions based on a single variable. Thus, it is possible to get the ideal value of  $T$  by applying the Fibonacci search method. A single valued convex function's optimal value was found using an automated Fibonacci search method. Keuster and Mize also have the coding for this procedure. By combining the elegance of mathematical derivations with the strength and power of a computer, an automated search technique on the exact cost equation enables the analyst to use a more practical approach to issue solution. Raafat has successfully dealt with failing inventory models using this method.

#### IV. Numerical Example :

The following example, taken from Mak's paper, shows the outcomes of the aforementioned methods for different values of the deterioration rate. Suppose:

$p = 8000$  units/yr;

$d = 2000$  units/yr;

$C_1 = \$5.0$  units/yr;

$C_2 = \$100$  units/yr;

$C_3 = \$200$  order;

$C_4 = \$400$  unit.

The methodology's findings show that employing Mak's cost expression considerably underestimates the real cost of the inventory system, but has no discernible effect on the other model parameters. Compared to Mak's inventory model, this methodology consistently yields a lower average total cost. The disparities become more noticeable as the deterioration rate grows in magnitude.

#### V. Conclusion :

The derivation and determination of the precise average total cost equation for the degrading production inventory model is the general solution methodology that is highlighted in this research. With the precise equation in hand, one can apply a computer search strategy, like the Fibonacci or Hooke and Jeeves search methods, to find the best value for the inventory model's other attributes. In declining inventory models, approximations are usually required in determining total carrying inventory in order to achieve "optimal" results. To create the "optimal" cost equation, a number of other numerical approximations are also required. The additional inventory parameters are then computed on this "optimal" equation using analytical and numerical methods. This research, however, demonstrates how automated search methods might be applied to more precisely and directly acquire the ideal outcome. Therefore, one can achieve the "optimal" answer within a predetermined interval of uncertainty rather than using optimisation techniques on the "approximate" cost equation to acquire the optimal inventory characteristics.

Table 1. Comparative results of the analysis

h	Results of this methodology				Results of Mak's methodology				Comparative results <sup>1</sup>		
	Q	T	K	D	Q	T	K	D	Q	K	D
0.00	473.28	0.2366	1690.31	0	473.28	0.2366	1690.31	0	473.28	1690.31	0 <sup>2</sup>
0.01	456.16	0.2279	1753.58	0.351	460.72	0.2302	1737.78	0.269	460.81	1753.68	0.359
0.02	441.08	0.2202	1814.22	0.650	449.17	0.2243	1783.69	0.507	449.34	1814.56	0.677
0.03	429.43	0.2143	1872.48	0.917	438.53	0.2189	1828.17	0.720	438.77	1873.18	0.962
0.04	415.77	0.2073	1928.56	1.136	428.67	0.2139	1871.33	0.911	428.98	1929.74	1.218
0.05	404.32	0.2015	1982.67	1.332	419.50	0.2092	1913.25	1.083	419.87	1984.39	1.450
0.07	384.91	0.1916	2685.60	1.662	402.97	0.2008	1993.71	1.381	403.44	2088.55	1.850
0.10	366.36	0.1821	2229.08	2.116	381.78	0.1900	2106.98	1.736	382.37	2233.63	2.329
0.15	329.12	0.1633	2442.47	2.443	353.49	0.1757	2279.33	2.163	354.24	2451.31	2.908
0.20	306.28	0.1518	2632.49	2.720	331.29	0.1644	2435.12	2.457	332.15	2645.09	3.308
0.25	290.38	0.1437	2803.93	2.950	313.30	0.1553	2577.56	2.666	314.23	2820.34	3.599

<sup>1</sup>T and T<sub>2</sub> were calculated using Mak's equations and then they were substituted in the exact expressions of this methodology.

<sup>2</sup>The following classical EPQ equations were used to calculate the initial values of T and T<sub>2</sub>:

$$T^* = \sqrt{\frac{2C_3}{C_1 d(1-d/p)}} \cdot \left( \frac{C_1 + C_2}{C_2} \right);$$

$$T_2^* = \sqrt{\frac{2C_3}{C_1 d(1-d/p)}} \cdot \left( \frac{C_2}{C_1 + C_2} \right).$$

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