



Study Of Heat Transfer In An Electrically Conducting Non-Newtonian Fluid Over A Linear Horizontal Stretching Sheet

Naveen Kumar N P
Department of Mathematics
Government First Grade College
18th Cross, Mallewshwaram, Bangalore, India.

Abstract

This paper concerns with a steady two-dimensional flow of an electrically conducting non newtoinan fluid over a heated linear stretching sheet. The flow is permeated by uniform transverse magnetic field the fluid viscosity is assumed to vary as a linear function of temperature. The effects of free convection and internal heat generation or absorption are also considered. Variable fluid properties flow and temperature dependent heat source/sink render the problem intractable. The shooting method is used to solve boundary value problems.

Key Words: stream function, stretching sheet, partial differential equations, shooting method.

Introduction

This paper concerns with a steady two-dimensional flow of an electrically conducting incompressible fluid over a heated stretching sheet. The flow is permeated by uniform transverse magnetic field the fluid viscosity is assumed to vary as a linear function of temperature. The effects of free convection and internal heat generation or absorption are also considered. Variable fluid properties flow and temperature dependent heat source/sink render the problem intractable. The shooting method is used to solve boundary value problems.

Key Words: Magnetic field, Stretching sheet, Shooting method.

Mathematical Formulation of the Problem

The steady two-dimensional flow of an incompressible, electrically conducting, non-Newtonian liquid over a linear and non-linear stretching sheet. The flow is generated by the action of two equal and opposite forces along the x-axis and the sheet is stretched with a velocity that is proportional to the distance from the origin.

Two different types of sheet velocities $u_w(x)$ are considered, namely linear and quadratic velocities,

Further, the sheet is assumed to warmer than the ambient liquid, i.e., $T_w(x) > T_\infty$.

The boundary layer equations governing the flow and heat transfer in a non newtonian liquid over a horizontal stretching sheet are as follows :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \lambda^* \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right), \quad (2)$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \frac{\partial}{\partial y} \left(\frac{k}{\rho C_p} \frac{\partial t}{\partial y} \right) + \frac{q'''}{\rho C_p}, \quad (3)$$

The following boundary conditions are made use :

$$u = u_w = cx, \quad v = v_c, \quad t = t_w = t + A \left(\frac{x}{L} \right)^\lambda \quad \text{at} \quad y = 0, \quad (4)$$

$$u \rightarrow 0, \quad t \rightarrow t_\infty \quad \text{as} \quad y \rightarrow \infty$$

where u and v are the velocity components along x and y directions respectively, t is the temperature of the fluid, t_w is the temperature of the sheet, t_∞ is the temperature of the liquid far away from the sheet, A is the constant, λ is the temperature parameter, L is the characteristic length, ρ is the density of the liquid, σ is the electrical conductivity of the fluid, C_p is the specific heat at constant pressure, k is the thermal conductivity of the liquid which is assumed to vary linearly with temperature and it is of the form,

$k = k_\infty \left[1 + \varepsilon \left(\frac{t - t_\infty}{t_w - t_\infty} \right) \right]$ with ε being a small parameter. The non-uniform heat source/sink q''' is modeled as

$$q''' = \frac{\rho k u_w(x)}{xK} \left[A^* (t_w - t_\infty) f' + (t - t_\infty) B^* \right], \quad (5)$$

where A^* and B^* are the coefficients of space and temperature dependent heat source/sink, respectively.

Here we make a note that the case :

- (i) $A^* > 0$, $B^* > 0$ corresponds to internal heat generation
- (ii) $A^* < 0$, $B^* < 0$ corresponds to internal heat absorption.

We now introduce the following dimensionless variables :

We have adopted the following boundary conditions:

$$X = \frac{x}{L}, \quad Y = \left(\frac{\rho U_0 L}{K} \right)^{\frac{1}{2}} \frac{y}{L}, \quad U = \frac{u}{U_0},$$

$$V = \left(\frac{\rho U_0 L}{K} \right)^{\frac{1}{2}} \frac{v}{U_0}, \quad V_C = \left(\frac{\rho U_0 L}{K} \right)^{\frac{1}{2}} \frac{v_c}{U_0},$$

$$E = \left(\frac{\rho U_0 L}{K} \right)^{\frac{1}{2}} \frac{Le}{2U_0}, \quad D = \frac{L^2 d}{U_0},$$

$$\text{Re}_L = \frac{\rho U_0 L}{K}, \quad T = \frac{t - t_\infty}{t_w - t_\infty}. \quad (6)$$

The boundary layer equation (1) – (3) on using (6) takes the following form :

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \lambda_1 \left(U \frac{\partial^3 U}{\partial X \partial Y^2} + V \frac{\partial^3 U}{\partial Y^3} \right), \quad (8)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} + \frac{UT\lambda}{X} = \frac{1}{\text{Pr}_L} \left\{ \varepsilon \left(\frac{\partial T}{\partial Y} \right)^2 + (1 + \varepsilon T) \frac{\partial^2 T}{\partial Y^2} \right\} + (1 + \varepsilon T)(\alpha f' + \beta T), \quad (9)$$

where $\text{Pr}_L = \frac{\rho C_p U_0 L}{k_\infty \text{Re}_L}$ is the uniform Prandtl number where $\text{Re}_L = \frac{\rho U_0 L}{K}$ is the Reynolds number, $\alpha = \frac{k_\infty A^*}{KC_p}$ is the space-dependent heat source/sink parameter, $\beta = \frac{k_\infty B^*}{KC_p}$ is the temperature-dependent heat source/sink parameter and $\lambda_1 = c\lambda^*$ is the viscoelastic parameter.

The boundary conditions given in (4) takes the form :

$$U = U_w = X, \quad V = V_C, \quad T = 1 \quad \text{at } Y = 0, \\ U \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } Y \rightarrow \infty. \quad (10)$$

Introducing the stream function $\psi(X, Y)$ that satisfies the continuity equation in the dimensionless form (7), we obtain :

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}, \quad (11)$$

using (11) equations in the boundary layer equations (8) and (9) can be written as

$$\frac{\partial \psi}{\partial Y} \frac{\partial^2 \psi}{\partial X \partial Y} - \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial^3 \psi}{\partial Y^3} + \lambda_1 \left(\frac{\partial \psi}{\partial Y} \frac{\partial^4 \psi}{\partial X \partial Y^3} - \frac{\partial \psi}{\partial X} \frac{\partial^4 \psi}{\partial Y^4} \right), \quad (12)$$

$$\frac{\partial \psi}{\partial Y} \frac{\partial T}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial T}{\partial Y} + \frac{\lambda T}{X} \frac{\partial \psi}{\partial Y} = \frac{1}{\text{Pr}_L} \left\{ \varepsilon \left(\frac{\partial T}{\partial Y} \right)^2 + (1 + \varepsilon T) \frac{\partial^2 T}{\partial Y^2} \right\} + (1 + \varepsilon T)(\alpha f' + \beta T), \quad (13)$$

The boundary conditions in (10) can be written in terms of stream function in the following form :

$$\frac{\partial \psi}{\partial Y} = X, \quad -\frac{\partial \psi}{\partial X} = V_c, \quad T = 1 \quad \text{at} \quad Y = 0, \quad (14)$$

$$\frac{\partial \psi}{\partial Y} \rightarrow 0, \quad T \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty.$$

In order to convert the partial differential equations (12) and (1) in to ordinary differential equations the following similarity transformation are adopted :

$$\psi(X, Y) = X f(Y),$$

$$T(X, Y) = \theta(Y), \quad (15)$$

Using the transformation (15) in the boundary layer equations (12) and (13) we have the following boundary value problems :

$$f''' + \lambda_1 (f' f''' - f f''') - (f')^2 + f f'' = 0, \quad (16)$$

$$(1 + \varepsilon \theta) \theta'' + \text{Pr}_x (f \theta' - \lambda \theta f') + \text{Pr}_x \{ (\alpha f' + \beta \theta) (1 + \varepsilon \theta) \} + \varepsilon (\theta')^2 = 0, \quad (17)$$

$$f(0) = -V_c, \quad f'(0) = 1, \quad \theta(0) = 1,$$

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad (18)$$

where $\text{Pr}_x = \frac{\rho C_p u_0 x}{k_\infty \text{Re}_x}$ is the generalized Prandtl number and $\text{Re}_x = \frac{\rho u_0 x}{k}$ is the local Reynolds number,

here the prime denote differentiations with respect to Y .

Method of Solution

The boundary value problems arising due to linear stretching sheet are solved numerically by shooting method. The method is illustrated as given below:

1. Decision on ∞
2. Converting BVP to IVP by choosing suitable initial condition for f & θ
3. The choice of $f''(0)$ & $\theta'(0)$ required for the solution of initial value problem by the classical, explicit Runge-Kutta method of four slopes.

The decision on an appropriate ' ∞ ' for the problem depends on the parameter values chosen. In view of this, for each parameter combination, the appropriate value of ' ∞ ' has to be decided.

Initially we chosen guess values $f''(0) = \alpha_1$ and $\theta'(0) = \beta_1$, the chosen guess values $f''(0) = \alpha_1$ and $\theta'(0) = \beta_1$ are not the most accurate values and hence there is a need to refine. We use the Newton-Raphson method for this purpose. We solve the equations (16) and (17) with initial conditions

$$f(0) = -V_c, \quad f'(0) = 1, \quad f''(0) = \alpha_1,$$

$$\theta(0) = 1, \quad \theta'(0) = \beta_1. \quad (19)$$

Using the classical explicit Runge-Kutta method of four slopes and obtain the solution at ‘ ∞ ’. The solution at ‘ ∞ ’ does not match with those given in the problem due to the crude choice of $f''(0)$ & $\theta'(0)$ and we write the initial value problems as follows

$$\begin{aligned}\frac{dF}{dY} &= F_1, \quad \frac{dF_1}{dY} = F_2, \quad \frac{dF_2}{dY} = F_3, \\ \frac{dF_3}{dY} &= \frac{1}{\lambda_1 F} (F_3 + \lambda_1 F_1 F_3 - (F_1)^2 + F F_2), \\ \frac{dI}{dY} &= I_1, \quad \frac{dI_1}{dY} = \frac{1}{1 + \varepsilon I} \left\{ -\text{Pr}_x (F I_1 - \lambda F_1 I) - \text{Pr}_x (1 + \varepsilon I) (\alpha F_1 + \beta I) - \varepsilon (I_1)^2 \right\}\end{aligned}\quad (20)$$

Subsequently the boundary conditions (19) takes the form :

$$\begin{aligned}F(0) &= -V_c, \quad F_1(0) = 1, \quad F_1(\infty) = 0, \quad F_2(0) = \alpha_1, \\ I(0) &= 1, \quad I_1(0) = \beta_1, \quad I(\infty) = 0,\end{aligned}\quad (21)$$

here $F = f(Y)$ & $I = \theta(Y)$.

The initial value problem (20) is integrated using the fourth order Runge-Kutta method. Newton-Raphson method is implemented to correct the guess values α_1 & β_1 . Appropriate ‘ ∞ ’ value of the solution is determined through the actual computation. It differs for each set of parameter values.

Results and Discussion

The boundary layer flow and heat transfer in an electrically conducting fluid over a stretching sheet with variable thermal conductivity is investigated in the presence of non-uniform heat source/sink. The effect of $\lambda_1, \text{Pr}, \alpha, \beta, \varepsilon$ and λ on flow and heat transfer is shown graphically in Figs. 1- 7.

Fig. 1 shows the effect of viscoelastic parameter λ_1 on velocity profile $f'(Y)$. From this graph it is evident that increasing values of λ_1 results in flattening of $f'(Y)$. It is clear from this plot that λ_1 contributes in producing considerable opposition to the flow. The boundary layer thickness decreases with increasing values of λ_1 .

Fig. 2 shows the effect of viscoelastic parameter λ_1 on temperature distribution. It is noticed that the temperature distribution is unchanged at the wall with change of values of the physical parameters and decays exponentially to zero. The temperature increases with the increasing values of viscoelastic parameter λ_1 . This is due to the fact that viscoelastic normal stress results in thickening of thermal boundary layer.

From Fig. 3, which illustrates the effect of Prandtl number Pr on the heat transfer, it is evident that large values of Prandtl number result in thinning of the thermal boundary layer.

Figs. 4 and 5 illustrate the effect of space-dependent heat source/sink parameter α and temperature-dependent heat source/sink parameter β , respectively on the temperature distribution. Before discussing the results we recollect the fact that $\alpha > 0, \beta > 0$ corresponds to internal heat absorption. The heat generation/absorption clearly depends on the axial flow and also on the boundary layer temperature T . It is the cumulative influence of the space-dependent and temperature-dependent heat source/sink parameter that

determines the extent to which the temperature falls or rises in the boundary layer region. From the plots it is clear that the energy is released for increasing values of α and $\beta > 0$, and this causes the magnitude of temperature to increase, where as energy is absorbed for decreasing values of $\alpha, \beta < 0$ resulting in temperature dropping significantly near the boundary layer. It is observed that the direction of the heat transfer is reversed for some negative values of α .

The effect of variable thermal conductivity parameter ε on temperature profiles is shown in Fig. 6. It is observed from this figure that increasing the values of ε results in increasing the magnitude of temperature causing thermal boundary layer thickening.

The effect of λ on the heat transfer is shown in Fig. 7 it is observed that, above some critical negative value λ_c , the increasing effect of λ is to decrease the magnitude of the temperature. There will be transfer of heat from sheet to the liquid for $\lambda > \lambda_c$. Below this critical value the effect of λ is opposite, i.e., if $\lambda < \lambda_c$ and the heat flow is from the liquid to sheet itself. This case is of least interest because the present investigation concerns cooling the sheet. Hence in Fig. 7 the values of λ are chosen above the critical value λ_c . When $\lambda = \lambda_c$, there is no heat transfer between the stretching surface and the ambient liquid.

Conclusion

- The viscoelastic normal stress results in thickening of thermal boundary layer.
- The individual effects of increasing λ_1 , α , β are to increase the magnitude of heat transfer. The opposite effect is observed for increasing values of Prandtl number Pr and temperature parameter λ .
- The variable thermal conductivity parameter ε increases the magnitude of temperature.
- The magnitude of λ dictates the direction of heat transfer.
- The effect of flow and temperature dependent heat source/sink parameters is to generate temperature for increasing positive values and absorb temperature for decreasing negative values. Hence flow and temperature-dependent heat sinks are better suited for cooling purposes.

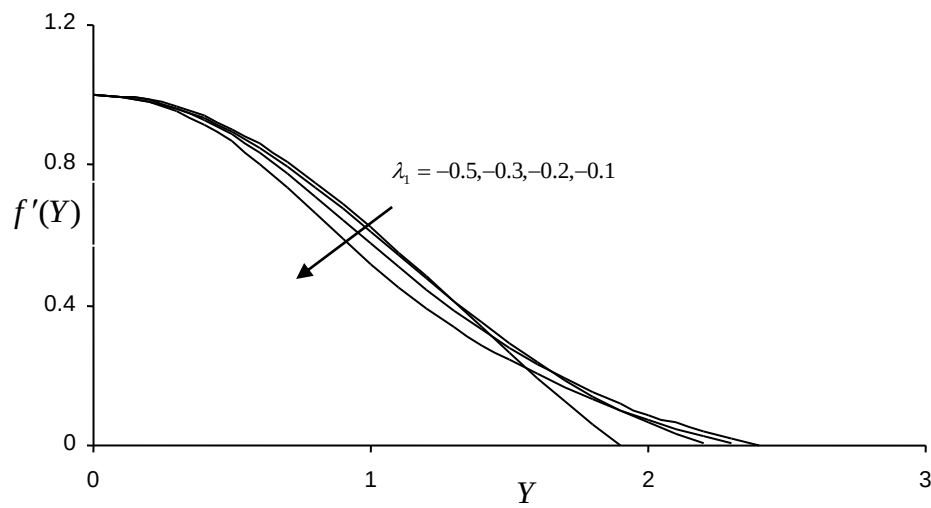


Fig 1: Effect of λ_1 on velocity profile $f'(Y)$ in case of linear stretching sheet problem.

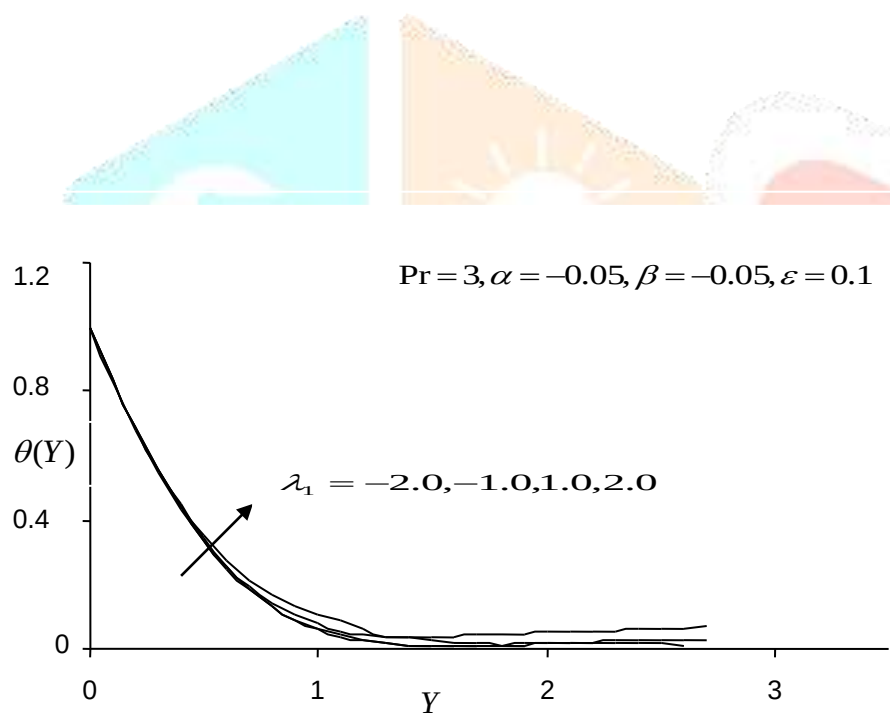


Fig. 2: Effect of λ_1 on temperature profile $\theta(Y)$ in case of linear stretching sheet problem.

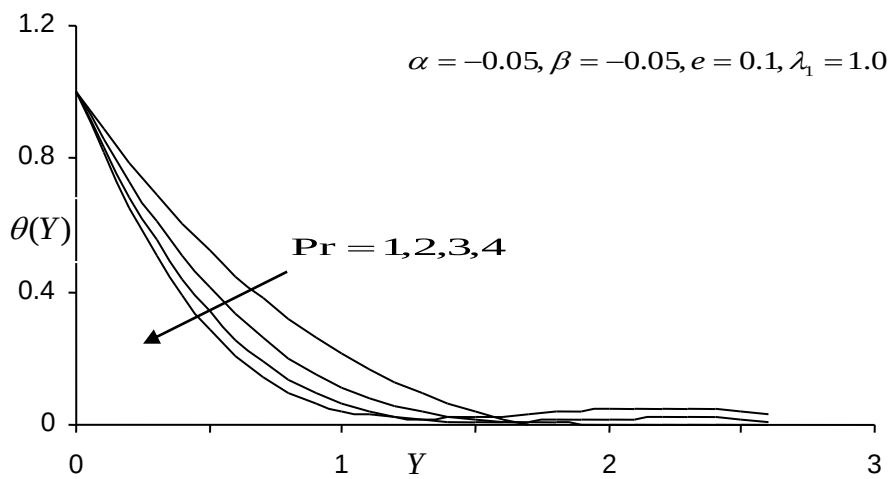


Fig. 3: Effect of Pr on temperature profile $\theta(Y)$ in case of linear stretching sheet problem.

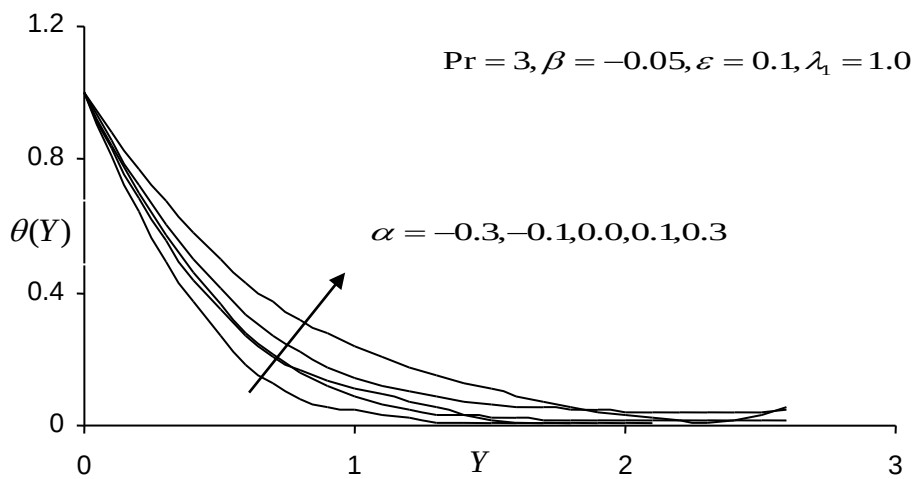


Fig. 4: Effect of α on temperature profile $\theta(Y)$ in case of linear stretching sheet problem.

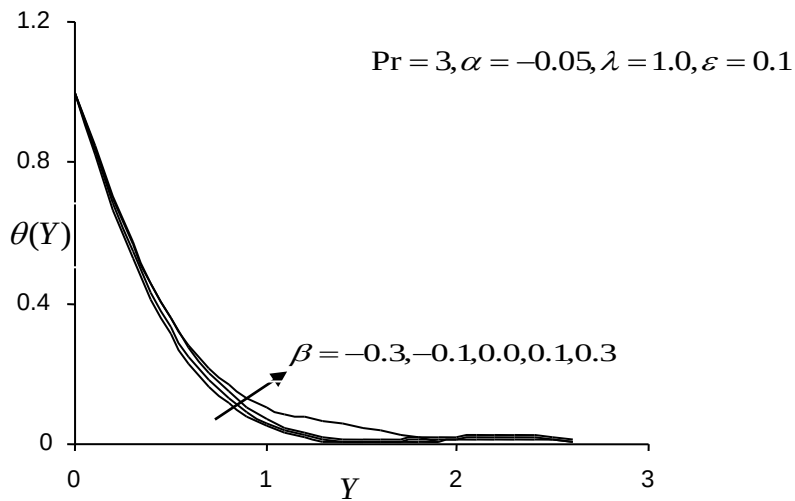


Fig. 5: Effect of β on temperature profile $\theta(Y)$ in case of linear stretching sheet problem.

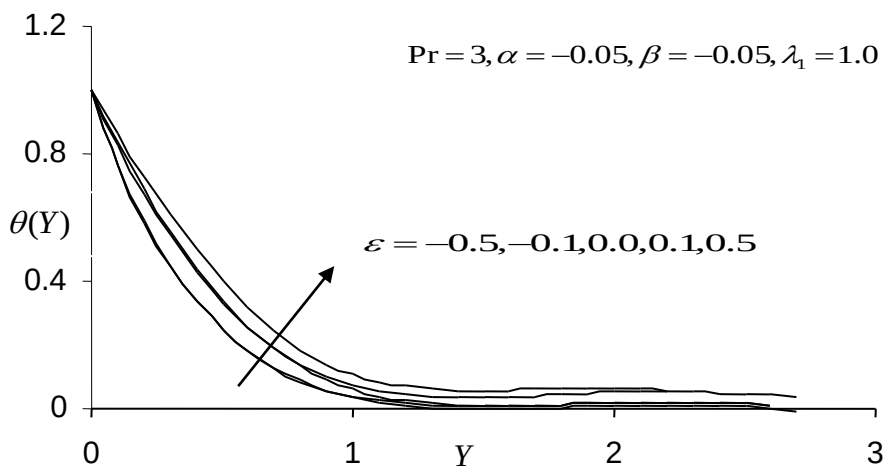


Fig. 6: Effect of ε on temperature profile $\theta(Y)$ in case of linear stretching sheet problem

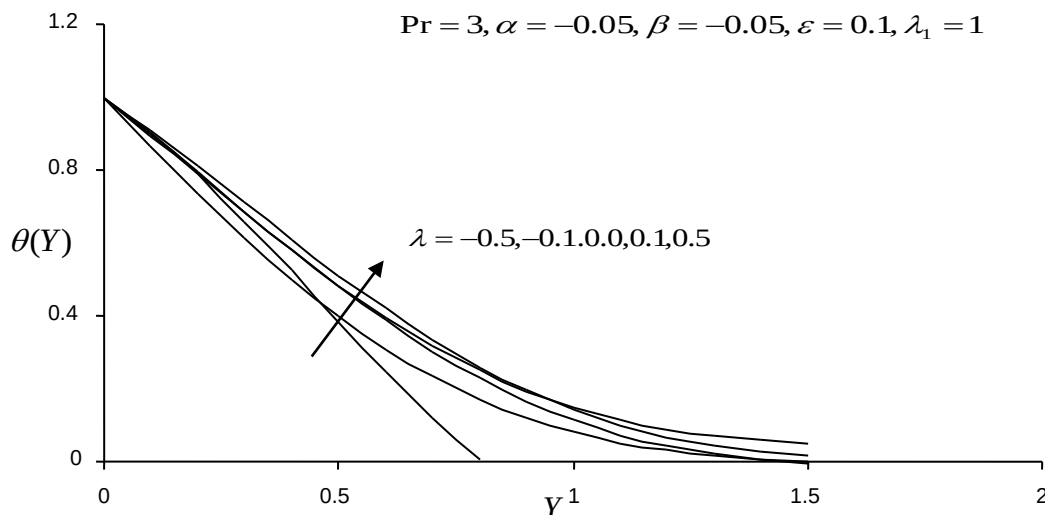


Fig. 7: Effect of λ on temperature profile $\theta(Y)$ in case of linear stretching sheet problem.

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