



Mathematical Model for the Analysis of carbon Dioxide Emission in Jharkhand.

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ABSTRACT

Global warming is most burning environmental curses ever to face with human society during last few decades. There has been a serious change in climate of the world because of this warming that has become a nightmare to scientists, researcher and academicians throughout the globe as this change leads to a fatal situation for the existence of life on this planet. Increased emission of different green house gases (GHG) is responsible for this alarming situation. This study is an attempt to develop a model which reflect the Green House Gases emission pattern in the Indian states of Jharkhand. Carbon dioxide (CO₂) is the leading green house gases which is responsible for Global Warming. Uncontrolled emission of the gas increases atmospheric temperature rapidly. Using historical data of carbon dioxide emission, a third-degree polynomial model has been developed analytically utilizing the method of least square. Quality of the proposed model is justified by the retrofitting process like regression analysis, residual analysis, coefficient of determination (R²) and adjusted R² etc. The developed model well represents the gas emission and future prediction can be made through the Instantaneous Rate of Change (IROC) curve. The prediction in long/short term evolutionary trend of the gas emission analysed by IROC analysis. Proposed model gives good estimates and may be utilized for understanding the emission pattern of other green GHGs. This study is quite helpful for strategic warning and future planning so that the precautions can be taken in near future.

Keywords: Green House Gas, Method of least square, Global Warming, Instantaneous Rate of Change (IROC), Residual analysis.

INTRODUCTION

The impact of CO₂ emissions in Jharkhand, like in any other region, is significant and multi-faceted. Jharkhand, being an industrialized state with a substantial reliance on coal-based industries and power generation, faces both local and global consequences of CO₂ emissions. Here are some of the impacts:

- 1. Air Quality:** CO₂ emissions, along with other pollutants emitted from industrial activities and transportation, contribute to poor air quality in Jharkhand. This pollution has adverse effects on human health, leading to respiratory issues, cardiovascular diseases, and other health problems among the population.
- 2. Climate Change:** CO₂ is a major greenhouse gas responsible for global warming and climate change. Jharkhand, like other regions, experiences the impacts of climate change, including erratic rainfall patterns, rising temperatures, and extreme weather events like floods and droughts. These changes can disrupt agricultural patterns, water availability, and overall ecosystem health, affecting the livelihoods of the people.
- 3. Ecological Impact:** CO₂ emissions can have direct and indirect effects on ecosystems. Increased CO₂ levels can contribute to changes in plant physiology, altering growth patterns and species composition. This can disrupt ecological balance and biodiversity in forests and other natural habitats in Jharkhand, affecting both flora and fauna.
- 4. Water Quality:** CO₂ emissions are linked to the phenomenon of acid rain, where sulfur dioxide and nitrogen oxides combine with atmospheric moisture to form sulfuric and nitric acids. Acid rain can pollute water bodies, soil, and vegetation, impacting aquatic life and overall water quality in rivers, lakes, and streams in Jharkhand.

To address these impacts, public awareness and participation are crucial in mitigating CO₂ emissions and adapting to the changing climate scenario in Jharkhand.

LITERATURE REVIEW

A series of studies have highlighted the significant growth in China's international shipping fuel consumption and greenhouse gas (GHG) emissions. Wu, Zheng, & Feng (2012) calculated and analyzed these trends, while Tian (2014) and To (2018) both emphasized the need for policy interventions to mitigate these emissions. Yang (2017) further underscored the urgency of this issue, predicting a substantial increase in CO₂ emissions from China's international marine transportation activities.

Several studies done on statistical models to analyze and predict carbon dioxide emissions in various states of India. Dubey (2023) and Basak (2014) both used non-linear least square and regression analysis methods to develop state-wise models for Gujarat, Maharashtra, Madhya Pradesh, and some northern and eastern states, respectively. Mondal (2021) extended this work to Himachal Pradesh, Punjab, and Haryana, using a fourth-degree polynomial curve and the Instantaneous Rate of Change (IROC) analysis. Mittal (2014) focused on coal combustion in thermal power plants, estimating emissions for the period of 2001/02 to 2009/10. These studies collectively provide valuable insights into the trends and potential future emissions of carbon dioxide in these regions.

This article is an attempt to develop a third-degree polynomial model utilizing the historical CO₂ emissions data in Jharkhand from 2004-05 to 2015-16 for characterizing the emission pattern of the Indian state.

DATA AND METHODOLOGY

Data collection

The GHG emission data (tonnes of CO₂ equivalent) from petroleum fuel combustion collected from the Annual Survey of Industries (ASI) datasets from 2004-05 to 2015-16, obtained from the Ministry of Statistics and Programme Implementation (MOSPI), are the primary source of information for the GHG estimations.

Method of least square

We formulated a third-degree polynomial model formulated for analysing CO₂ emission of Jharkhand. We followed the work of Jin et al, Mondal et al, and Basak and Nandi to develop the polynomial model. Proposed third degree polynomial model for emission of the gas is

$$y = f(x) = \delta x^3 + \gamma x^2 + \beta x + \alpha \quad [1]$$

where y is emission of carbon dioxide in tonnes of carbon equivalent and x represents time in years.

With the help of the given data $\{(x_1, y_1), (x_2, y_2), \dots \dots \dots (x_n, y_n)\}$ we may define the error associated by

$$\omega(\alpha, \beta, \gamma, \delta) = \sum_{i=1}^n (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3)^2 \quad [2]$$

is a function of five variables α, β, γ and δ . For minimizing the error and estimating corresponding α, β, γ and δ multivariate calculus is used to have

$$\frac{\partial \omega}{\partial \alpha} = 0, \quad \frac{\partial \omega}{\partial \beta} = 0, \quad \frac{\partial \omega}{\partial \gamma} = 0, \quad \frac{\partial \omega}{\partial \delta} = 0.$$

Differentiating (2) partially and equating to zero we get

$$-2 \sum (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3) = 0$$

$$-2 \sum x_i (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3) = 0$$

$$-2 \sum x_i^2 (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3) = 0$$

$$-2 \sum x_i^3 (y_i - \alpha - \beta x_i - \gamma x_i^2 - \delta x_i^3) = 0$$

Now corresponding normal equations are:

$$\sum y_i = n\alpha + \beta \sum x_i + \gamma \sum x_i^2 + \delta \sum x_i^3$$

$$\sum y_i x_i = \alpha \sum x_i + \beta \sum x_i^2 + \gamma \sum x_i^3 + \delta \sum x_i^4$$

$$\sum x_i^2 y_i = \alpha \sum x_i^2 + \beta \sum x_i^3 + \gamma \sum x_i^4 + \delta \sum x_i^5$$

$$\sum x_i^3 y_i = \alpha \sum x_i^3 + \beta \sum x_i^4 + \gamma \sum x_i^5 + \delta \sum x_i^6$$

For given set of points $(x_i, y_i); (i=1, 2, \dots, n)$, the equations can be solved for α, β, γ and δ to find estimated $\alpha^*, \beta^*, \gamma^*$ and δ^* . It has been found that in all the cases, the value of the second order derivatives evolves to be positive at the points α, β, γ and δ . These satisfy the minimization criteria of ω .

Thus, the third degree fitted polynomial of carbon dioxide emission is estimated as

$$y = f(x) = \delta^* x^3 + \gamma^* x^2 + \beta^* x + \alpha^* \quad [3]$$

Instantaneous rate of change of emission

To compute the rate of change of the gas, the derivative of equation (3) is presented as

$$\frac{dy}{dx} = \frac{df(x)}{dx} = \delta^* x^2 + \gamma^* x + \beta^* \quad [4]$$

The equation (4) is utilized to predict the emitted CO₂ at a particular year.

EVALUATION CRITERIA OF MODEL PERFORMANCE

Equation (4) may be used for estimating emission of CO₂ for medium or short terms of time. The goodness of fit is verified with different statistical tools like R², R² (adjusted) and residual analysis.

Coefficient of determination (R²)

The coefficient of determination R² is defined as the proportion of the total response variation that is explained by the model. It provides an overall measure of how well the model fits. The general definition of the coefficient of determination is

$$R^2 = 1 - \frac{SS_{err}}{SS_{tot}}$$

where $SS_{tot} = \sum_i (y_i - \bar{y})^2$,

$$SS_{reg} = \sum_i (f_i - \bar{y})^2, SS_{reg} = \sum_i (y_i - f_i)^2$$

Here, SS_{tot} = Total sum of square (proportional to the sample variance),

SS_{reg} = the regression sum of squares or the explained sum of square and SS_{err} = the sum of squares of residuals, also called the residual sum of square. y_i and f_i are observed and estimated values of CO₂ emission.

Adjusted coefficient of determination

The adjusted R² is defined as:

$$\begin{aligned} \bar{R}^2 &= 1 - (1 - R^2) \frac{n-1}{n-p-1} \\ &= R^2 - (1 - R^2) \frac{p}{n-p-1} \end{aligned}$$

where p is the total number of regressors in the model excluding the constant term and n is the sample size. It is another advanced measure helps us to understand how good the model fit with the observed data.

RESULT AND DISCUSSION

The study represents an intuitive analysis of CO₂ emission combusting of petroleum fuels from different industries in Jharkhand from 2004-05 to 2015-16. Clearly the increasing emission trend observed in Figure 1.

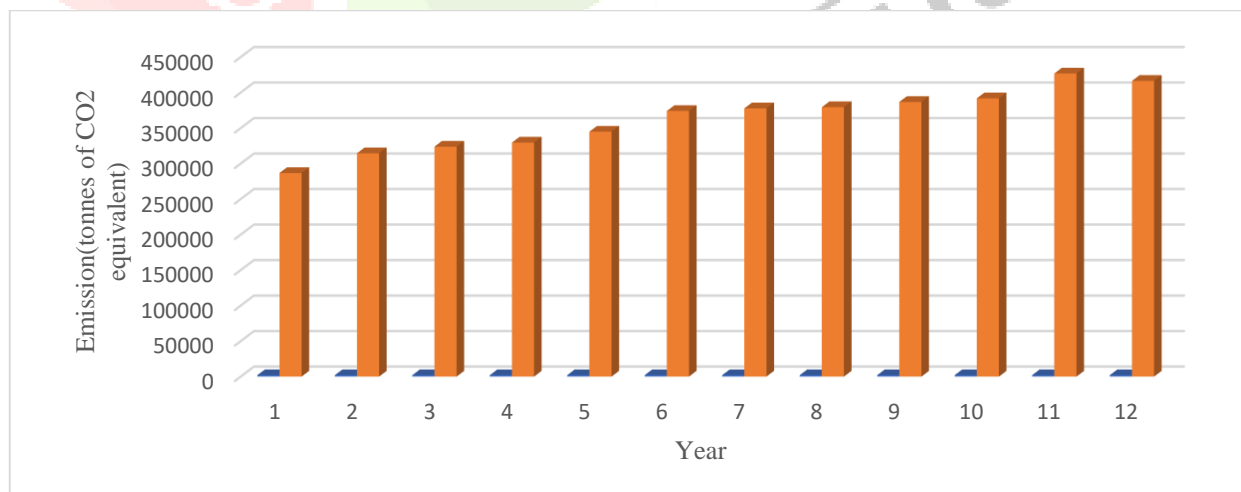


Figure 1. Bar diagram of CO₂ emission in Jharkhand

According to the real emission data, this paper draws a scatter plot to visualize the actual situation of emission pattern in Figure 2.

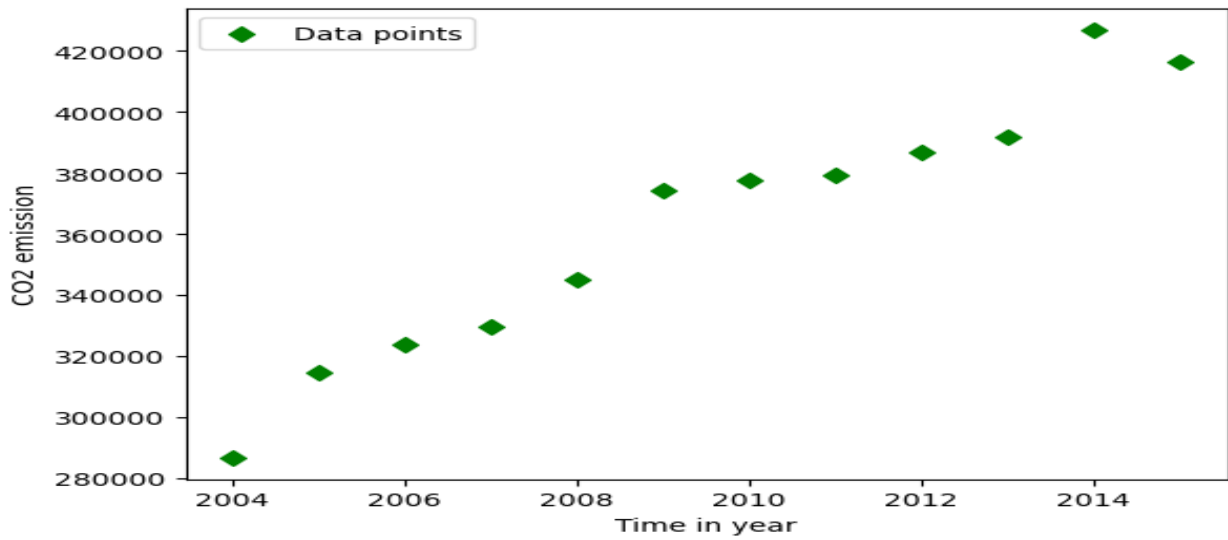


Figure 2. Real CO₂ emission data points in Jharkhand

Using nonlinear least square method the third-degree polynomial model is developed and is represented by the following equation.

$$E(x) = -424868539684.24 + 633475167.36x - 314839.67x^2 + 52.16x^3$$

where x represents time in year and E represents emission.

A graphical display of real emission data and fitted curve is presented in Figure 3. Here points representing actual data and solid line depicted polynomial curve. Which shows increasing emission trend of CO₂ from petroleum fuels in Jharkhand.

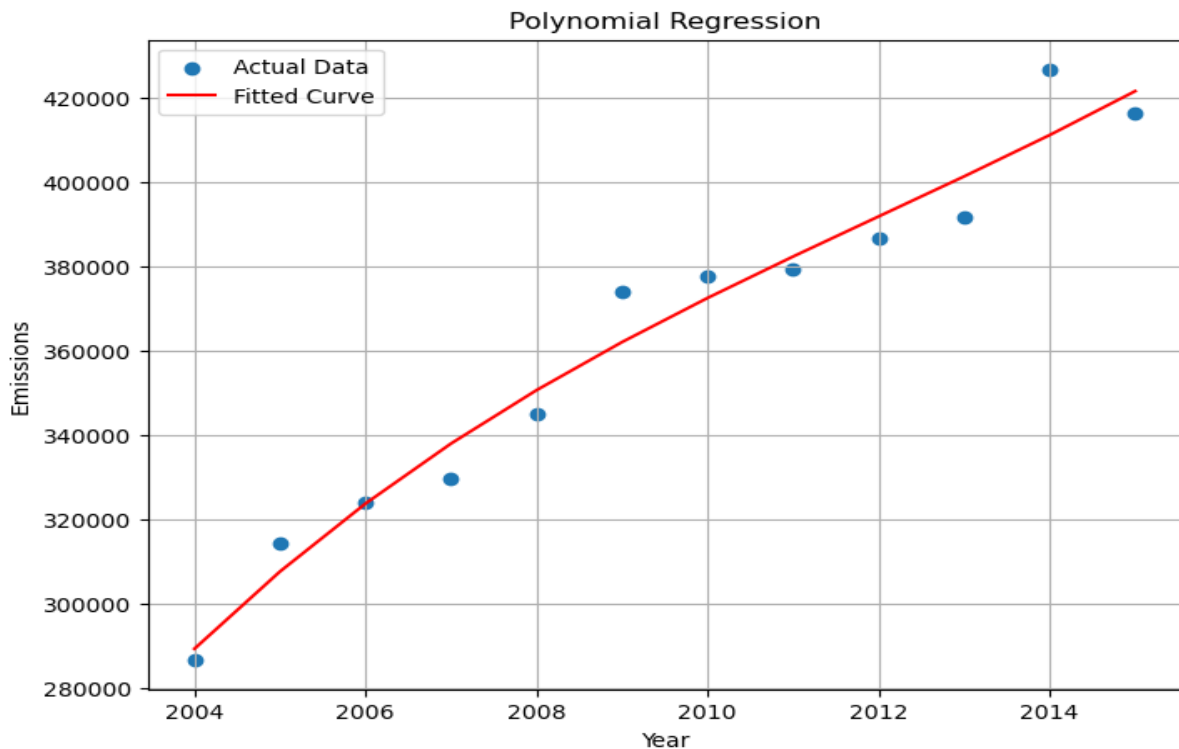


Figure 3. Graphical presentation of polynomial model of CO₂ emission in Jharkhand

From Figure 3 it is observed that the developed dynamic polynomial model of Jharkhand state matches well with the real data of CO₂ emission in Jharkhand. The efficacy of the model is measured by R squared and adjusted R squared.

| Measures | R ² | Adjusted R ² |
|----------|----------------|-------------------------|
| Values | 0.9637 | 0.9501 |

The value of R squared is 0.9637 representing our model capturing 96 % (Approximately) variability of emission data. Which suggest that we have chosen an appropriate model to characterize the emission pattern of the Indian state indicating various insights for environmental research and policy formulation.

Following table certify the goodness of fit of the proposed model and its efficacy. Low percentage error indicating acceptance of fitted values of the emission data.

Table of Original Values, Fitted Values, Difference, and Percentage Error:

| <i>Original Values</i> | <i>Fitted Values</i> | <i>Difference</i> | <i>Percentage Error (%)</i> |
|------------------------|----------------------|-------------------|-----------------------------|
| 286669.7537 | 289243.530823 | -2573.777123 | -0.897820 |
| 314410.3306 | 307582.723206 | 6827.607394 | 2.171559 |
| 323807.2685 | 323722.969788 | 84.298712 | 0.026034 |
| 329493.2951 | 337977.227844 | -8483.932744 | -2.574842 |
| 344912.3590 | 350658.455383 | -5746.096383 | -1.665958 |
| 374105.0275 | 362079.610535 | 12025.416965 | 3.214449 |
| 377785.9413 | 372553.650574 | 5232.290726 | 1.384988 |
| 379440.7865 | 382393.533752 | -2952.747252 | -0.778184 |
| 386784.4278 | 391912.217346 | -5127.789546 | -1.325749 |
| 391770.7143 | 401422.659729 | -9651.945429 | -2.463672 |
| 426864.3169 | 411237.818176 | 15626.498724 | 3.660765 |
| 416410.8254 | 421670.650940 | -5259.825540 | -1.263134 |

Instantaneous variation of CO₂ emissions in Jharkhand

To determine the rate of change of emission in a particular year we need to analyze instantaneous rate of change (IROC). The equation of IROC is given by

$$\frac{dy}{dx} = 156.48 x^2 - 629679.34 x + 633475167.36$$

Where y denotes emission and x denotes time in year. The graphical display is illustrated in Figure 4.

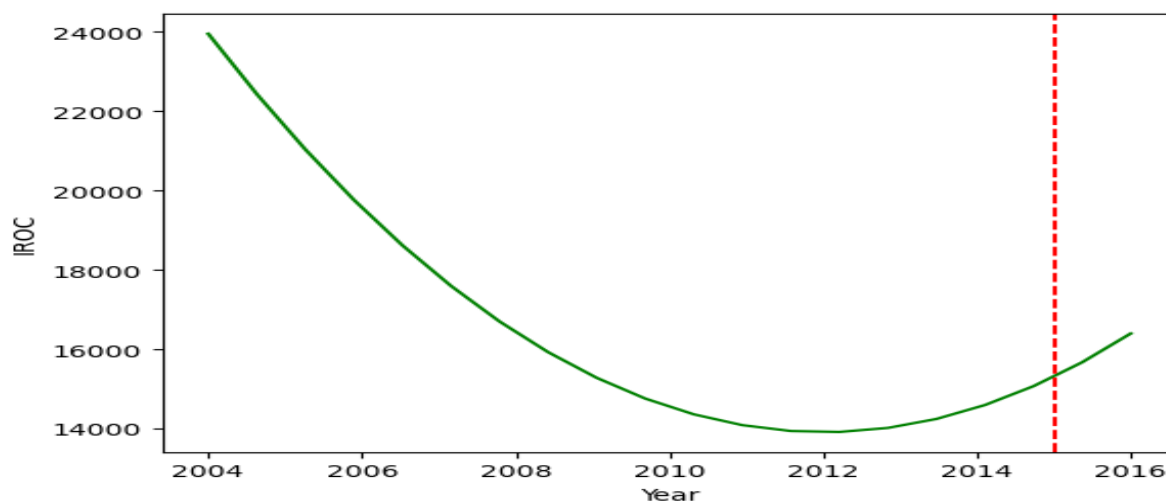


Figure 4. Instantaneous variation of CO₂ emission in Jharkhand

From the figure it is observed that IROC curve is upwards and indicates that CO₂ emission is growing rapidly from 2012 onwards. Using this future prediction of the gas emission may be obtained at a particular year.

CONCLUSION

Based on the historical data (2004-05 to 2015-16) of GHG emission (tonnes of CO₂ equivalent) from petroleum fuel combustion this article is an attempt to develop a third-degree polynomial model to describe the CO₂ emission dynamics in Jharkhand. Numerical values of R-squared and adjusted R-squared values are 0.9637 and 0.9501 which attest the effectiveness of the model. This model may be utilized to study the emission dynamics of other GHGs. The result of the study indicates the rapid growing emission in the state and will create an alarming situation in near future. The article may be treated as a blue print of environmental research in future. To reduce the CO₂ emission immediate actions on emission strategies and policies are required for restraining the situation in Jharkhand.

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