IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

FIXED POINT FOR TWO PAIRS OF WEAKLY COMPATIBLE MAPPINGS IN B–METRIC SPACE

Varsha D. Borgaonkar[1], K. L. Bondar [2], S. M. Jogdand [3]

[1]P.G. Department of Mathematics, N.E.S. Science College, Nanded, India - 431 602

[2]P.G. Department of Mathematics, G. V. Institute of Science and Humanities, Amravati, India - 444 604

[3]Department of Mathematics, Shri Shivaji College of Arts, Commerce and Sci., Kandhar, Nanded, India - 431 714

1

Abstract: The aim of this paper is to discuss weakly compatible mappings and to obtain the fixed point theorems for two pairs of weakly compatible mappings in b-metric space. The b-metric need not be a continuous function. Here in this paper, the b-metric under consideration is a continuous function.

Keywords. b–Metric space, Compatible mapping, Weakly compatible mappings, Fixed Point, coincidence point , Cauchy's Sequence.

AMS Classification: 47H10, 54H25.

1. INTRODUCTION.

The Banach fixed Point theorem is very popular and useful theorem in Mathematics as well as in other subjects. In 1989, Bakhtin [1] introduced the concept of generalized b–metric spaces. Boriceanu [2], Mehmat Kir [3] extended the fixed point theorem in b–metric space. Borkar [4] obtained the common fixed point theorem for non expansive type mapping. Czerwik [5-6] presented the generalization of Banach fixed point theorem in b-metric spaces. Using this idea many researchers presented a generalization of the renowned Banach fixed point theorem in b-metric space. Agrawal [7] presented the existence and uniqueness theorem in b-–Metric Space. Chopade [8] given common fixed point theorems for contractive type mapping in metric space. Borgaonkar V.D. and K.L. Bondar [9-10] has obtained the fixed point theorems in b -metric spaces. Roshan [11] obtained common fixed point of four maps in b–Metric space. Suzuki [12] obtained some basic inequalities and it's applications in a b–Metric space.

To generalize the fixed point theorems, the concept of commuting mappings is proven to be very useful. Jungck [13] has generalized the Banach's Contraction Principle and proved a fixed point theorem for commuting mappings. Later, many researchers have obtained the fixed point theorems for commuting mappings. Sessa [14] has introduced the notion of weakly commuting mapping and obtained the fixed point results for such mappings. Later Jungck [15] generalized the concept of commuting mappings and he introduced the notion of compatible mappings. After that many fixed point results are obtained for compatible mappings. Jungck et.al. [16] defined the compatible mappings of type (A). In (1998), Jungck and Rhodes [17] have given the concept of weakly compatible mappings. Many researchers have obtained the fixed point theorems for weakly compatible mappings assuming the continuity of at least one mapping in various metric spaces.

In this paper, we will obtain the fixed point theorem for two pairs of weakly compatible mappings in b-metric space for a continuous b-metric.

2. SOME BASIC DEFINITIONS AND PRELIMINARIES

Definition 2.1: Let X be a non-empty set. A function $\delta: X \times X \longrightarrow \mathbb{R}$ is called as a metric provided that for all $u, v, w \in X$ we have,

(i) $\delta(u, v) \geq 0$
(ii) $\delta(u, v) = 0$ if and only if $u = v$
(iii) $\delta(u, v) = \delta(v, u)$
(iv) $\delta(u, v) \leq \delta(u, w) + \delta(w, v)$

A pair (X, δ) is called as metric space.

Definition 2.2: By Czerwik [5], Let X be a non-empty set and $s \geq 1$ be a given real number. A function $\delta: X \times X \longrightarrow \mathbb{R}$ is called as a b- metric provided that for all $u, v, w \in X$ we have,

(i) $\delta(u, v) \geq 0$
(ii) $\delta(u, v) = 0$ if and only if $u = v$
(iii) $\delta(u, v) = \delta(v, u)$
(iv) $\delta(u, v) \leq s\{\delta(u, w) + \delta(w, v)\}$

A pair (X, δ) is called as b- metric space. b-metric space is an extension of the usual metric space.

Remark 2.1: If $s = 1$, then the b-metric space is a usual metric space.

Example 2.1: If $X = \mathbb{R}$, be the set of all real numbers and $d(u, v) = |u - v| \ \forall u, v \in \mathbb{R}$ be a usual metric defined on $\mathbb{R}$, then $\delta(u, v) = (u - v)^2$ is a b-metric on $\mathbb{R}$ with $s = 2$.

Example 2.2: If (X, d) be a metric space and $\delta(u, v) = d(u, v)^k \ \forall u, v \in X$, where $k > 1$ is a real number. Clearly (X, δ) is a b- metric space with $s = 2^{k-1}$.

Example 2.3: By Boriceanu [2], Let $A = \{0, 1, 2\}$ and $\delta: A \times A \rightarrow \mathbb{R}$ is defined as,

i) $\delta(0, 2) = \delta(2, 0) = m \geq 2$
ii) $\delta(0, 1) = \delta(1, 0) = \delta(2, 1) = \delta(1, 2) = 1$
iii) $\delta(0, 0) = \delta(1, 1) = \delta(2, 2) = 0$

Here, $\delta(a, b)$ is a metric on A with $s = \frac{m}{2}$.

Definition2.3: By Boriceanu [2], Let (X, δ) be a b-metric space then a sequence $\{u_n\}$ in X is said to be a convergent sequence if there exist $u \in X$ such that for all $\epsilon > 0$ there exists $n(\epsilon) \in N$ such that for $n \geq n(\epsilon)$ we have $\delta(u_n, u) < \epsilon$. In this case we write $\lim_{n \to \infty} u_n = u$

Definition2.4: By Boriceanu [2], Let (X, δ) be a b-metric space then a sequence $\{u_n\}$ in X is called as Cauchy sequence if for all $\epsilon > 0$ there exists $n(\epsilon) \in N$ such that for $m, n \geq n(\epsilon)$ we have $\delta(u_n, u_m) < \epsilon$.

Definition2.4: By Boriceanu [2], Let (X, δ) be a b-metric space then X is said to be complete if every Cauchy sequence in X is a convergent sequence in X.

Definition 2.5: Let X be any nonempty set then two self mappings $P: X \rightarrow X$ and $Q: X \rightarrow X$ are said to be weakly compatible if, $PQ(x) = QP(x)$ whenever, $Px = Qx$ for some $x \in X$.

Example 2.3: Let $X = R$. Define $P, Q: X \rightarrow X$ as $P(x) = x^2 + x - 4$ and $Q(x) = x$. Here, $P(2) = 2$ and $Q(2) = 2$, Hence, $QP(2) = 2$ and $PQ(2) = 2$.
$\therefore$ P and Q are weakly compatible.

Example 2.4: Let $X = R$. Define $P, Q: X \rightarrow X$ as $P(x) = x^2 - x - 1$ and $Q(x) = -x$. Here, $P(1) = -1$ and $Q(1) = -1$ and $QP(1) = -1$ Hence, $PQ(1) = -1$.
$\therefore$ P and Q are weakly compatible.

Remark 2.2: The b-metric under consideration in this chapter is a continuous function.

Lemma 2.1: Let (X, δ) be a complete b-metric space with $s \geq 1$, and let $\{x_n\}$ be a sequence in X. Assume that there exist $r \in [0,1)$ satisfying,

$$\delta(x_{n+1}, x_{n+2}) \leq r\delta(x_n, x_{n+1}) \quad \text{for any } n \in N.$$

Then $\{x_n\}$ is Cauchy sequence in X.

3. MAIN RESULT.

Theorem 3.1: Let P, Q, R and S be self-mappings of a complete b-metric space (X, d) with $s \geq 1$ such that,

(3.1) $P(X) \subset S(X)$ and $Q(X) \subset R(X)$.
(3.2)
$$\begin{aligned}[1 + \alpha d(Rx, Sy)]d(Px, Qy) \leq\ & \alpha \max\{d(Qy, Sy). d(Px, Rx), d(Px, Sy). d(Rx, Qy)\}\\ &+ \beta \max\{d(Rx, Sy), d(Sy, Qy)\}\\ &+ \frac{\gamma}{2}[d(Px, Sy) + d(Px, Rx)],\end{aligned}$$
holds for all $x, y \in X$,

where, $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ satisfying, $\beta + 2\gamma s < \frac{1}{2}$.
(3.3) The pairs (P, R) and (Q, S) are weakly compatible.
(3.4.) One of the P, Q, R and S is continuous.
Then prove that P, Q, R and S have a unique common fixed point in X.

Proof: Existence of Fixed Point:

Let $x_0 \in X$ be an arbitrary point. For $x_0 \in X$, choose a point $x_1 \in X$, such that, $Px_0 = Sx_1$. Also, for $x_1 \in X$, choose a point $x_2 \in X$, such that, $Qx_1 = Rx_2$. By continuing this process, we define a sequence $\{y_n\}$ in X as,

$$y_{2n} = Px_{2n} = Sx_{2n+1} \quad \text{and} \quad y_{2n+1} = Qx_{2n+1} = Rx_{2n+2}, \ \forall \ n = 0,1,2, \dots.$$

Firstly, we will prove that, $\{y_n\}$ is a Cauchy sequence in X.
Put $x = x_{2n+2}$ and $y = x_{2n+1}$ in condition (3.2) then we get,

$$\begin{aligned}[1 + \alpha\ d(Rx_{2n+2}, Sx_{2n+1})]d(Px_{2n+2}, Qx_{2n+1}) \leq\ & \alpha \max\{d(Qx_{2n+1}, Sx_{2n+1}). d(Px_{2n+2}, Rx_{2n+2}),\\ & d(Px_{2n+2}, Sx_{2n+1}). d(Rx_{2n+2}, Qx_{2n+1})\}\\ &+\beta \max\{d(Rx_{2n+2}, Sx_{2n+1}), d(Sx_{2n+1}, Qx_{2n+1})\}\\ &+\frac{\gamma}{2}[d(Px_{2n+2}, Sx_{2n+1}) + d(Px_{2n+2}, Rx_{2n+2})]\end{aligned}$$

$$\begin{aligned}[1 + \alpha\ d(y_{2n+1}, y_{2n})]d(y_{2n+2}, y_{2n+1}) \leq\ & \alpha \max\{d(y_{2n+1}, y_{2n}). d(y_{2n+2}, y_{2n+1}),\\ & d(y_{2n+2}, y_{2n}). d(y_{2n+1}, y_{2n+1})\}\\ &+\beta \max\{d(y_{2n+1}, y_{2n}), d(y_{2n}, y_{2n+1})\}\\ &+\frac{\gamma}{2}[d(y_{2n+2}, y_{2n}) + d(y_{2n+2}, y_{2n+1})]\end{aligned}$$

$$d(y_{2n+1},y_{2n+2})+\alpha\ d(y_{2n},y_{2n+1}).d(y_{2n+1},y_{2n+2}) \le \alpha\ \max\{d(y_{2n},y_{2n+1}).d(y_{2n+1},y_{2n+2}), d(y_{2n},y_{2n+2}).0\}+\beta d(y_{2n},y_{2n+1}) +\frac{\gamma s}{2}d(y_{2n},y_{2n+1})+\frac{\gamma s}{2}d(y_{2n+1},y_{2n+2}) +\frac{\gamma}{2}d(y_{2n+1},y_{2n+2})$$

$$d(y_{2n+1},y_{2n+2})+\alpha\ d(y_{2n},y_{2n+1}).d(y_{2n+1},y_{2n+2}) \le \alpha d(y_{2n},y_{2n+1}).d(y_{2n+1},y_{2n+2}) +\beta d(y_{2n},y_{2n+1})+\frac{\gamma s}{2}d(y_{2n},y_{2n+1}) +\frac{\gamma s+\gamma}{2}d(y_{2n+1},y_{2n+2})$$

$$\left[1-\left(\frac{\gamma s+\gamma}{2}\right)\right]d(y_{2n+1},y_{2n+2})\le\left(\beta+\frac{\gamma s}{2}\right)d(y_{2n},y_{2n+1})$$

$$\therefore d(y_{2n+1},y_{2n+2})\le\left[\frac{\beta+\frac{\gamma s}{2}}{1-\left(\frac{\gamma s+\gamma}{2}\right)}\right]d(y_{2n},y_{2n+1})$$

$$\therefore d(y_{2n+1},y_{2n+2})\le rd(y_{2n},y_{2n+1})$$

where $r=\left[\frac{\beta+\frac{\gamma s}{2}}{1-\left(\frac{\gamma s+\gamma}{2}\right)}\right]<1\ \because \beta+2\gamma s<1$

$\therefore$ In general, we have, for any $n\in N$,

$$\therefore d(y_{n+1},y_{n+2})\le rd(y_n,y_{n+1})$$

Therefore, by Lemma (2.1), $\{y_n\}$ is a Cauchy sequence in X, hence it converges to a point y_0 in X.

$$\therefore \lim_{n\to\infty} y_n=y_0$$

$$\therefore \lim_{n\to\infty} y_{2n}=\lim_{n\to\infty} y_{2n+1}=y_0$$

$\therefore$ The subsequences Px_{2n}, Sx_{2n+1}, Qx_{2n+1} and Rx_{2n} converges to y_0.
Now, suppose P is continuous. Therefore, the sequences P^2x_{2n} and PRx_{2n} converges to Py_0. As the mappings P and R are weakly compatible therefore by proposition 2.8. in [19], We have,

$$\lim_{n\to\infty} RRx_{2n}=Py_0.$$

Now, put $x=Rx_{2n}$ and $y=x_{2n+1}$ in condition (3.2) then we get,

$$[1+\alpha\ d(RRx_{2n},Sx_{2n+1})]d(PRx_{2n},Qx_{2n+1}) \le \alpha\ \max\{d(Qx_{2n+1},Sx_{2n+1}).d(PRx_{2n},RRx_{2n}), d(PRx_{2n},Sx_{2n+1}).d(RRx_{2n},Qx_{2n+1})\} +\beta\ \max\{d(RRx_{2n},Sx_{2n+1}),d(Sx_{2n+1},Qx_{2n+1})\} +\frac{\gamma}{2}[d(PRx_{2n},Sx_{2n+1})+d(PRx_{2n},RRx_{2n})].$$

Taking lim as $n\to\infty$, on both sides then we get,

$$[1+\alpha\ d(Py_0,y_0)]d(Py_0,y_0) \le \alpha\ \max\{d(y_0,y_0).d(Py_0,Py_0),d(Py_0,y_0).d(Py_0,y_0)\} +\beta\ \max\{d(Py_0,y_0),d(y_0,y_0)\} +\frac{\gamma}{2}[d(Py_0,y_0)+d(Py_0,Py_0)]$$

$$\therefore d(Py_0,y_0)+\alpha\ d(Py_0,y_0).d(Py_0,y_0) \le \alpha\ d(Py_0,y_0).d(Py_0,y_0) +\beta\ d(Py_0,y_0)+\frac{\gamma}{2}d(Py_0,y_0)$$

$$\therefore d(Py_0, y_0) \leq \beta\ d(Py_0, y_0) + \frac{\gamma}{2} d(Py_0, y_0)$$

$$\therefore \left(1 - \beta - \frac{\gamma}{2}\right) d(Py_0, y_0) \leq 0$$

$$\therefore \left(1 - \left(\beta + \frac{\gamma}{2}\right)\right) d(Py_0, y_0) \leq 0$$

as $\beta + \frac{\gamma}{2} < \beta + 2\gamma s < 1$ therefore $1 - \left(\beta + \frac{\gamma}{2}\right) > 0$

$$0 \leq d(Py_0, y_0) \leq 0$$

$$\Rightarrow \mathrm{d}(Py_0, y_0) = 0$$

$$\Rightarrow Py_0 = y_0$$

$\therefore y_0$ is fixed point of mapping P.
We have $P(X) \subset S(X)$, therefore, there exists a point $y_1 \in X$, such that $y_0 = Sy_1$.
Now, put $x = Rx_{2n}$ and $y = y_1$ in condition (3.2), then we get,

$$\begin{aligned}[1 + \alpha\ d(RRx_{2n}, Sy_1)]d(PRx_{2n}, Qy_1) &\leq \alpha \max\{d(Qy_1, Sy_1).d(PRx_{2n}, RRx_{2n}),\\ &\quad d(PRx_{2n}, Sy_1).d(RRx_{2n}, Qy_1)\}\\ &\quad + \beta \max\{d(RRx_{2n}, Sy_1), d(Sy_1, Qy_1)\}\\ &\quad + \frac{\gamma}{2}[d(PRx_{2n}, Sy_1) + d(PRx_{2n}, RRx_{2n})].\end{aligned}$$

Taking lim as $n \to \infty$ on both sides then we get,

$$\begin{aligned}[1 + \alpha\ d(Py_0, y_0)]d(Py_0, Qy_1) &\leq \alpha \max\{d(Qy_1, y_0).d(Py_0, Py_0),\\ &\quad d(Py_0, y_0).d(Py_0, Qy_1)\}\\ &\quad + \beta \max\{d(Py_0, y_0), d(y_0, Qy_1)\}\\ &\quad + \frac{\gamma}{2}[d(Py_0, y_0) + d(Py_0, Py_0)].\end{aligned}$$

But, $Py_0 = y_0$ and $Sy_1 = y_0$ then we get,

$$\begin{aligned}\therefore [1 + \alpha\ d(y_0, y_0)]d(y_0, Qy_1) &\leq \alpha \max\{d(Qy_1, y_0).d(y_0, y_0), d(y_0, y_0).d(y_0, Qy_1)\}\\ &\quad + \beta \max\{d(y_0, y_0), d(y_0, Qy_1)\}\\ &\quad + \frac{\gamma}{2} d(y_0, y_0) + \frac{\gamma}{2} d(y_0, y_0)\end{aligned}$$

$$\therefore d(y_0, Qy_1) \leq \beta\ d(y_0, Qy_1)$$

$$\therefore 0 \leq (1 - \beta) d(y_0, Qy_1) \leq 0$$

$$\Rightarrow d(y_0, Qy_1) = 0 \qquad \because \quad \beta < 1$$

$$\therefore Qy_1 = y_0$$

$$\therefore Qy_1 = Sy_1 = y_0.$$

But, Q and S are weakly compatible, therefore $QSy_1 = SQy_1$

$$\therefore Qy_0 = Sy_0.$$

Put, $x = x_{2n}$ and $y = y_0$ in condition (3.2) then we get,

$$\begin{aligned}[1+\alpha\ d(Rx_{2n},Sy_0)]d(Px_{2n},Qy_0) \leq\ & \alpha\ \max\{d(Qy_0,Sy_0).d(Px_{2n},Rx_{2n}),\\ & d(Px_{2n},Sy_0).d(Rx_{2n},Qy_0)\}\\ & +\beta\ \max\{d(Rx_{2n},Sy_0),d(Sy_0,Qy_0)\}\\ & +\frac{\gamma}{2}d(Px_{2n},Sy_0)+\frac{\gamma}{2}d(Px_{2n},Rx_{2n}).\end{aligned}$$

Taking lim as $n\to\infty$ on both sides then we get,

$$\begin{aligned}[1+\alpha\ d(y_0,Sy_0)]d(y_0,Qy_0) \leq\ & \alpha\ \max\{d(Qy_0,Sy_0).d(y_0,y_0),d(y_0,Sy_0).d(y_0,Qy_0)\}\\ & +\beta\ \max\{d(y_0,Sy_0),d(Sy_0,Qy_0)\}\\ & +\frac{\gamma}{2}d(y_0,Sy_0)+\frac{\gamma}{2}d(y_0,y_0)\end{aligned}$$

$$\begin{aligned}d(y_0,Qy_0)+\alpha\ d(y_0,Sy_0).d(y_0,Qy_0) \leq\ & \alpha\ d(y_0,Sy_0).d(y_0,Qy_0)\\ & +\beta\ d(y_0,Sy_0)+\frac{\gamma}{2}d(y_0,Sy_0)\end{aligned}$$

$$\therefore d(y_0,Qy_0)\leq\left(\beta+\frac{\gamma}{2}\right)d(y_0,Sy_0)$$

$$\therefore d(y_0,Sy_0)\leq\left(\beta+\frac{\gamma}{2}\right)d(y_0,Sy_0)\qquad \because \text{ by } (5.3.1)$$

$$\therefore\left(1-\left(\beta+\frac{\gamma}{2}\right)\right)d(y_0,Sy_0)\leq 0$$

$$\therefore 0\leq d(y_0,Sy_0)\leq 0\qquad \because\quad \beta+\frac{\gamma}{2}<\beta+2\gamma s<1$$

$$\therefore d(y_0,Sy_0)=0$$

$$\therefore Sy_0=y_0.$$

Therefore, by (3.1) we have, $Qy_0=y_0$.
Hence y_0 is fixed point of both S and Q.
Now, $Q(X)\subset R(X)$ therefore there exist a point $y_2\in X$, such that $Ry_2=y_0$.
Put $x=y_2$ and $y=x_{2n+1}$ in condition (3.2.) then we get,

$$\begin{aligned}[1+\alpha\ d(Ry_2,Sx_{2n+1})]d(Py_2,Qx_{2n+1}) \leq\ & \alpha\ \max\{d(Qx_{2n+1},Sx_{2n+1}).d(Py_2,Ry_2),\\ & d(Py_2,Sx_{2n+1}).d(Ry_2,Qx_{2n+1})\}\\ & +\beta\ \max\{d(Ry_2,Sx_{2n+1}),d(Sx_{2n+1},Qx_{2n+1})\}\\ & +\frac{\gamma}{2}[d(Py_2,Sx_{2n+1})+d(Py_2,Ry_2)].\end{aligned}$$

Taking lim as $n\to\infty$ on both sides, then we get,

$$\begin{aligned}[1+\alpha\ d(y_0,y_0)]d(Py_2,y_0) \leq\ & \alpha\ \max\{d(y_0,y_0).d(Py_2,y_0),d(Py_2,y_0).d(y_0,y_0)\}\\ & +\beta\ \max\{d(y_0,y_0),d(y_0,y_0)\}\\ & +\frac{\gamma}{2}[d(Py_2,y_0)+d(Py_2,y_0)]\end{aligned}$$

$$d(Py_2,y_0)\leq\gamma d(Py_2,y_0)$$

$$(1-\gamma)d(Py_2,y_0)\leq 0$$

but $(1-\gamma)\neq 0$, therefore we get,

$$0\leq d(Py_2,y_0)\leq 0$$

$$Py_2=y_0$$

$$\therefore Py_2 = Ry_2 = y_0.$$

The mappings P and R are weakly compatible mappings.

$$\therefore Py_2 = Ry_2$$

$$\Rightarrow P\mathrm{R}y_2 = RPy_2$$

$$\Rightarrow Py_0 = Ry_0$$

$$\Rightarrow y_0 = Ry_0 \quad \because Py_0 = y_0$$

$\therefore y_0$ is fixed point of mapping R.
Hence, the mappings P, Q, R and S have a common fixed point y_0.

Uniqueness of Fixed Point:
Suppose y'_0 be another common fixed point of P, Q, R and S. We have,

$$Py'_0 = Qy'_0 = Ry'_0 = Sy'_0 = y'_0.$$

Put $x = y_0$ and $y = y'_0$ in condition (3.2) then we get

$$\begin{aligned}[1 + \alpha\ d(Ry_0, Sy'_0)]d(Py_0, Qy'_0) \quad &\leq \alpha\ \max\{d(Qy'_0, Sy'_0).d(Py_0, Ry_0),\\ &\quad d(Py_0, Sy'_0).d(Ry_0, Qy'_0)\}\\ &+\beta\ \max\{d(Ry_0, Sy'_0), d(Sy'_0, \mathrm{Q}y'_0)\}\\ &+\frac{\gamma}{2}[d(Py_0, Sy'_0) + d(Py_0, Ry_0)]\end{aligned}$$

$$\begin{aligned}[1 + \alpha\ d(y_0, y'_0)]d(y_0, y'_0) \quad &\leq \alpha\ \max\{d(y'_0, y'_0).d(y_0, y_0), d(y_0, y'_0).d(y_0, y'_0)\}\\ &+\beta\ \max\{d(y_0, y'_0), d(y'_0, y'_0)\}\\ &+\frac{\gamma}{2}[d(y_0, y'_0) + d(y_0, y_0)]\end{aligned}$$

$$d(y_0, y'_0) + \alpha\ d(y_0, y'_0).d(y_0, y'_0) \leq \alpha\ d(y_0, y'_0).d(y_0, y'_0) + \beta\ d(y_0, y'_0) + \frac{\gamma}{2}d(y_0, y'_0)$$

$$\therefore \left[1 - \left(\beta + \frac{\gamma}{2}\right)\right] d(y_0, y'_0) \leq 0$$

$$\therefore 0 \leq d(y_0, y'_0) \leq 0 \quad \because \beta + \frac{\gamma}{2} < 1$$

$$\therefore d(y_0, y'_0) = 0$$

$$\therefore y_0 = y'_0$$

Hence, the mappings P, Q, R and S have a unique common fixed point y_0 in X.

Corollary 3.1: Let P, Q, R and S be the self maps of a complete b–metric space (X, d) with $s \geq 1$ satisfying condition (3.1) and

$$d(Px, Qy) \leq \beta\ \max\ \{d(Rx, Sy), d(Sy, Qy)\} + \frac{\gamma}{2}[d(Px, Sy) + d(Px, Rx)]$$

where, $0 < \beta < 1$ and $0 < \gamma < 1$ are such that $\beta + 2\gamma s < 1$. Suppose one of the mappings P, Q, R and S is continuous. If the pairs (P, R) and (Q, S) are weakly compatible then P, Q, R and S have unique common fixed point in X.

4. DISCUSSION AND THE CONCLUDING REMARKS

In this paper, we have proved the existence and uniqueness of common fixed points for two pairs of weakly compatible mappings in b–metric space.

5. REFERENCES

1. Bakhtin I. A., The Contraction Mapping Principle in almost Metric Spaces, Funct. Anal. Unisco, Gauss. Ped. Inst., vol. **30,** pp.26-37, (1989).
2. Boriceanu M., Fixed Point theory for multivalued generalized contraction on a set with two b-Metric, studia, univ Babes, Bolya: Math, vol. Liv **3**, pp.1-14, (2009).
3. Mehmat Kir Kiziltune, Hukmi., On Some Wellknown Fixed Point Theorems in b–Metric Space, Turkshi Journal of Analysis and Number Theory, vol. **1**, pp. 13-16, (2013).
4. Borkar V. C., et. Al., Common Fixed Point for nonaxpansive type mappings with application , Acta Cinecia India , vol. **4**, pp.674-682, (2010).
5. Czerwik S., Contraction Mappings In b–Metric Spaces, Acta, Mathematica ,et. Informatica Universities Ostraviensis, vol. **1**, pp.5-11, (1993).
6. Czerwik S., Non-linear Set Valued Contraction Mappings in b–Metric Spaces. Atti sem Maths, FIQ Univ. Modena., vol. **46**, pp.263-276.(1998).
7. Agrawal Swati , K. Qureshi and Jyoti Nema., A fixed Point Theorem for b-Metric Space, International Journal of Pure and Applied Mathematical Sciences, vol. **9**, pp.45-50, (2016).
8. Chopade P. U., et. Al., Common Fixed Point Theorem for Some New Generalized Contractive mappings, Int. Journal of Math. Analisis, vol. **4**, pp.1881-1890, (2010).
9. Borgaonkar V. D. and Dr. K. L. Bondar, Common Fixed Point theorem for Two Mappings in b-Metric Space, The Mathematics Student, vol. **90 (3-4)**, pp. 19-27, (2021).
10. Borgaonkar V. D. and Dr. K. L. Bondar, Existence and Uniqueness of Fixed Point for a Mapping in b-Metric Space, International Journal of All Research Education and Scientific Methods, vol. **10 (9)**, pp. 285-291, (2022).
11. Roshan J. R., et. Al., Common Fixed Point of Four Maps in b–Metric Spaces, Hacettep Journal of Mathematics and Statistics, vol. **43**, pp.613-624, (2014).
12. Suzuki Tomonari, Basic Inequality on a b-Metric Space and Its Applications, Journal of Inequalities and Applications, pp.2-11, (2017).
13. Jungck G., Commuting Maps and Fixed Points, Am. Math. Mon., vol. 83(4), pp.261-263, (1976).
14. Sessa S., On a weak commutativity condition of mappings in fixed point considerations, publ. Inst. math, vol. **32(46)**, pp.149-153, (1982).
15. Jungck G., Compatible mappings and common fixed points, International Journal of math. Math. Sci., vol. **9(4)**, pp.771-779, (1986).
16. G. Jungck, P. P. Murthy, Y. J. cho., Compatible Mappings of Type(A) and Common Fixed Point Theorems, Mathematica Japonica, vol. **38(2)**, pp. 381–390, (1993).
17. G. Jungck, Billy Rhoades., Fixed Point for Set-Valued Function Without Continuity, Indian Journal of Pure and Applied Mathematics, vol. **29(3)**, pp.227–238, (1998).

18. Borgaonkar V. D. et. al. Common Fixed Point theorem for Two Mappings in bi- b-Metric Space, Advances in Mathematics: Scientific Journal, **11(1)**, 25–34, (2022).

19. R. P. Pant., Common Fixed Points of Sequence of Mappings, Ganita, vol. **47(2)**, pp.43–49, (1996).

20. V. Popa, H. K. Pathak., Common Fixed Point Theorems for Compatible Mappings and Compatible Mappings of type (A), Math. Bech., vol. **49**, pp. 109–114, (1997).