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SOLUTION TO FUZZY FREDHOLM, VOLTERRA DIFFERNETIAL AND INTEGRAL EQUATIONS BY THE METHOD OF SUCCESSIVE APPROXIMATION

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Abstract

In the mathematical modeling of real world phenomena, we encounter two inconveniences. The first is caused by the excessive complexity of the model. As the complexity of the system being modeled increases, our ability to make precise and yet relevant beyond which precision and significance become almost mutually exclusive characteristics. A fuzzy set is a function from a set into a lattice or as a special case, into the interval. Using it, one can model the meaning of vague notions and also certain kinds of human reasoning. Fuzzy logic is currently genuinely settled in different fields and state space control is a typical method for ideal procedure control. The vast majority of the pages here have recently presented these extremely same techniques. The mixing of state space with Fuzzy logic for nonlinear systems towards ideal control is addressed in the rest of the pages with creator's very own novel, basic figuring techniques.

Keywords: fuzzy, set, Neumann, Series, volterra, integral, equation.

1. INTRODUCTION

In the mathematical modeling of real world phenomena, we encounter two inconveniences. The first is caused by the excessive complexity of the model. As the complexity of the system being modeled increases, our ability to make precise and yet relevant beyond which precision and significance become almost mutually exclusive characteristics. As a result, we are either not able to formulate the mathematical model or the model is too complicated to be useful in practice. The second inconvenience relates to the indeterminacy caused by our inability to differentiate events in a real situation exactly and therefore to define instrumental notions in precise form. This indeterminacy is not an obstacle, when we use natural language, because its main property is the vagueness of its semantics and therefore capable of working with vague notions. Classical mathematics, on the other hand, cannot cope with such vague notions. It is therefore necessary to have some mathematical apparatus to describe vague and uncertain notions and thereby help to overcome the foregoing obstacles in the mathematical modelling of imprecise real world systems.

The rise and development of new fields such as general system theory, robotics, artificial intelligence and language theory, force us to be engaged in specifying imprecise notions. The basic idea of fuzzy set theory is simple and natural. A fuzzy set is a function from a set into a lattice or as a special case, into the interval. Using it, one can model the meaning of vague notions and also certain kinds of human reasoning. Fuzzy set theory and its applications have been extensively developed since the 1970s and industrial interest in fuzzy control has dramatically increased since 1990. There are several books dealing with these aspects. An exact description of any real world phenomenon is virtually impossible and one needs to accept this fact and adjust to it. The inexactness of the description is not a liability but is a blessing because it makes for greater efficiency. A fuzzy set is a membership function which describes the gradual transition from membership to non membership and is a subjective one. Spaces of such fuzzy sets are function spaces with special properties.

2. SOLUTION TO FUZZY DIFFERNETIAL AND INTEGRAL EQUATIONS

The conditions by which there exist a novel solution for the fuzzy integral condition and the conditions by which a solitary fuzzy integral condition is limited have been appeared in the past part. Different strategies by which a solution to the fuzzy integral condition is gotten are being viewed as now. A useful strategy for acquiring better approximations to the fixed point is given by the Banach fixed point hypothesis. This system is emphasis. A discretionary u_0 in a given set is picked and a succession u_0, u_1………is determined recursively from a connection of the structure $u_{n+1} = T\,u_n, n = 0,1,2,\ldots$..This implies a self-assertive u_0 is picked and $u_1 = T\,u_0 u_2 = Tu_1$is resolved progressively and assembly are acquired by applying Banach fixed point hypothesis for fuzzy sets.

2.1 Neumann Series and Approximation

Here the fuzzy Fredholm integral condition of the subsequent kind is taken and attempts to tackle it. An iterative plan dependent on the technique for progressive approximations is utilized for this. The methodology of Carl Neumann is followed right now. Consider first the condition

$$u(t) = v(t) + \int_a^b K(t,s)u(s)ds \quad (1)$$

Where K(t,s) is genuine and ceaseless and thus limited. The given capacity $v\,(t) \epsilon\, E^n$ is additionally consistent.

On the off chance $v\,(t) \epsilon\, E^n$ that is a ceaseless capacity, at that point the integral $\int_a^b K\,(t,s)u\,(s)ds$ is well characterized and is a nonstop capacity of t, which we mean by $Ku(s)$or by $K\,u$. Since $u\,\epsilon\, E^n$ we utilize the parametric portrayal of u

That is $u(t) = (a(r,t), b(r,t)$and since $v\,\epsilon\, E^n$ we can compose $v(t) = \big(c(r,t), d(r,t)\big)$, where $0 < r \leq 1$ At that point condition (5.1) can be written in its parametric structure

$$(c(r,t), d(r,t)) + \int_a^b K(t,s)[a(r,s), b(r,s)]ds. \quad (2)$$

At that point we can compose our condition in the structure

$$[a(r,t), b(r,t)] = [c(r,t), d(r,t)] + \int_a^b K(t,s)[a(r,s), b(r,s)]ds$$

Applying the strategy for progressive approximations for unraveling this, we take the capacity $u_{0(t)} = [a_0\,(r,t), b_0$(r,t)]=[0,0] for the main estimate. The progressively better approximations are

$$[a_1(r,t), b_1(r,t)] = [c(r,t), d(r,t)]\,,$$

$$[a_2(r,t), b_2(r,t)] = [c(r,t), d(r,t)] + K\,[c(r,t), d(r,t)]$$

$$[a_3(r,t), b_3(r,t)] = [c(r,t), d(r,t)] + K\,[c(r,t), d(r,t)] + K^2\,[c(r,t), d(r,t)]$$

3. SOLUTION OF FUZZY VOLTERRA INTEGRAL EQUATION BY THE METHOD OF SUCCESSIVE SUBSTITUTION

Consider the fuzzy Volterra integral equation of second kind

$$u(t) = f(t) + \lambda \int_a^t K(t,s)u(s)ds. \quad (3)$$

f(t) and $u(t) \epsilon E^n$,where

- ✓ The piece K(t,s) is genuine and consistent in the square shape $R, a \le t \le b, a \le s \le b$.

Consider $|K(t,s)| \le p$, is the most extreme incentive in R.

- ✓ The capacity f(t) is genuine and consistent in an interim $a \le t \le b$. consider $|f(t)| \le Q$, where Q is the most extreme incentive in the interim
- ✓ λ is a non-zero numerical parameter.

Subbing the obscure capacity u(s) under an integral sign structure the equation (2) itself, we have

$$u(t) = f(t) + \lambda \int_a^t K(t,s) \left[f(s) + \lambda \int_a^s K(s,s_1)u(s_1)ds_1 \right] ds$$

Where

$$f(t) = [c(r,t), d(r,t)]$$

$$u(s_1) = [a(r,s_1), b(r,s_1)]$$

Again

$$u(t) = f(t) + \lambda \int_a^t K(t,s)f(s)ds + \lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1)u(s_1)ds, ds$$

Performing the operations successively for $u(s_1)$, we have

$$u(t) = f(t) + \lambda \int_a^t K(t,s)f(s)ds + \lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1)$$

4. SOLUTION OF THE FUZZY FREDHOLM INTEGRAL EQUATION BY THE METHOD OF SUCCESSIVE SUBSTITUTIONS

Consider the fuzzy Fredholm integral equation of second kind below,

$$u(t) = f(t) + \lambda \int_a^b K(t,s)u(s)ds \quad (4)$$

Where

- The kernel K(t,s) is real and continuous in the rectangle $R, a \le t \le b, a \le s \le b$.

Consider $K(t,s) \leq p$,where p is the maximum value in R.

- The function f(t) is real and continuous in an interval $I, a \leq t \leq b$.Consider $|f(t) \leq Q$, where Q is the maximum value in the interval.
- λ is a non-zero numerical parameter.

Solution: As there exists a continuous solution u(t) substituting the unknown function under an integral sign from the equation (4) itself, we obtain

$$[a(r,t),b(r,t)] = [c(r,t),d(r,t)] + \lambda\int_a^b K(t,s)\Big[[c(r,s),d(r,s)]$$

$$+ \lambda\int_a^b K(s,s_1)[a(r,s_1),b(r,s_1)]ds_1\Big]ds$$

i.e.,

$$[a(r,t),b(r,t)] = [c(r,t),d(r,t)] + \lambda\int_a^b K(t,s)[c(r,s),d(r,s)]ds$$

$$+ \lambda^2\int_a^b K(t,s)\int_a^b K(s,s_1)[a(r,s_1),b(r,s_1)]ds_1 ds$$

5. SOLUTION OF FUZZY VOLTERRA INTEGRAL EQUATION OF SECOND KIND BY THE METHOD OF SUCCESSIVE APPROXIMATION

A Volterra fuzzy Integral equation of second $u(t) = f(t) + \lambda\int_a^t K(t,s)u(s)$has one and only one solution, given by the relation $u(t) = f(t) + \lambda\int_0^t R(t,s,\lambda)f(s)ds$where the resolvent kernel $R(t,s,\lambda)$ is the sum of the series $(R(t,s,\lambda) = K(t,s) + \sum_{j=1}^{\infty}\lambda^j K_j(t,s)$,convergent for all values of λ.

Consider the Volterra Integral equation of second kind

$$u(t) = f(t) + \lambda\int_0^t K(t,s)u(s)ds \quad (5)$$

Where the kernel K(t,s) is a continuous function for $0 \leq t \leq a, 0 \leq s \leq a$ and the function f(t) is continuous for $0 \leq t \leq a$.

Let $f(t)\epsilon E^n$and hence $u(t)\epsilon E^n$

Let $f(t) = [c(r,t),d(r,t)]$ and $u(t) = [a(r,t),b(r,t)]$ at r-level.

Consider an infinite power series in ascending powers of λ as

$$[a(r,t),b(r,t)] = [a_0(r,t),b_0(r,t)] + \lambda[a_1(r,t),b_1(r,t)] \quad (6)$$

$$+ \lambda^2[a_2(r,t),b_2(r,t)] + \cdots + \lambda^2[a_n(r,t),b_n(r,t)] + \cdots$$

Let the series (6) is a solution of the integral equation (5), then

$$[a_0(r,t),b_0(r,t)]+\lambda[a_1(r,t),b_1(r,t)]+\lambda^2[a_2(r,t),b_2(r,t)]+\cdots$$

$$+\lambda^n[a_n(r,t),b_n(r,t)]+\cdots$$

$$=[c(r,t),d(r,t)]+\lambda\int_0^t K(t,s)\Big[[a_0(r,s),b_0(r,s)]\Big] \quad (7)$$

$$+\lambda[a_1(r,s),b_1(r,s)]$$

$$+\lambda^2[a_2(r,s),b_2(r,s)]+\cdots+\lambda^n[a_n(r,s),b_n(r,s)]ds$$

Equating the coefficients of like power ofλ, we get

$$[a_0(r,t),b_0(r,t)]=[c(r,t),d(r,t)]$$

$$[a_1(r,t),b_1(r,t)]=\int_0^t K(t,s)[a_0(r,s),b_0(r,s)]ds$$

$$[a_2(r,t),b_2(r,t)]=\int_0^t K(t,s)[a_1(r,s),b_1(r,s)]ds$$

$$[a_n(r,t),b_n(r,t)]=\int_0^t K(t,s)\Big[a_{n-1}(r,s),b_{n-1}(r,s)\Big]ds$$

Thus get an approximation of the function$u_n(t)$. It may be shown that the series (7) joins consistently in t and t for any λ and $t\epsilon[0,a)$,under these suspicions concerning f(t) and K(t, s), its aggregate is a novel solution of the equation (5).

6. CONCLUSION

Fuzzy logic is currently genuinely settled in different fields and state space control is a typical method for ideal procedure control. The vast majority of the pages here have recently presented these extremely same techniques. The mixing of state space with Fuzzy logic for nonlinear systems towards ideal control is addressed in the rest of the pages with creator's very own novel, basic figuring techniques. Control execution enhancement, creation improvement, vitality streamlining and such related desires especially need in enormous procedure control plants. Because of the gigantic idea of procedures and creation, this angle is felt especially in enormous plant computerization. At the point when procedure ideal evaluations are never again linearly identified with the factors as the early day quadratic execution list portrayed in books and writing, how might one handle the ideal control issue for exceptionally nonlinear systems has involved concern.

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