IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Evaluation of Eigenvalues of Linear Electrical Circuits Based On Characteristic Equation

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Abstract

In this paper ,the eigenvalues in the time-domain, we can say ,indirectly the poles of system functions in the s-domain, are used to determine the dynamics of the systems in circuit analysis of linear circuit. This paper deals with an efficient method to determine the eigenvalues of linear electrical circuits. It is based on characteristic equation obtained by applying nodal and mesh analysis. Its applications are simpler than the state-space representation. This paper gives the dominant method for evaluation of Eigenvalues.

Keywords: eigenvalues; linear electrical circuits; mesh analysis; nodal analysis.

Characteristic Equations of the Electrical Circuits

1.1State Space Method:

Consider the linear continuous-time electrical circuit described by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ are the state and input vectors and $A \in \Re^{nxn}$, $B \in \Re^{nxm}$. It is well-known [1, 2, 16-20] that any standard linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources can be described by the equation (1). Usually as the state variables $x_1(t)$, …, $x_n(t)$ (the components of the vector x(t)) the currents in the coils and voltages on the capacitors are chosen.

Here the electrical Circuit equation (1) is called (internally) positive if

$x(t) \in \Re^n_+$, for any initial condition $x(0) \in \Re^n_+$ and every $u(t) \in \Re^m_+$, $t \in [0, +\infty)$.

The electrical circuit Equation (1) is positive if and only if

$$A \in M_n,\ B \in \Re^{n \times m}_+ . \tag{2}$$

The positive electrical circuit Equation(1) for u(t) = 0 is called asymptotically stable if

$$\lim_{t \to \infty} x(t) = 0 \text{ for all } x(0) \in \Re^n_+ \tag{3}$$

The positive electrical circuit equation(1) is asymptotically stable if and only if

$$\operatorname{Re}\lambda_k < 0 \text{ for } k = 1,\dots,n,$$

where λ_k is the eigenvalue of the matrix $A \in M_n$ and

$$\det[I_n\lambda - A] = (\lambda - \lambda_1)(\lambda - \lambda_2)\dots(\lambda - \lambda_n) \quad (4)$$

1.2 Mesh Method

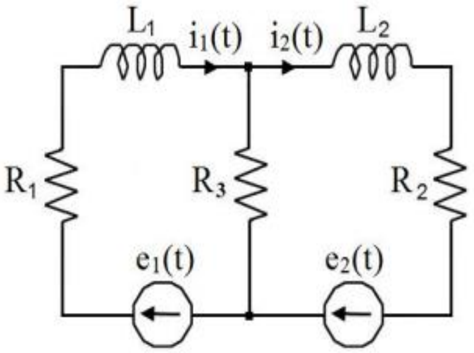
Any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources in transient states can be also analyzed by the use of the mesh method [2, 16]. Using the mesh method and the Laplace transform we can describe the electrical circuit in transient states by the equation

$$Z(s)X(s) = E(s) \quad 5(a)$$

where $X(s) = \mathcal{L}[x(t)] = \int_0^\infty x(t)e^{-st}dt$ ($\mathcal{L}$ is the Laplace operator),

$$Z(s) = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) & \cdots & Z_{1n}(s) \\ Z_{21}(s) & Z_{22}(s) & \cdots & Z_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1}(s) & Z_{n2}(s) & \cdots & Z_{nn}(s) \end{bmatrix}, \quad E(s) = \begin{bmatrix} E_1(s) \\ E_2(s) \\ \vdots \\ E_n(s) \end{bmatrix}. \quad 5(b)$$

For example for the electrical circuit with given resistances R1 , R2 , R3 , inductances L1 , L2 and voltage sources e_1, e_2 shown in Fig(1) using the mesh method we obtain the following



Fig(1) Electrical Circuit

Using the Kirchhoff's laws for the electrical circuit we obtain the equations

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + R_3(i_1 - i_2)$$

$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + R_3(i_2 - i_1) \quad (6)$$

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_1 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B_1 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad 7(a)$$

Where

$$A_1 = \begin{bmatrix} -\frac{R_1+R_3}{L_1} & \frac{R_3}{L_1} \\ \frac{R_3}{L_2} & -\frac{R_2+R_3}{L_2} \end{bmatrix}, \; B_1 = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}. \quad 7(b)$$

The electrical circuit is positive since A1 $\in M_2$ and $B_1 \in \Re^{2x2}$. The characteristic equation of the electrical circuit has the form

$$\det[I_2 s - A_1] = \begin{vmatrix} s+\frac{R_1+R_3}{L_1} & -\frac{R_3}{L_1} \\ -\frac{R_3}{L_2} & s+\frac{R_2+R_3}{L_2} \end{vmatrix}$$

$$= s^2 + \left(\frac{R_1+R_3}{L_1} + \frac{R_2+R_3}{L_2}\right)s + \frac{R_1(R_2+R_3)+R_2R_3}{L_1L_2} = 0 \quad (8)$$

Using the mesh method and the Laplace transform we obtain

$$\begin{bmatrix} R_1+R_3+sL_1 & -R_3 \\ -R_3 & R_2+R_3+sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \quad 9(a)$$

where $I_k(s) = \mathcal{L}[i_k(t)]$, $E_k(s) = \mathcal{L}[e_k(t)]$, $k = 1,2$

In this case we have

$$Z(s) = \begin{bmatrix} R_1+R_3+sL_1 & -R_3 \\ -R_3 & R_2+R_3+sL_2 \end{bmatrix}, \; X(s) = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}, \; E(s) = \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \quad 9(b)$$

Note that

$$\det Z(s) = \begin{vmatrix} R_1+R_3+sL_1 & -R_3 \\ -R_3 & R_2+R_3+sL_2 \end{vmatrix}$$

$$= L_1L_2s^2 + [(R_1+R_3)L_2 + (R_2+R_3)L_1]s + R_1(R_2+R_3) + R_2R_3 \quad (10)$$

and after multiplication by 1 /L_1 L_2 we obtain

$$\det Z(s) = L_1L_2 \det[I_2 s - A_1] \quad (11)$$

From (11) we have the following conclusion

It means we can say that in state space method the characteristic equation (8) of the electrical circuit can be also obtained by computation of the determinant of the matrix Z(s) in the mesh method.

1.3 Node method:

Any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources in transient states can be also analyzed by the use of the node method. Using the node method and the Laplace transform we can describe the electrical circuit in transient states by the equation [2, 16].

$$Y(s)V(s) = I_z(s) \quad 12(a)$$

Where

$$Y(s)=\begin{bmatrix} Y_{11}(s) & Y_{12}(s) & \cdots & Y_{1q}(s) \\ Y_{21}(s) & Y_{22}(s) & \cdots & Y_{2q}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{q1}(s) & Y_{q2}(s) & \cdots & Y_{qq}(s) \end{bmatrix},\ V(s)=\begin{bmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_q(s) \end{bmatrix},\ I_z(s)=\begin{bmatrix} I_{z1}(s) \\ I_{z2}(s) \\ \vdots \\ I_{zq}(s) \end{bmatrix}, \quad 12(b)$$

q is the number of linearly independent nodes, Y_{ij} (s) and V_i(s) , i, j =1,...,q are Laplace transforms of conductances and current sources of the electrical circuit, respectively

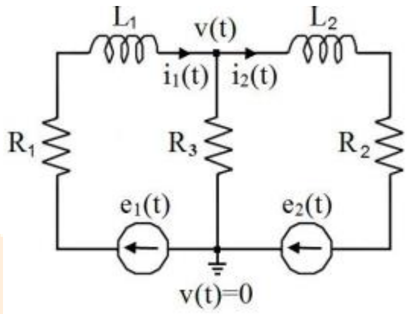


Fig.(2) Electrical Circuit

For example for the electrical circuit shown in Fig.(2) using the node method we obtain

$$Y(s)V(s)=I_z(s) \quad 13(a)$$

$$V(s)=\mathcal{L}[v(t)],\ E_k(s)=\mathcal{L}[e_k(t)],\ k=1,2$$

and

$$Y(s)=Y_{11}(s)=\frac{1}{R_1+sL_1}+\frac{1}{R_2+sL_2}+\frac{1}{R_3},$$

$$I_z(s)=-\frac{E_1(s)}{R_1+sL_1}+\frac{E_2(s)}{R_2+sL_2}. \quad 13(b)$$

Note that

$$\det Y(s)=Y(s)=\frac{1}{R_1+sL_1}+\frac{1}{R_2+sL_2}+\frac{1}{R_3}$$
$$=\frac{L_1L_2s^2+[(R_1+R_3)L_2+(R_2+R_3)L_1]s+R_1(R_2+R_3)+R_2R_3}{(R_1+sL_1)(R_2+sL_2)R_3} \quad (14)$$

And after multiplication by

$$\frac{(R_1+sL_1)(R_2+sL_2)R_3}{L_1L_2}$$

we obtain

$$\det Y(s)=\frac{L_1L_2}{(R_1+sL_1)(R_2+sL_2)R_3}\det[I_2s-A_1]. \quad (15)$$

From equation(15) We have the following conclusion.

It means we can say that in Mesh Method the characteristic equation (8) of the electrical circuit can be also obtained by computation of the determinant of the matrix Y (s) in the node method.

System equations:

In general, in the time-domain, the eigenvalues are determined from the system matrix in the state-space representation of circuit equations.5–8 Although the state space method, based on the graph theoretical approach, has minimum variables, It involves an intensive mathematical process and has major limitations in the formulation of circuit equations. Some of these limitations arise because the state variables are capacitor voltages and inductor currents. Not every circuit element can be easily included into the state equations. It has a structure of differential equations. Especially, there are some restrictions in the analysis of active circuits. Therefore, students of electrical engineering generally have these difficulties in obtaining the state-space representation of system equations. It is not always suitable to use this method for obtaining both eigenvalues and transfer functions. In this study, it is shown that the eigenvalues can be easily determined according to nodal and mesh equations in the s-domain, through more efficient and understandable analysis methods in circuit analysis courses. It has a structure of algebraical equations. There are no restrictions in the formulation of circuit equations. In this paper the nodal and mesh analysis methods with virtual sources for some special cases in circuit analysis are used.

The system equations in the algebraical structure, in the Laplace domain, obtained by using nodal or mesh analysis method, relating to a linear circuit are

$$\mathbf{AX(s)= BU(s)} \qquad \textbf{(1)}$$

Where, A, B are coefficient matrices, U(s) is the source vector, X(s) is the unknown vector. The frequency-dependent elements (inductor, capacitor) can be entered in the form having s(sL, sC) or 1/s(1/sL, 1/sC) into the system equations. Matrix A is also called the characteristic matrix. By taking the inverse of matrix A, solutions of the system equations are given as in equation(2), as follows

$$\mathbf{X}(s) = \mathbf{A}^{-1}\mathbf{B}\mathbf{U}(s) = \left(\frac{1}{\det(\mathbf{A})}\mathrm{Adj}(\mathbf{A})\right)\mathbf{B}\mathbf{U}(s) \qquad \textbf{(2)}$$

where det(A) denotes the determinant of matrix A, and adj are notes the adjoint matrix. It is obvious that solutions of equation(2) are fractional. The determinant of the characteristic matrix (A) has also fractional and polynomial form as in equation(3). Q(s) represents the numerator of the determinant and R(s) represents the denominator of the determinant.

$$\det(\mathbf{A}) = \frac{Q(s)}{R(s)} \qquad \textbf{(3)}$$

The determinant expression in equation (3) is substituted into equation (2)

$$\mathbf{X}(s) = \left(\frac{1}{\frac{Q(s)}{R(s)}}\mathrm{Adj}(\mathbf{A})\right)\mathbf{BU}(s)$$

$$\mathbf{X}(s) = \frac{\mathrm{Adj}(\mathbf{A}) * \mathbf{B} * R(s)}{Q(s)}\mathbf{U}(s) \qquad \textbf{(4)}$$

According to Equation(4), all variables relating to any circuit have the same denominator, Q(s). It is the numerator of determinant of the coefficient matrix (A) in equation (3).

Therefore, Q(s) polynomial is also called the characteristic equation. The eigenvalues, indirectly poles, of any circuit are obtained from the roots of the characteristic polynomial, Q(s) = 0. In the proposed approach, the system equations in the form of equation(1) are obtained algebraically by nodal or mesh analysis in the Laplace (s) domain. The characteristic equation, Q(s), is determined in terms of the numerator of determinant of the coefficient matrix (A) relating to system equations. Later, the eigenvalues of the circuit are easily obtained from the roots of the characteristic equation.

Applications:

Example 1. Consider the circuit shown in Fig.(1). Element values are R = 2 Ω, C1 = C2 = 3 F, L = 5 H. Mesh equations in the s-domain are as follows. The variables of the method are I_1, I_2 mesh currents.

$$1 \rightarrow R(I_1) + \frac{1}{sC_1}(I_1 - I_2) = U(s)$$

$$2 \rightarrow \frac{1}{sC_1}(I_2 - I_1) + sL(I_2) + \frac{1}{sC_2}(I_2) = 0$$

Let's rearrange the system equations in matrix form.

$$\mathbf{AX}(s) = \mathbf{BU}(s)$$

$$\begin{bmatrix} R + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & sL + \frac{1}{sC_1} + \frac{1}{sC_2} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{U}(s)$$

(5)

After substituting the element values into the system equations, the determinant of the coefficient matrix (A) is obtained as follows.

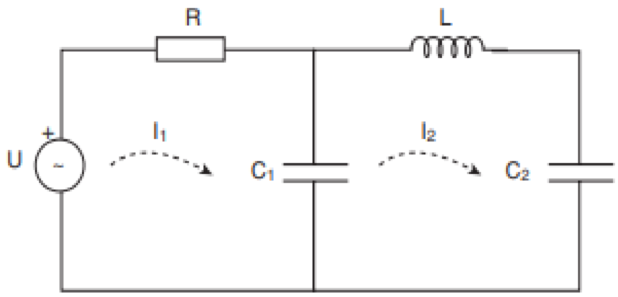


Fig.(1) Circuit for example 1

$$\det(\mathbf{A}) = \frac{Q(s)}{R(s)} = \frac{90s^3 + 15s^2 + 12s + 1}{9s^2}$$

(6)

The characteristic equation: $Q(s) = 90s^3 + 15s^2 + 12s + 1$

After solving the system equations in (5), the circuit variables, mesh currents, are obtained as follows.

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{45s^3 + 6s}{90s^3 + 15s^2 + 12s + 1} \\ \frac{3s}{90s^3 + 15s^2 + 12s + 1} \end{bmatrix} \mathrm{U(s)}$$

(7)

It can be easily seen that all variables relating to the circuit have the same denominator, the characteristic equation (Q(s)). The roots of characteristic equation (Q(s)) give the eigenvalues of the third order circuit: α1 ≅ −0.0394 + 0.3534i, α2 ≅ −0.0394 − 0.3534i, α3 = −0.0879. The circuit is stable because all eigenvalues are located on the left-hand side of the complex
s-plane.

Conclusion:

The difficulty of determining the eigenvalues in circuit analysis courses depends on obtaining the system equations. In general, the eigenvalues are determined from a state variables method having a structure of differential equations and some restrictions in obtaining the system equations. In this paper, it is shown how the eigenvalues of linear circuits can be determined according to the characteristic equation created by nodal and mesh analysis, in algebraical form. In terms of complexity, the proposed method is simpler and more understandable than the commonly used form of state equations. The method is general and can be easily applied to all possible active and passive circuits. It has no restrictions. The main advantages of the method are efficient, systematic, and understandable in terms of advances in student learning. Students can easily determine the eigenvalues and, moreover, can write a computer program about computation of the eigenvalues by employing the presented method and also the problem of calculation of the characteristic equations of the standard positive and linear electrical circuits has been addressed.

Acknowledgement:

First and foremost, I would like to thank God, the almighty for his blessings I would like to thank my wife Sarita Srivastava for standing besides me throughout my career while writing this paper. Also I would like to thank my sons Er.Satyam Srivastava and Dr.Kumar Yash Srivastava and my daughter Dr.Shriya Srivastava.

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